Supplementary Material: Perceptually Based Downscaling of Images

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1 Solving the Problem in Equation 9

For simplicity of the equations, we make the following definitions $\mathbf{e} := \mathbf{M}^{1/2}\mathbf{d}$, $\mathbf{b} := \mathbf{M}^{-1/2}\mathbf{m}$, $c^2 := \alpha^2\mu_h^2 + \gamma^2\sigma_h^2$, $\mathbf{f} := \mathbf{M}^{-1/2}\mathbf{a}$. Then, the problem in Equation 5 of the paper can be rewritten as

$$\begin{aligned} \max_{\mathbf{e}} \quad & \mathbf{f}^T\mathbf{e} \\ \mathbf{b}^T\mathbf{e} = \alpha\mu_h, \quad & ||\mathbf{e}||^2 = c^2. \end{aligned} \tag{1}$$

We solve this problem with the method of Lagrange multipliers. Hence, we optimize the following function

$$F(\mathbf{e}, \lambda_1, \lambda_2) = \mathbf{f}^T\mathbf{e} - \lambda_1(\mathbf{b}^T\mathbf{e} - \alpha\mu_h) - \lambda_2(||\mathbf{e}||^2 - c^2). \tag{2}$$

Taking the derivatives with respect to $\mathbf{e}$, λ_1, and λ_2 gives us

$$\mathbf{e} = \frac{-\mathbf{f} - \lambda_1\mathbf{b}}{2\lambda_2} \tag{3}$$

$$-(\mu_h + \lambda_1) = 2\alpha\mu_h\lambda_2 \tag{4}$$

$$\mathbf{a}^T\mathbf{l} + 2\lambda_1\mu_h + \lambda_1^2 = 4c^2\lambda_2^2. \tag{5}$$

Combining the last two equations, we can solve for λ_1 and λ_2 as

$$\lambda_1 = \frac{-\mu_h \pm \alpha\mu_h\sqrt{\mathbf{a}^T\mathbf{l} - \mu_h^2}}{\gamma\sigma_h} \tag{6}$$

$$\lambda_2 = \mp\frac{1}{2}\frac{\sqrt{\mathbf{a}^T\mathbf{l} - \mu_h^2}}{\gamma\sigma_h}. \tag{7}$$

Substituting these into the expression for $\mathbf{e}$ gives us

$$\mathbf{e} = \frac{-\mathbf{f} - (-\mu_h \pm \frac{\alpha\mu_h\sigma_l}{\gamma\sigma_h})\mathbf{b}}{\frac{\mp\sigma_l}{\gamma\sigma_h}}. \tag{8}$$

Hence, we get the solution

$$\mathbf{d} = \alpha\mu_h\mathbf{1} \pm \frac{\gamma\sigma_h}{\sigma_l}(\mathbf{l} - \mu_h\mathbf{1}), \tag{9}$$

where $\mathbf{1}$ denotes the vector of ones. In order to decide on the sign, we recall that we would like to maximize the covariance and hence $\mathbf{a}^T\mathbf{d}$. Substituting the expression for $\mathbf{d}$, we can see that this dot product is maximized for the positive sign.

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