Practical Probability
Applying pGCL to Lattice Scheduling

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Overview

In this talk, I present a **verified lattice scheduler**, that eliminates leakage via a shared cache, while guaranteeing non-starvation. In addition, this work:

- Applies our existing pGCL package for Isaelle.
- Presents a multilevel probabilistic refinement proof.
- Integrates with the seL4 proof.

Outline

- Lattice Scheduling
- The Probabilistic Scheduler
  - Refinement
- Lottery Scheduling
  - Data Refinement
- seL4 Integration
- Non-Leakage
- Summary

- Consider a system with two classification tags: A and B. Information tagged with A may only be seen by an agent cleared to see A, likewise for B.

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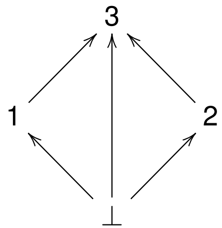
- Any output from an agent clear for A is tagged A, likewise for B.
- There are four possible clearances: A, B, A and B, and nothing. These are **domains**.
- The who-may-talk-to-whom order is a lattice:

$$
\begin{tikzcd}
 & \{A, B\} & \\
\{A\} \arrow[ur] & & \{B\} \arrow[ul] \\
 & \{\,\} \arrow[ul] \arrow[ur] \arrow[uu] &
\end{tikzcd}
$$

$$
\begin{tikzcd}
 & 3 & \\
1 \arrow[ur] & & 2 \arrow[ul] \\
 & \perp \arrow[ul] \arrow[ur] \arrow[uu] &
\end{tikzcd}
$$

- For brevity, label the domains and then forget the sets.



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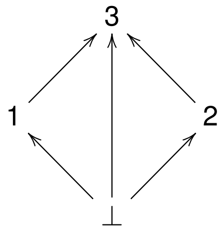
- Enforcing rules for explicit communication in such a system is a well-studied problem.



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- Enforcing rules for explicit communication in such a system is a well-studied problem.
- **Implicit** communication is harder.

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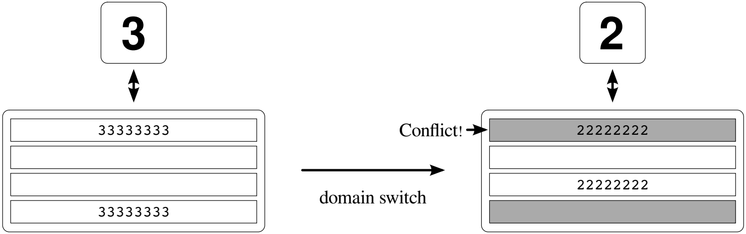
- For brevity, label the domains and then forget the sets.
- Enforcing rules for explicit communication in such a system is a well-studied problem.
- **Implicit** communication is harder.
- We're specifically concerned with covert channels due to sharing hardware.

The Cache Channel



Even if two domains are unable to communicate, they leave detectable traces in the machine state.

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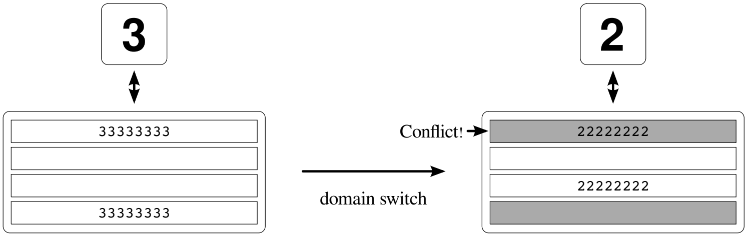

Even if two domains are unable to communicate, they leave detectable traces in the machine state.
For example, 2 cannot read 3's cache lines, but it can infer where they are, by timing its own memory accesses.

The Cache Channel

How do we mitigate this channel?

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- We could flush the cache everytime

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- Transition up as long as possible. . .

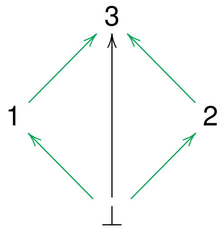
The Cache Channel

How do we mitigate this channel?

