Scheduling in the Random-Order Model

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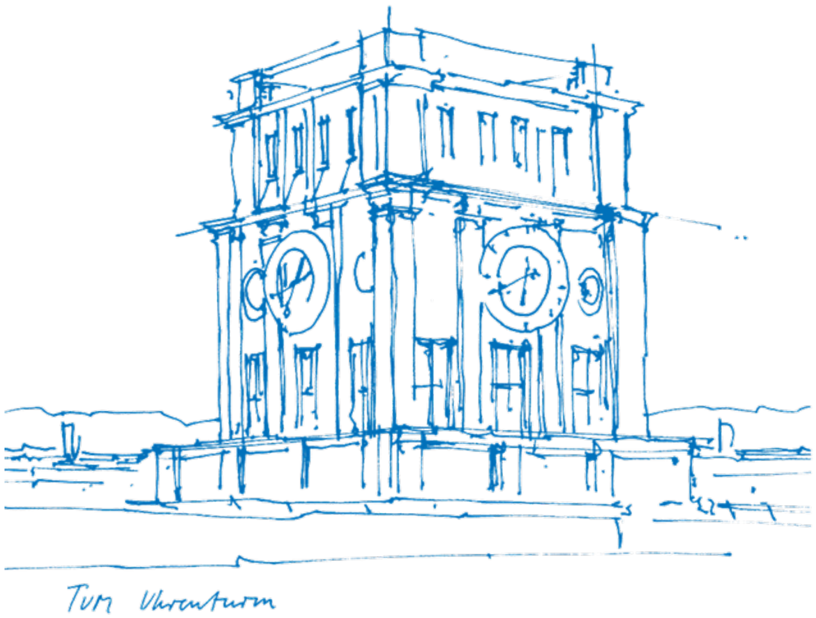
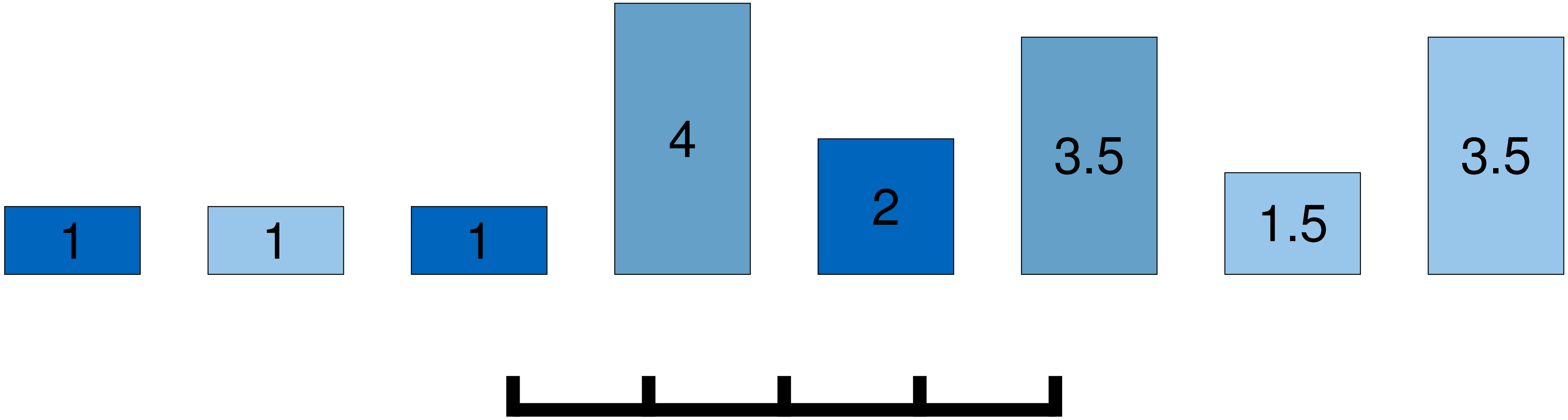
Department of Computer Science
Chair of Algorithms and Complexity



Makespan Minimization

Task: Assign **jobs** to **machines**.



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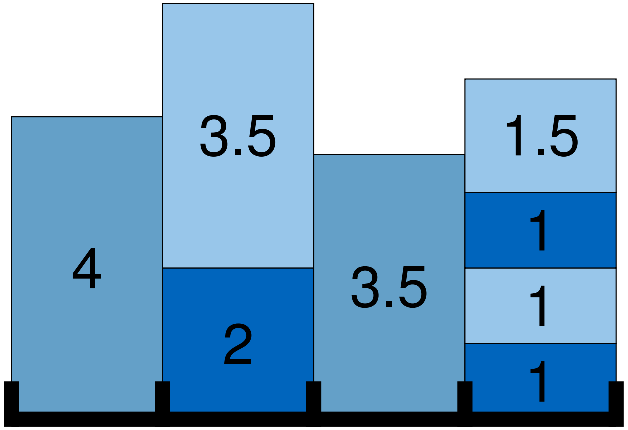

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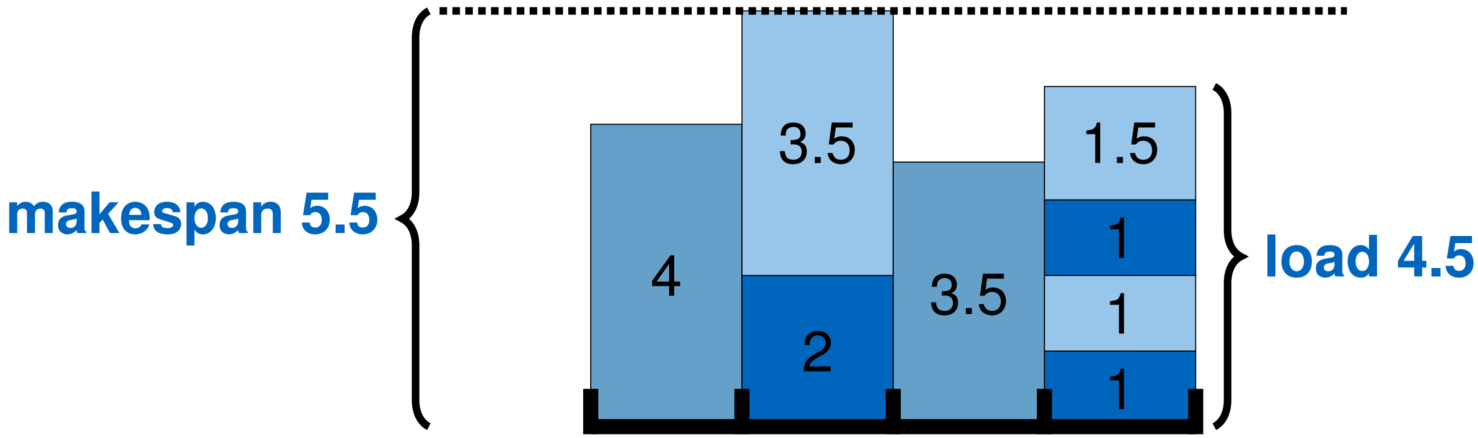

