The All-Best-Swap-Edge Problem on Tree Spanners

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Abstract

Given a 2-edge connected – possibly weighted – undirected graph G with n vertices and m edges, a σ-tree spanner is a spanning tree T of G in which the ratio between the distance in T of any pair of vertices and the corresponding distance in G is upper bounded by σ. The minimum value of σ for which T is a σ-tree spanner of G is also called the *stretch factor* of T. We address the fault-tolerant scenario in which each edge e of a given tree spanner may temporarily fail and has to be replaced by a *best swap edge*, i.e. an edge that reconnects $T - e$ at a minimum stretch factor. In this paper, we survey all the results that are known for this problem.

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1 Introduction

Given a 2-edge connected – possibly weighted – undirected graph G with n vertices and m edges, a σ-tree spanner is a spanning tree T of G in which the ratio between the distance in T of any pair of vertices and the corresponding distance in G is upper bounded by σ. The minimum value of σ for which T is a σ-tree spanner of G is also called the *stretch factor* of T. The stretch factor of a tree spanner is a measure of how the all-to-all distances degrade w.r.t. the underlying communication graph if we want to sparsify it. Therefore, tree spanners find several applications in the network design problem area as well as in the area of distributed algorithms (see also [5, 6] for some additional practical motivations).

Unfortunately, tree-based network infrastructures are highly sensitive to even a single transient link failure, since this always results in a network disconnection. Furthermore, when these events occur, the computational costs for rearranging the network flow of information from scratch (i.e., recomputing a new tree spanner with small stretch factor, reconfiguring the routing tables, etc.) can be extremely high. Therefore, in such cases it is enough to promptly reestablish the network connectivity by the addition of a *swap* edge, i.e. a link that temporarily substitutes the failed edge.

In this paper we address the fault-tolerant scenario in which each edge e of a given tree spanner may undergo a transient failure and has to be replaced by a *best swap edge*, i.e. an edge that reconnects $T - e$ at a minimum stretch factor. More precisely, we aim at designing both time and space efficient algorithms that computes all the best swap edges (ABSE for short), i.e., a best swap edge for every edge of T. The ABSE problem on tree spanners has been introduced by Das et al. in [4], where the authors designed two algorithms for both the weighted and the unweighted case, running in $O(m^2 \log n)$ and $O(n^3)$ time, respectively, and using $O(m)$ and $O(n^2)$ space, respectively. The result was later improved in [1], where the authors provided two efficient linear-space solutions for both the weighted and the unweighted case, running in $O(m^2 \log \alpha(m, n))$ and $O(mn \log n)$ time, respectively. Subsequently, in [2] the authors designed a very clever recursive algorithm that uses centroid-decomposition techniques and lower envelope data structures to solve the ABSE problem on tree spanners in $O(n^2 \log^4 n)$ time and $O(n^2 + m \log^2 n)$ space. Finally, very recently, an $O(n^2)$ time and space algorithm that computes all the best swap edges in unweighted graphs has been designed in [3]. Even though the algorithm in [3] works only for unweighted graphs and improves the time and space complexities of the algorithm in [2] only by a polylogarithmic factor, we stress the fact that designing an $o(n^2)$ time and space algorithm would be considered a breakthrough in this field.

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