- We could flush the cache everytime . . . expensive!
- We don't **need** to flush when transitioning up.
- Transition up as long as possible. . . then flush and start again.

This is **Lattice Scheduling**

$$
\begin{tikzcd}
 & 3 & \\
1 \arrow[ur] & & 2 \arrow[ul] \\
 & \bot \arrow[ul] \arrow[ur] \arrow[uu] &
\end{tikzcd}
$$



- The schedule relation S, is a subset of the up transitions.

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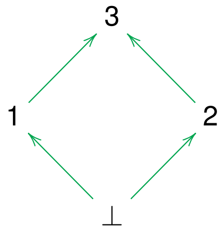

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- This schedule is incomplete: There's no way to leave 3.

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$$

- The schedule relation S, is a subset of the up transitions.
- This schedule is incomplete: There's no way to leave 3.
- We must add downward transitions, but how?

$$\begin{tikzcd}
 & 3 & \\
1 \arrow[ur] & & 2 \arrow[ul] \\
 & \perp \arrow[ul] \arrow[ur] &
\end{tikzcd}$$

- Designate a downgrader, $\perp$.

$$
\begin{tikzcd}
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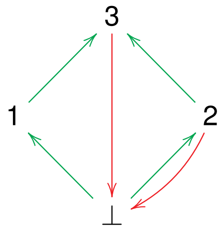

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Lemma (Downgrading)

If S allows a downward transition, it is to the downgrader, $\perp$:

$$\frac{(c, n) \in S \qquad \textit{clearance } c \not\subseteq \textit{clearance } n}{n = \perp}$$

The Lattice Scheduler

We'll verify a scheduler written in pGCL, an imperative, probabilistic language:

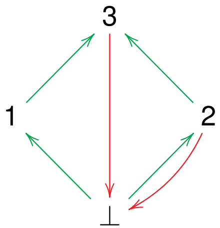
$$\textbf{record}\ \text{stateA} = \text{current\_domain} :: \textit{dom\_id}$$

$$\text{scheduleS} =$$

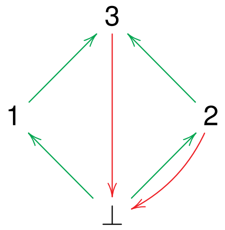
$$\quad c \text{ is current\_domain in}$$

$$\quad \text{current\_domain} :\in (\lambda\_.\ \{n.\ (c, n) \in S\})$$

This program selects a new domain nondeterministically from among those with a valid transition from the current.



- We want to **refine** this to a realistic implementation.



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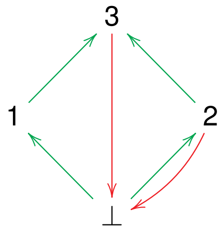

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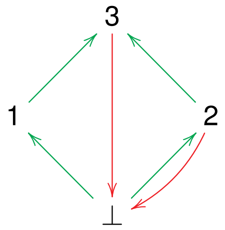

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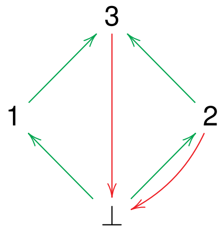
- The specification permits starvation.



- We want to **refine** this to a realistic implementation.
- The refinement may produce any trace permitted here.
- For example: $\perp, 2, \perp, 2, \ldots$.
- The specification permits starvation.
- Randomisation gives us a neat solution.

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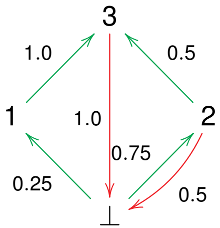
Random Transitions



- Assign a probability to each transition such that $T\ (c, n) > 0$ only if $(c, n) \in S$.
- Outgoing probabilities sum to 1 (or less).
- The previous trace now has probability 0!