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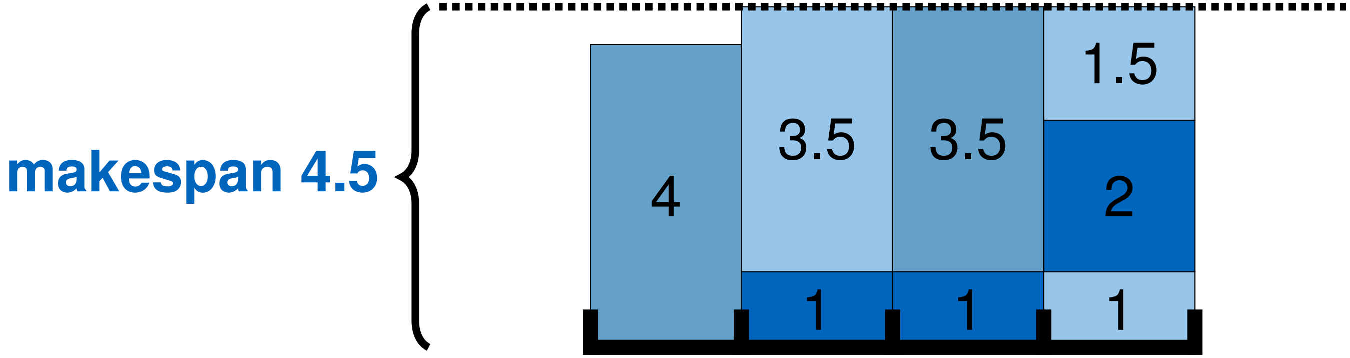
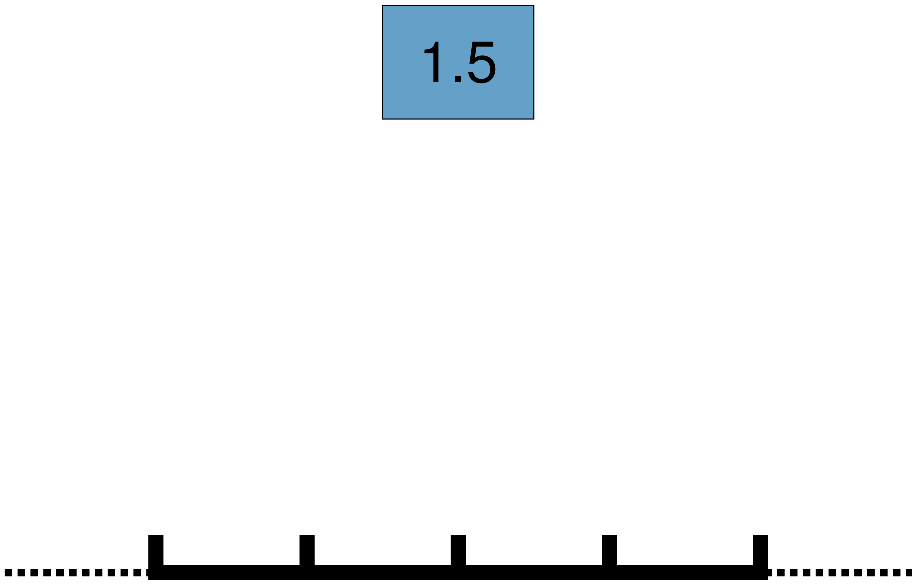


Online Algorithm

Jobs are revealed one by one and assigned immediately.



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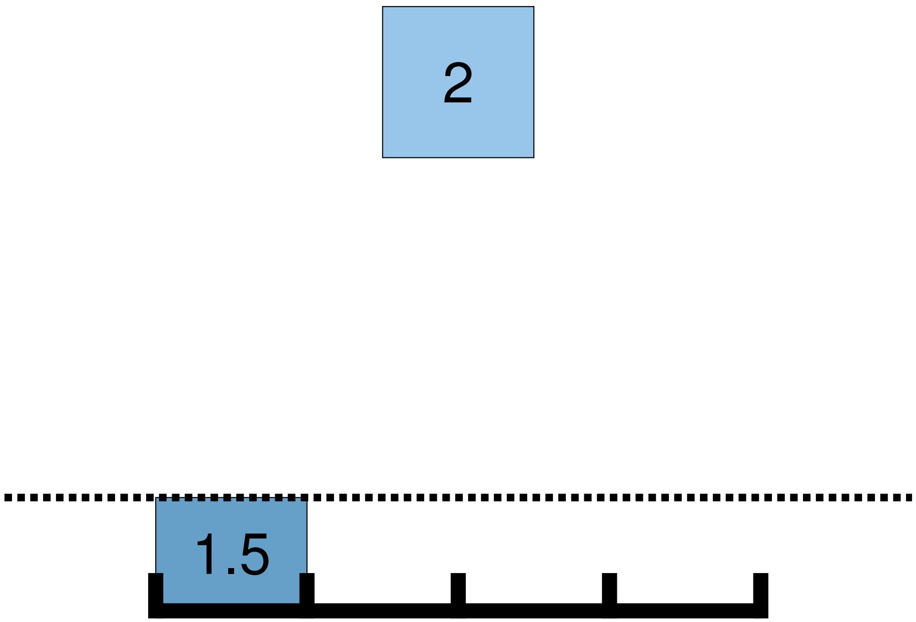

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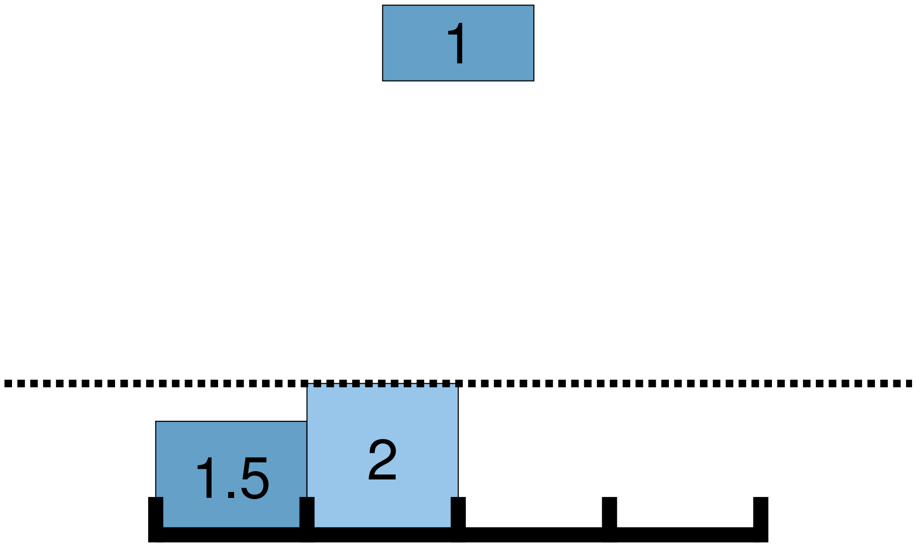

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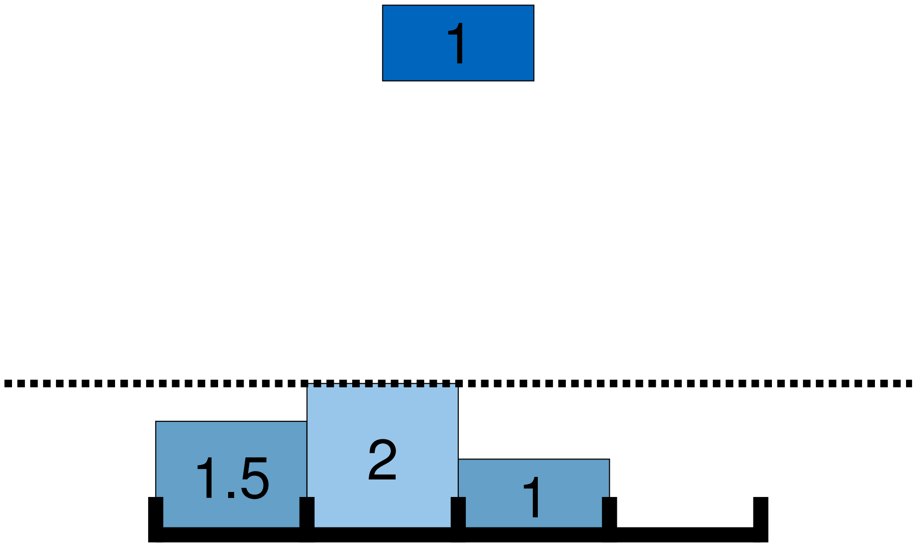

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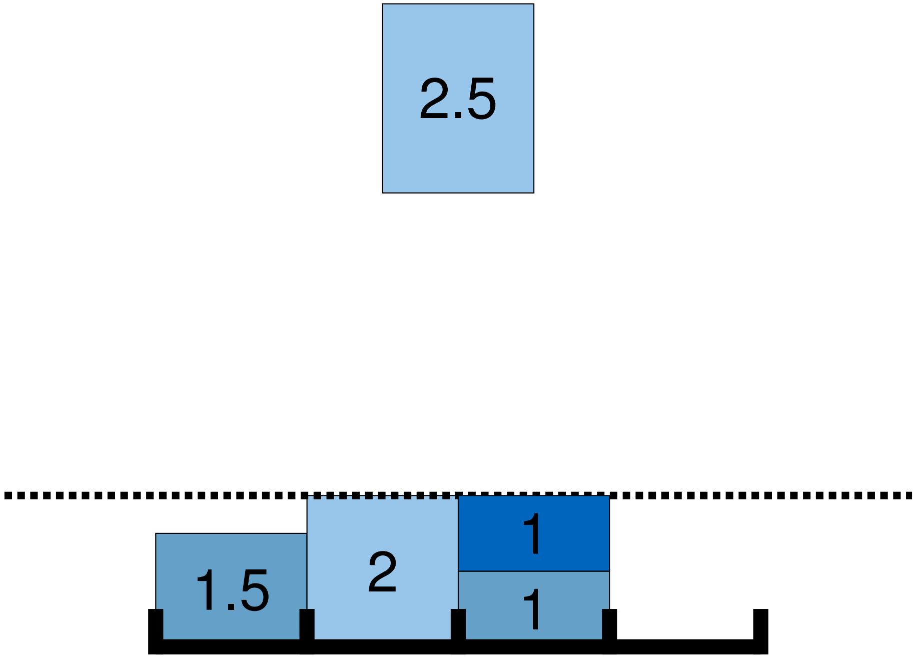

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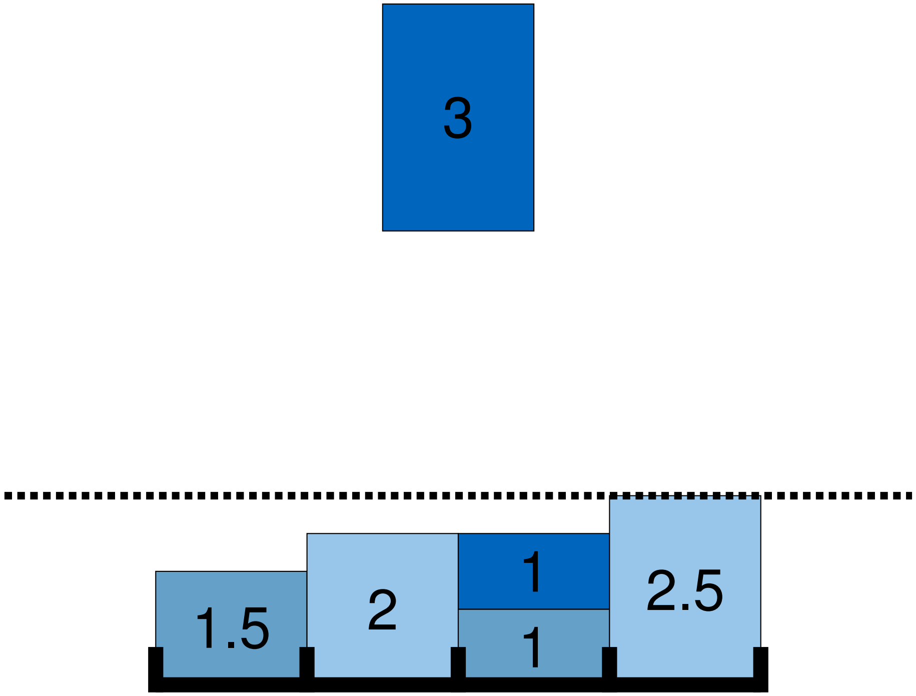

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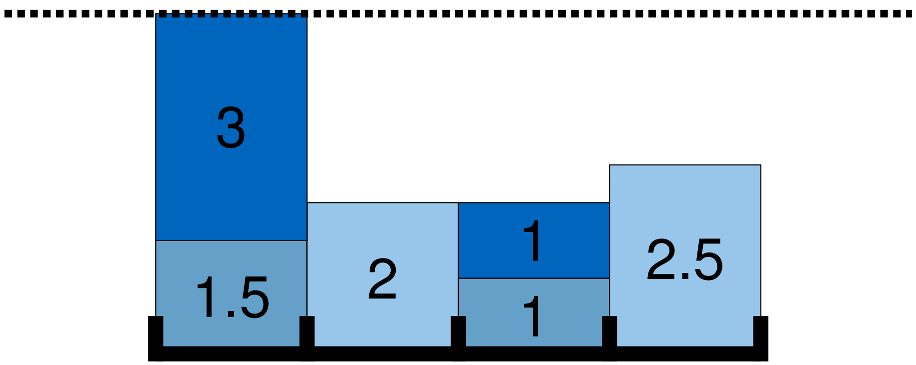
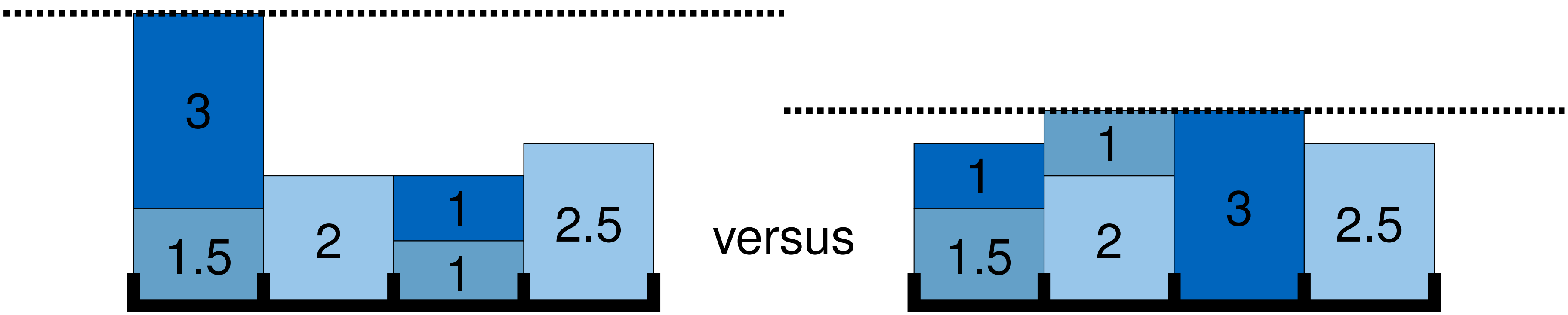


Competitive ratio

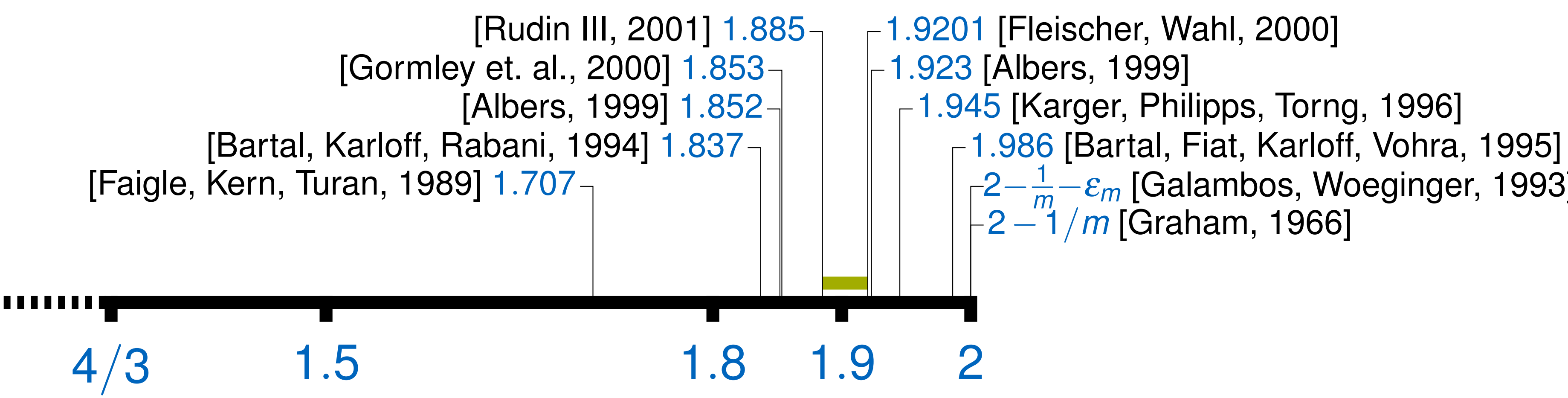
The **worst** ratio an **adversary** can cause.