A Probabilistic Scheduler

scheduleT $=$

c is current_domain in

current_domain $:\in (\lambda\_.\ \{\bot, 1, 2, 3\}$ at $(\lambda\_\ n.\ T\ (c, n))$

A Probabilistic Scheduler

$$\text{scheduleT} =$$
$$\quad c \text{ is current\_domain in}$$
$$\quad \text{current\_domain} :\in (\lambda\_.\ \{\bot, 1, 2, 3\} \text{ at } (\lambda\_\ n.\ T\ (c, n))$$

Lemma (Non-starvation)

Taking at least 8 steps from any *initial domain, we reach* any *final domain with non-zero probability:*

$$\forall s.\ 0 < wp\ scheduleT^{8+n}\ (in\_dom\ d_f)\ s$$

Note that predicates (expectations) in pGCL are real-valued.

Refinement in pGCL

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- Entailment ($\vdash$) generalises ($\vDash$), which is really just $\leq$:

$$\text{False} \rightarrow \text{True} \qquad 0 \leq 1$$

$$\lambda x.\ \text{False} \vdash \lambda x.\ \text{True} \qquad \lambda x.\ 0 \vDash \lambda x.\ 1$$

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- Predicates are lifted to **expectations**:

$$«P» = \lambda x.\ \text{if } P\ x \text{ then } 1 \text{ else } 0$$

- We reason about **weakest-preexpectations**:

$$Q \vDash \text{wp prog } R$$

Refinement in pGCL

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Definition

Program b *refines* program a, written $a \sqsubseteq b$, exactly when all expectation-entailments on a also hold on b:

$$\frac{P \vDash \text{wp } a\ Q}{P \vDash \text{wp } b\ Q}$$

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Lemma

The transition scheduler refines the lattice scheduler:

$$\mathit{scheduleS} \sqsubseteq \mathit{scheduleT}$$

First Refinement

$$\begin{tikzcd} \textsc{Downgrading} \arrow[r, dotted] & \text{scheduleS} \arrow[d, "\sqsubseteq"] \\ & \text{scheduleT} \end{tikzcd}$$

- Downgrading is preserved by refinement,

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$$\begin{tikzcd} \textsc{Downgrading} \arrow[r, dotted] \arrow[dr, dotted] & \text{scheduleS} \arrow[d, "\sqsubseteq"] \\ \textsc{Non-Starvation} \arrow[r, dotted] & \text{scheduleT} \end{tikzcd}$$

- Downgrading is preserved by refinement, and therefore holds for scheduleT.
- Non-starvation holds **only** for scheduleT.

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The Lottery Scheduler

Our scheduler is so far very abstract. The next step is to **implement** the randomisation. We use a lottery:

- We only need a uniform random choice from $\mathbb{Z}_{32}$.
- Each option is assigned some number, x, of tickets.
- The chance of winning is $\frac{x}{2^{32}}$.
- We need to assume that the **lottery relation** holds:

$$T\ (c, n) = 2^{-32} \|\{t.\ \text{lottery}\ (\text{domains}\ s\ c)\ t = n\}\|$$

- Different state spaces: need more than simple refinement.

The Lottery Scheduler

$$\textbf{record}\ \text{domain} = \text{lottery} :: 32\ \text{word} \Rightarrow \mathit{dom\_id}$$

$$\begin{aligned}\textbf{record}\ \text{stateC} = {} & \text{current\_domain} :: \mathit{dom\_id} \\ & \text{domains} :: \mathit{dom\_id} \Rightarrow \text{domain}\end{aligned}$$

$$\begin{aligned}\text{scheduleM}\ t = \text{do}\ & c \leftarrow \text{gets current\_domain} \\ & \mathit{dl} \leftarrow \text{gets domains} \\ & \text{let}\ n = \text{lottery}\ (\mathit{dl}\ c)\ t\ \text{in} \\ & \text{modify}\ (\lambda s.\ s(\!|\text{current\_domain} := n|\!)) \\ \text{od} &\end{aligned}$$

$$\begin{aligned}\text{scheduleC} = {} & t\ \text{from}\ (\lambda s.\ \text{UNIV})\ \text{at}\ 2^{-32}\ \text{in} \\ & \text{Exec}\ (\text{scheduleM}\ t)\end{aligned}$$