Literature

Adversarial deterministic algorithms.

[Günther, Maurer, Megow and Wiese, 2013] and [Chen, Ye, Zhang, 2015] approximate the optimum online algorithm.



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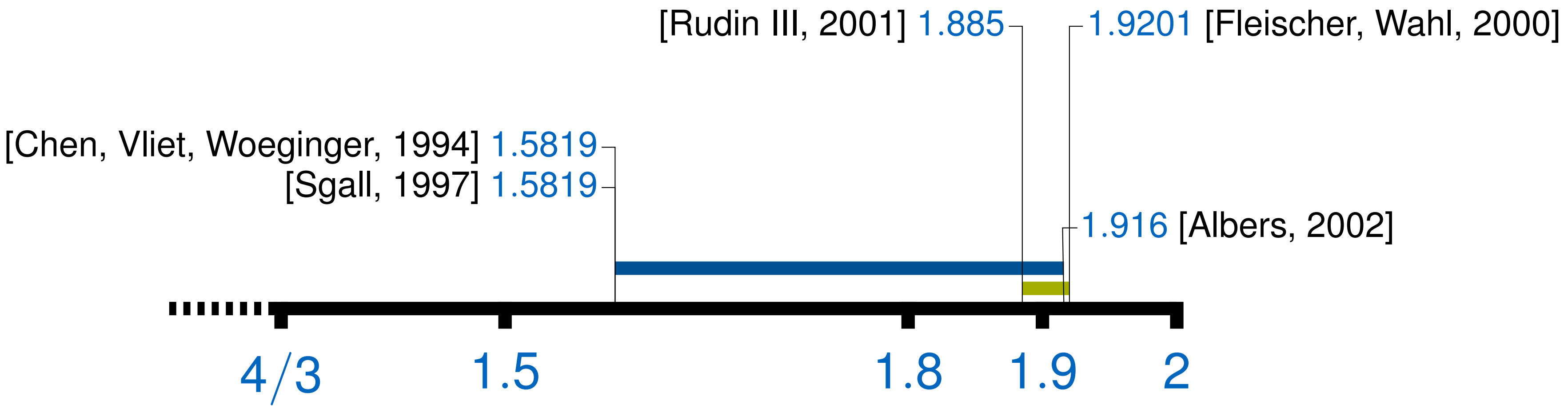

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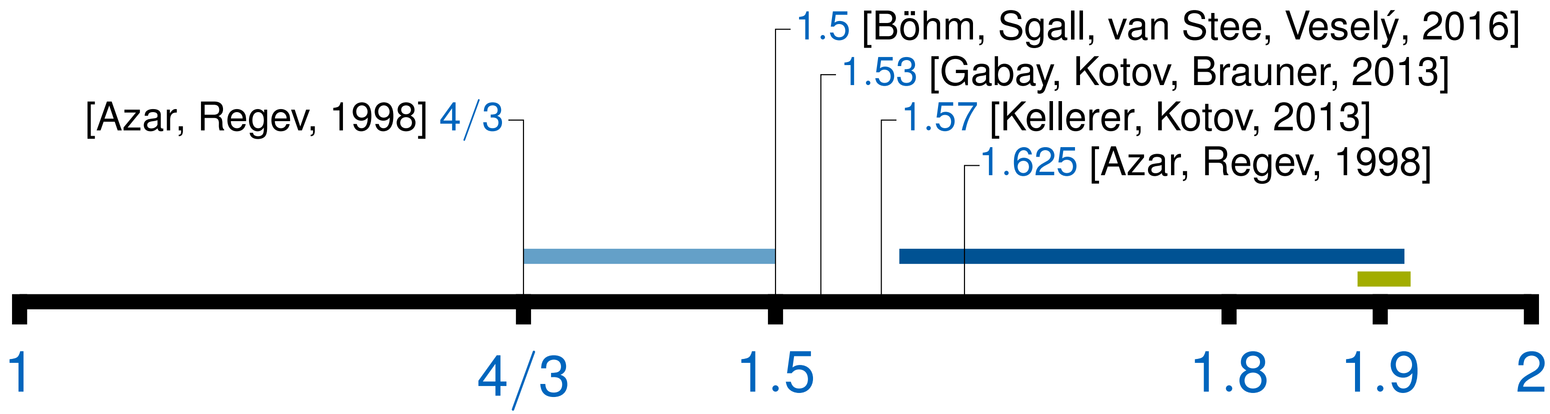

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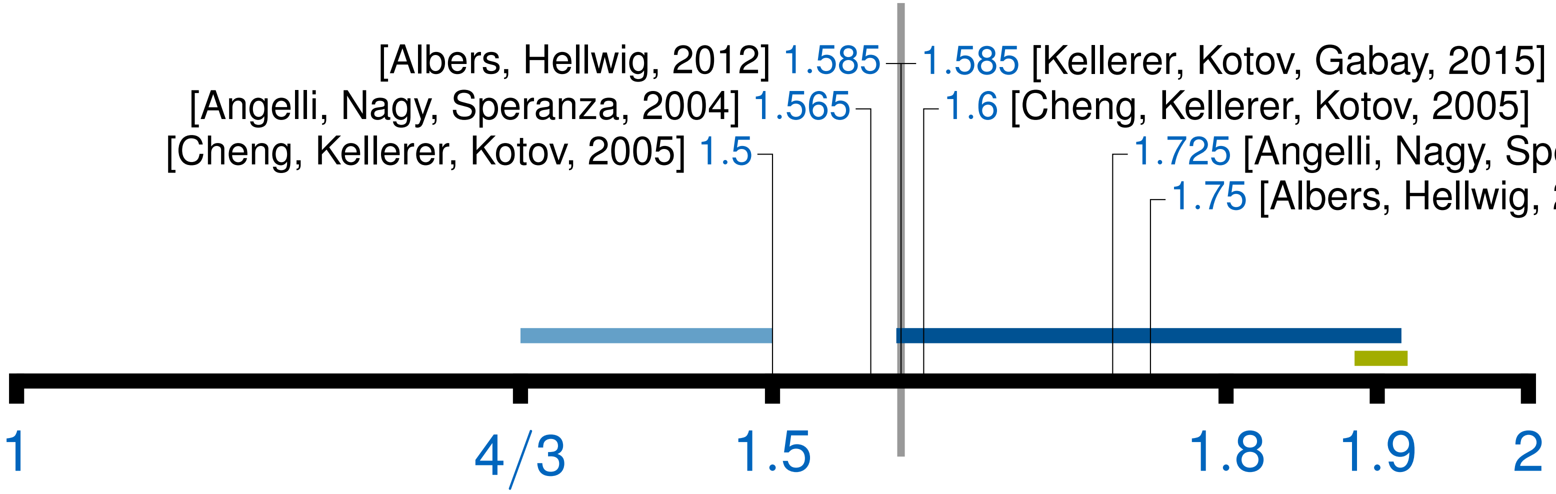

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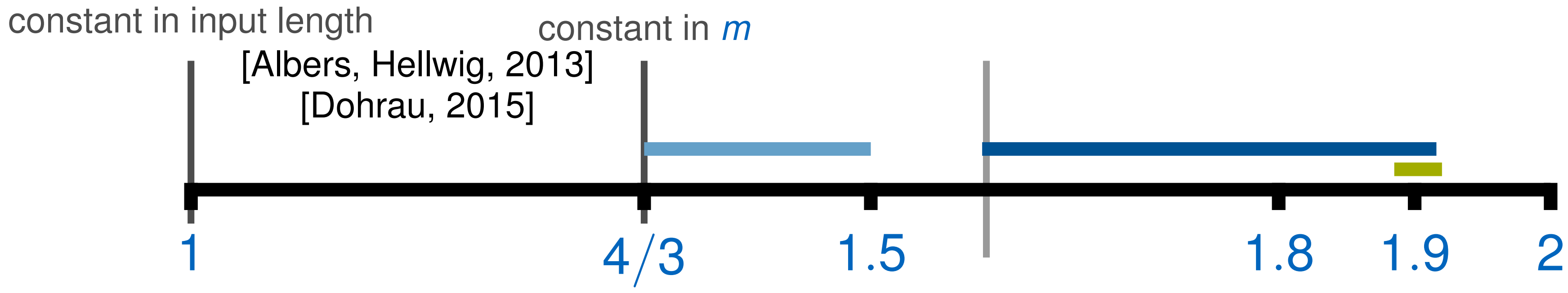

I made some corrections here.

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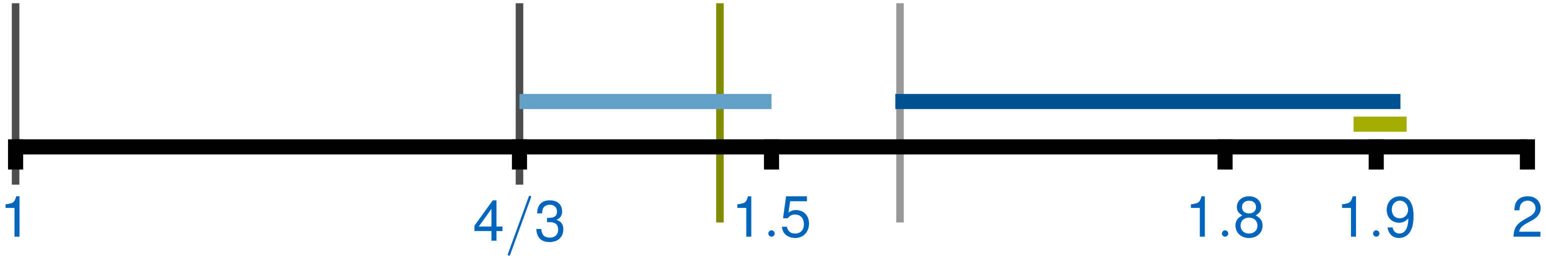
Buffer Reordering.

1.4659 [Englert, Özmen, Westermann, 2008]



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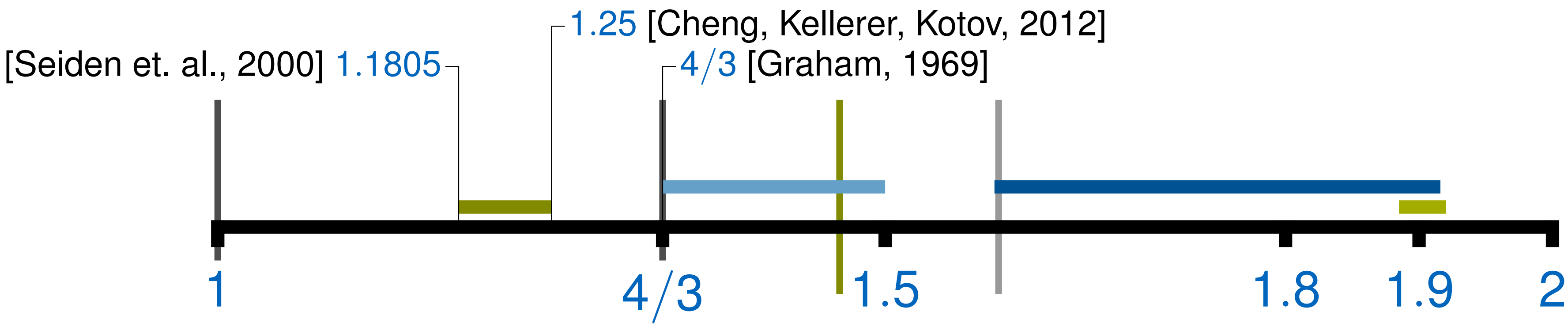
Job processing times ordered decreasingly.



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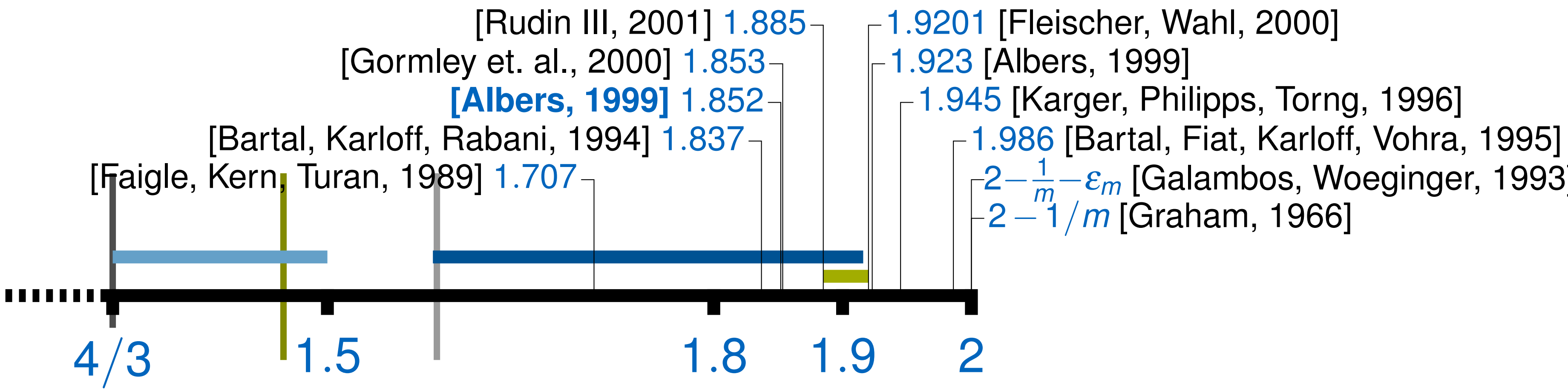
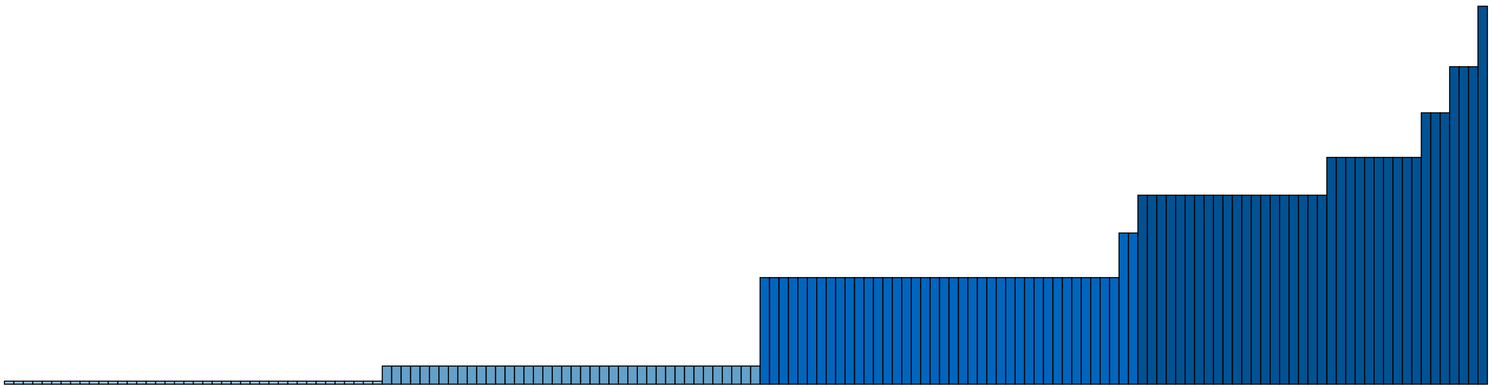


Is **worst-case analysis** too pessimistic?



(Lower bound of [Albers, 1999] for $m = 40$ machines.)