Data Refinement

Definition (Probabilistic Data Refinement)

Program b, on state type σ, refines program a, state τ, given precondition $G : \sigma \to$ Bool and under projection $\theta : \sigma \to \tau$, written $a \sqsubseteq_{G,\theta} b$, exactly when any expectation entailment on a implies the same for b, on the projected state and with a guarded pre-expectation:

$$\frac{P \vDash \text{wp}\ a\ Q}{«G» \mathbin{\&\&} (P \circ \theta) \vDash \text{wp}\ b\ (Q \circ \theta)}$$

$$a \mathbin{\&\&} b = \max\ (a + b - 1)\ 0$$

Correspondence

Definition (Probabilistic Correspondence)

Programs *a* and *b* are said to be in *probabilistic correspondence*, pcorres θ *G a b*, given condition *G* and under projection θ if, for any post-expectation *Q*, the guarded pre-expectations coincide:

$$\text{«}G\text{»} \,\&\&\, (\text{wp}\ a\ Q \circ \theta) = \text{«}G\text{»} \,\&\&\, \text{wp}\ b\ (Q \circ \theta)$$

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Lemma

The specifications scheduleT and scheduleC correspond given condition LR and under projection ϕ:

$$\mathit{pcorres}\ \phi\ \mathit{LR}\ \mathit{scheduleT}\ \mathit{scheduleC}$$

Second Refinement

$$\begin{tikzcd}
\textsc{Downgrading} \arrow[r, dotted] & \text{scheduleS} \arrow[d] \\
\textsc{Non-Starvation} \arrow[r, dotted] & \text{scheduleT}
\end{tikzcd}$$

Second Refinement

$$\begin{tikzcd} \textsc{Downgrading} \arrow[r, dotted] & \text{scheduleS} \arrow[d] \\ \textsc{Non-Starvation} \arrow[r, dotted] & \text{scheduleT} \arrow[d, Rightarrow, "\phi\text{,LR}"] \\ & \text{scheduleC} \end{tikzcd}$$

- The double arrow represents correspondence.

Second Refinement

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$$

- The double arrow represents correspondence.
- Correspondence composes with refinement.
- Downgrading and non-starvation are preserved.

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The Nondeterministic State Monad

- The seL4 specification is written using a **nondeterministic state monad**.
- We can embed this cleanly into pGCL.
- In fact, we just used it: scheduleM and Exec.
- L4.verified used a particular notion of nondeterministic correspondence.
- We know how to lift these results, probabilistically:

Lemma (Lifting Correspondence)

Given correspondence between M and M′: with

$$\mathit{corres\_underlying}\ \{(s, s').\ s = \phi\ s'\}\ \mathit{True}\ \mathit{rrel}\ G\ (G \circ \phi)\ M\ M'$$

and standard side-conditions:

$$\mathit{no\_fail}\ G\ M \qquad \mathit{empty\_fail}\ M \qquad \mathit{empty\_fail}\ M'$$

and that M is deterministic on the image of the projection,

$$\forall s.\ \exists (r, s').\ M\ (\phi\ s) = \{(\mathit{False}, (r, s'))\}$$

then we have probabilistic *correspondence:*

$$\mathit{pcorres}\ \phi\ (G \circ \phi)\ (\mathit{Exec}\ M)\ (\mathit{Exec}\ M')$$

Lemma

If the kernel preserves the lottery relation,

$$\{\!|LR|\!\}\ \mathit{stepKernel}\ \{\!|\lambda\_.\ LR|\!\}$$

and the current domain,

$$\{\!|\lambda s.\ CD\ s = d|\!\}\ \mathit{stepKernel}\ \{\!|\lambda\_\ s.\ CD\ s = d|\!\}$$

and is total,

$$\mathit{no\_fail}\ \top\ \mathit{stepKernel} \quad \mathit{empty\_fail}\ \mathit{stepKernel}$$

then with the concrete scheduler, it refines the transition scheduler:

$$\mathit{scheduleT} \sqsubseteq_{LR,\phi} \mathit{stepKernel};;\mathit{scheduleC}$$