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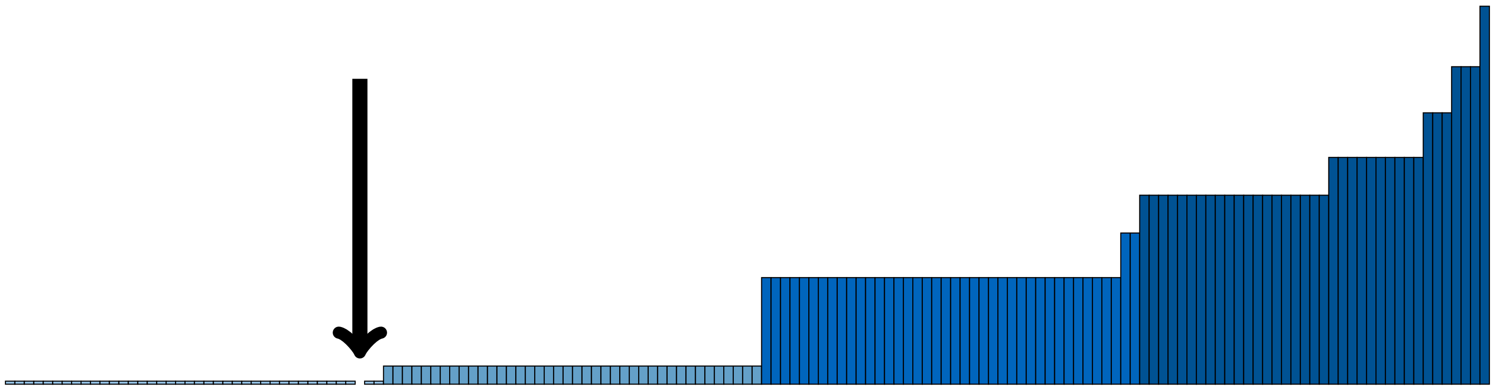

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The argument of Albers does not hold anymore if we

- Delete any job. (The lower bound would be 1.714)

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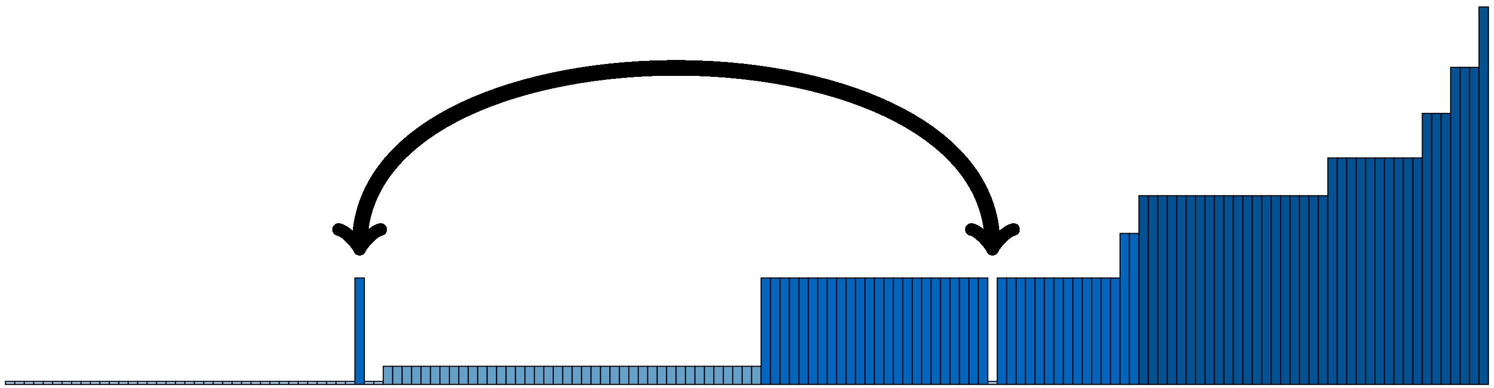

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The argument of Albers does not hold anymore if we

- Delete any job.
- Swap (non-identical) jobs.

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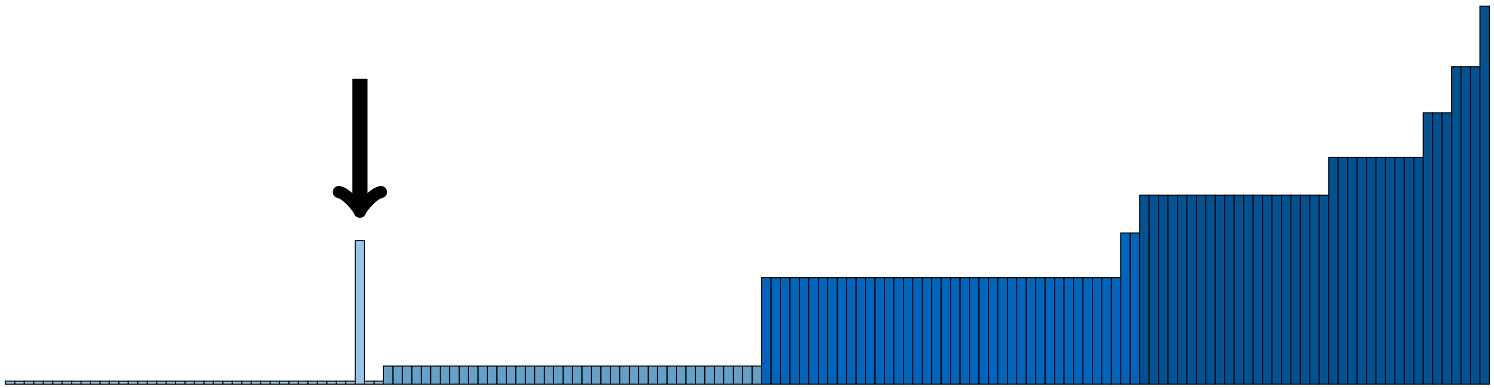

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- Delete any job.
- Swap (non-identical) jobs.
- Change a job size significantly.

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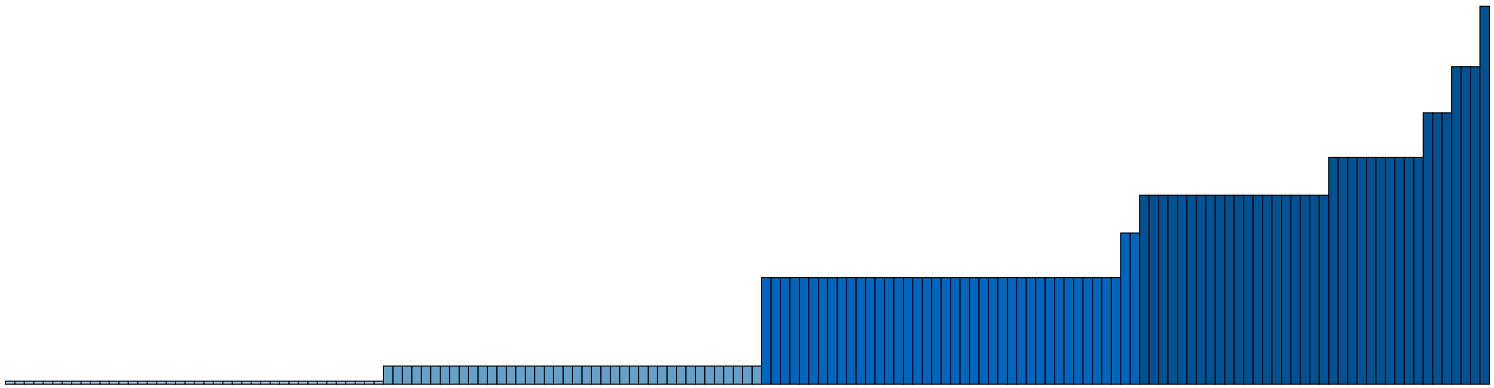

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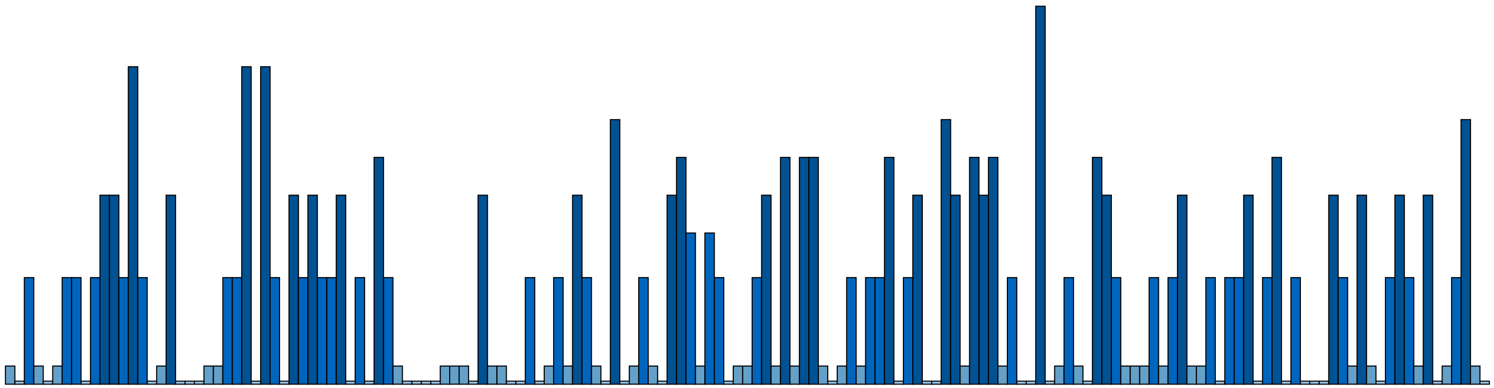
The argument of Albers does not hold anymore if we

- Swap (non-identical) jobs. **How important is job order?**

Random-Order Analysis



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Adversary chooses **job set**, order is **uniformly random**.

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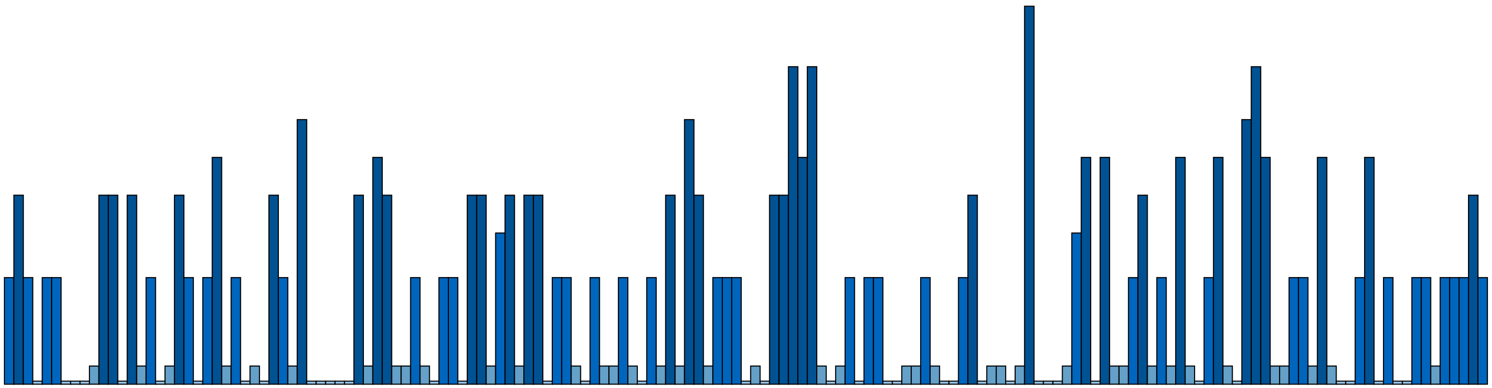

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Expected makespan of A versus **optimum makespan**.

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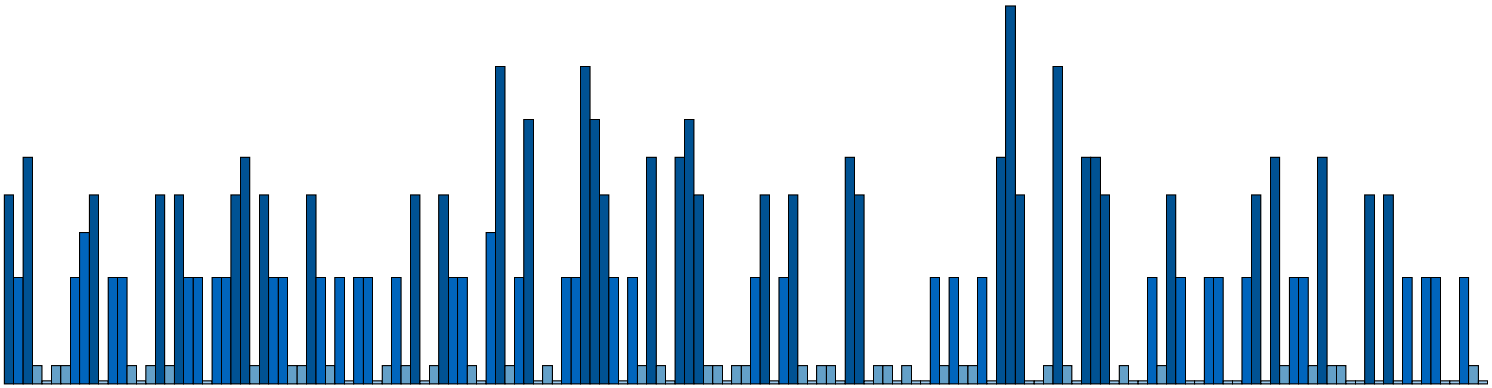

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Competitive ratio: Worst ratio the adversary may cause.

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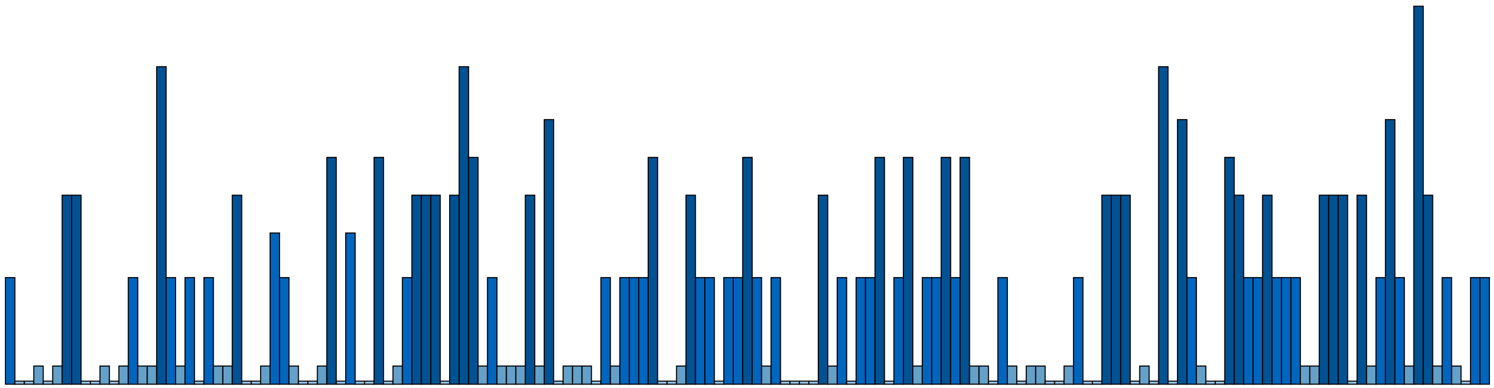

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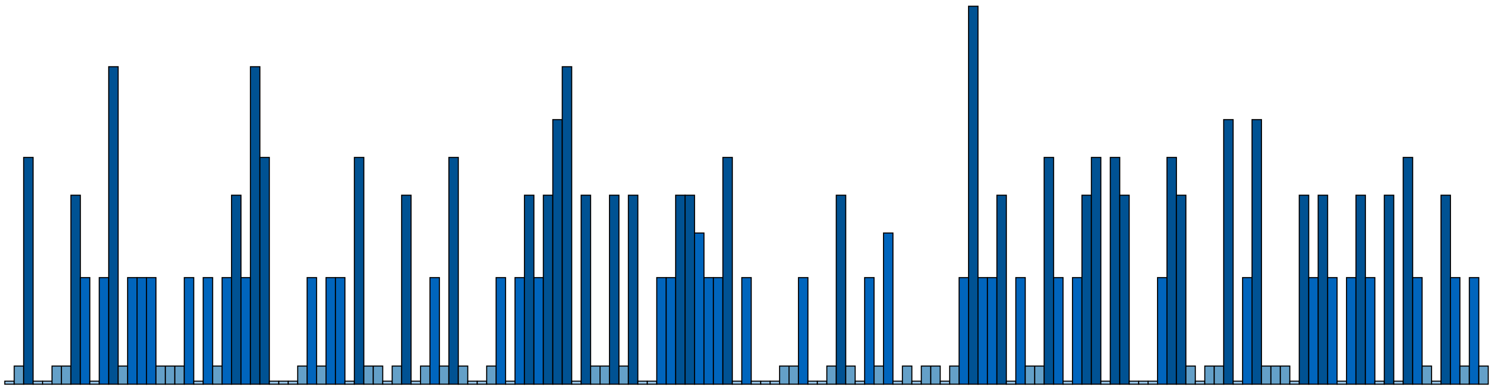

We are **nearly c-competitive** iff we are $(c + o_m(1))$-competitive with probability $1 - o_m(1)$ after random permutation and have a constant competitive ratio on worst-case sequences.