Composed Refinement

$$
\begin{tikzcd}
& \textsc{Downgrading} \arrow[r, dotted] & \text{scheduleS} \arrow[d] \\
& \textsc{Non-Starvation} \arrow[r, dotted] & \text{scheduleT} \arrow[d, Rightarrow, "\phi\text{,LR}"] \\
\text{callKernelD} = = = \text{stepKernel; ; scheduleC} \arrow[d, Rightarrow] & & \text{scheduleC} \arrow[ll, Rightarrow, "\phi\text{,LR}"'] \\
\text{callKernelH} \arrow[d, Rightarrow] & & \\
\text{callKernelC} & &
\end{tikzcd}
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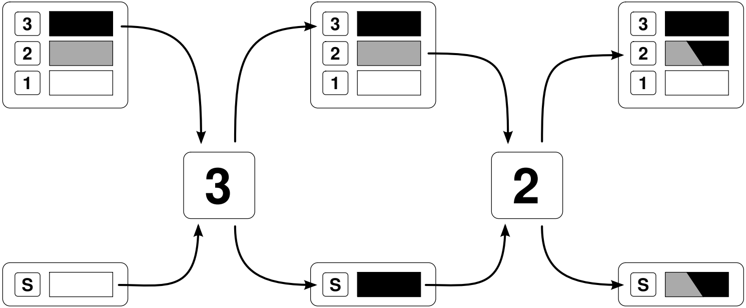
Hardware Model

Ultimately, we want to know that our scheduler eliminates leakage via the cache. We append a machine model:

$$\textbf{record}\ (\mathit{sh}, \mathit{pr})\ \text{machine} = \text{private} :: \mathit{dom\_id} \Rightarrow \mathit{pr}$$
$$\text{shared} :: \mathit{sh}$$

- A private state per domain.
- A shared state between domains (the cache).
- Domains are underspecified, but may only update their own private state and the shared state.

Leakage via Shared State



- Propagating taint takes at least 2 steps.
- A single-step policy isn't enough.

Lemma (Non-leakage)

If the clearance of domain h is not entirely contained within that of domain l,

$$\textit{clearance } h \not\subseteq \textit{clearance } l$$

then any function of the state after execution, which depends only on elements within l's clearance,

$$Q \circ \textit{mask } l$$

is invariant under modifications to h's private state (as represented by replace):

$$\begin{aligned} & wp\ (runDom;;scheduleT)^n\ (Q \circ mask) = \\ & \quad (wp\ (runDom;;scheduleT)^n\ (Q \circ mask)) \circ (replace\ h\ p) \end{aligned}$$

Final Refinement

$$
\begin{tikzcd}
\textsc{Downgrading} \arrow[r, dotted] & \mathsf{scheduleS} \arrow[d] & \textsc{Non-Leakage} \arrow[d, dotted] \\
\textsc{Non-Starvation} \arrow[r, dotted] & \mathsf{scheduleT} \arrow[r, Rightarrow, "{\psi,\mathsf{LR}}"'] \arrow[d, Rightarrow, "{\phi,\mathsf{LR}}"] & \mathsf{runDom\ ;;\ scheduleT} \arrow[d, Rightarrow, "{\phi,\mathsf{LR}}"] \\
\mathsf{stepKernel\ ;;\ scheduleC} & \mathsf{scheduleC} \arrow[l, Rightarrow, "{\phi,\mathsf{LR}}"] \arrow[r, Rightarrow, "{\psi,\mathsf{LR}}"'] & \mathsf{runDom\ ;;\ scheduleC}
\end{tikzcd}
$$

- We've have non-leakage for the probabilistic scheduler (scheduleT), and it is preserved by refinement.
- We now have all 3 properties for the concrete implementation.

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- Probabilistic programs need not be harder to verify than traditional ones.

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- Probabilistic results can compose well with large existing proofs.

NICTA

Questions?