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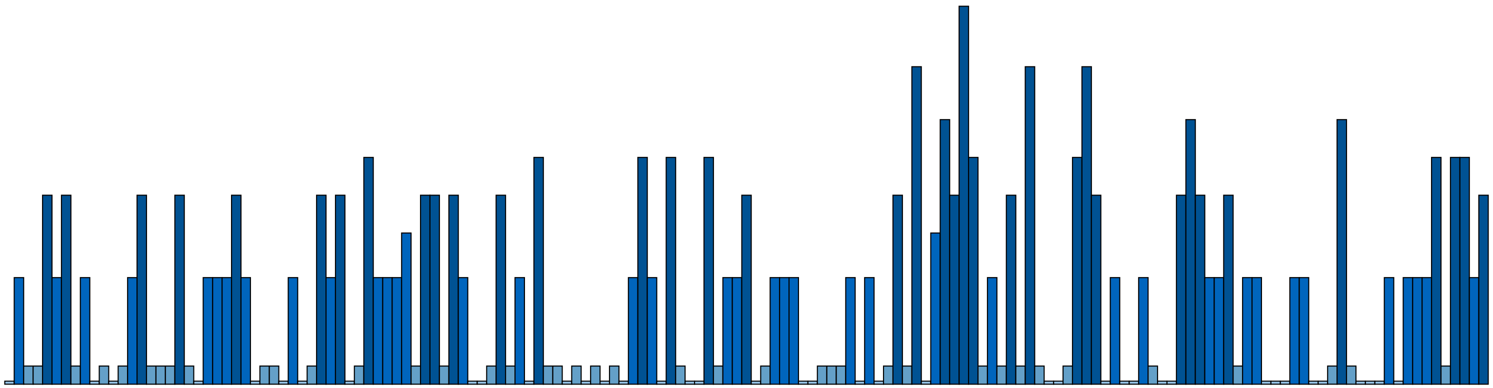

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A **nearly c-competitive** online algorithm is **c-competitive in the random-order model** for $m \to \infty$.

The Random-Order model

Scheduling for the random-order model is not well researched yet:

- [Osborn and Torng, 2008] show Greedy remains 2-competitive.

The Random-Order model

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- [Osborn and Torng, 2008] show Greedy remains 2-competitive.
- [Göbel, Kesselheim and Tönnis, 2015] and [Molinaro, 2017] analyze different scheduling problems.

Our results

Our new algorithm is **nearly 1.8478-competitive**.

We also provide lower bounds.

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Profit from random-order arrival.

- The **Load Lemma**. Identifying **time measures**.

Profit from random-order arrival.

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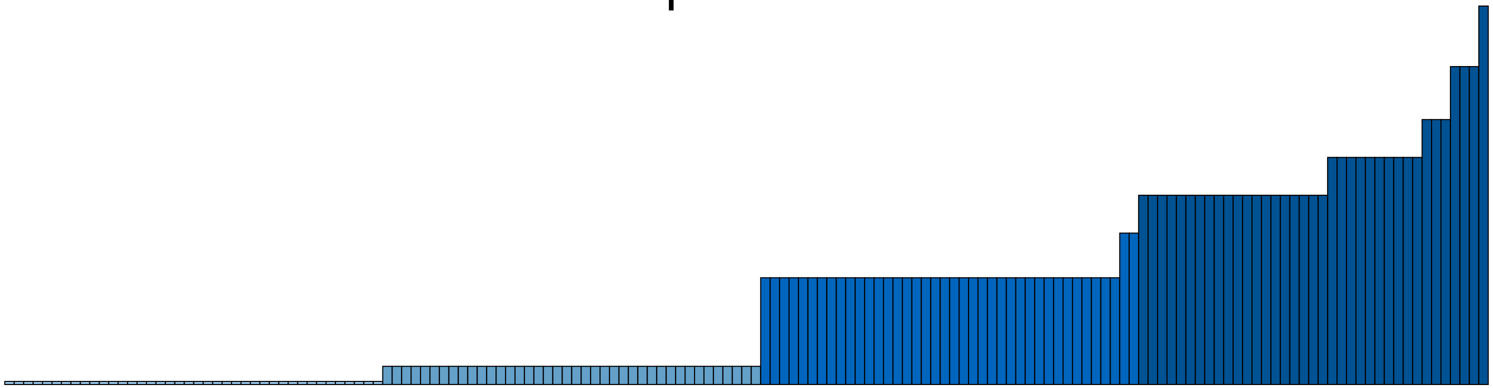
- **Profit**. A nearly 1.8478-competitive algorithm.

Different time measures.

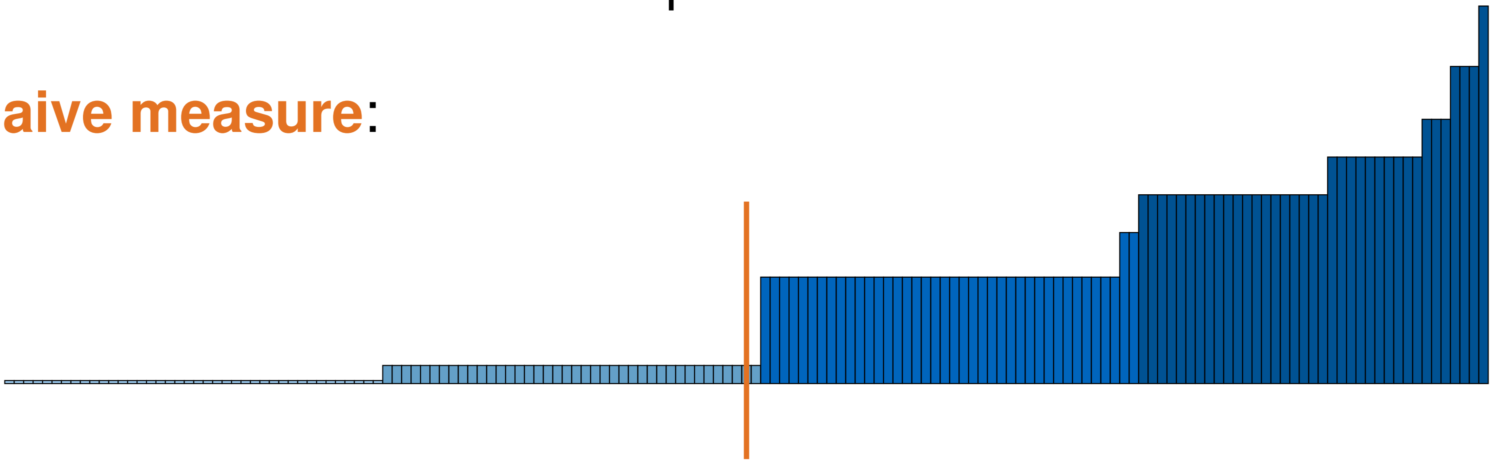
When have we seen half the sequence?



Different time measures.

When have we seen half the sequence?

The **naive measure**:



Different time measures.

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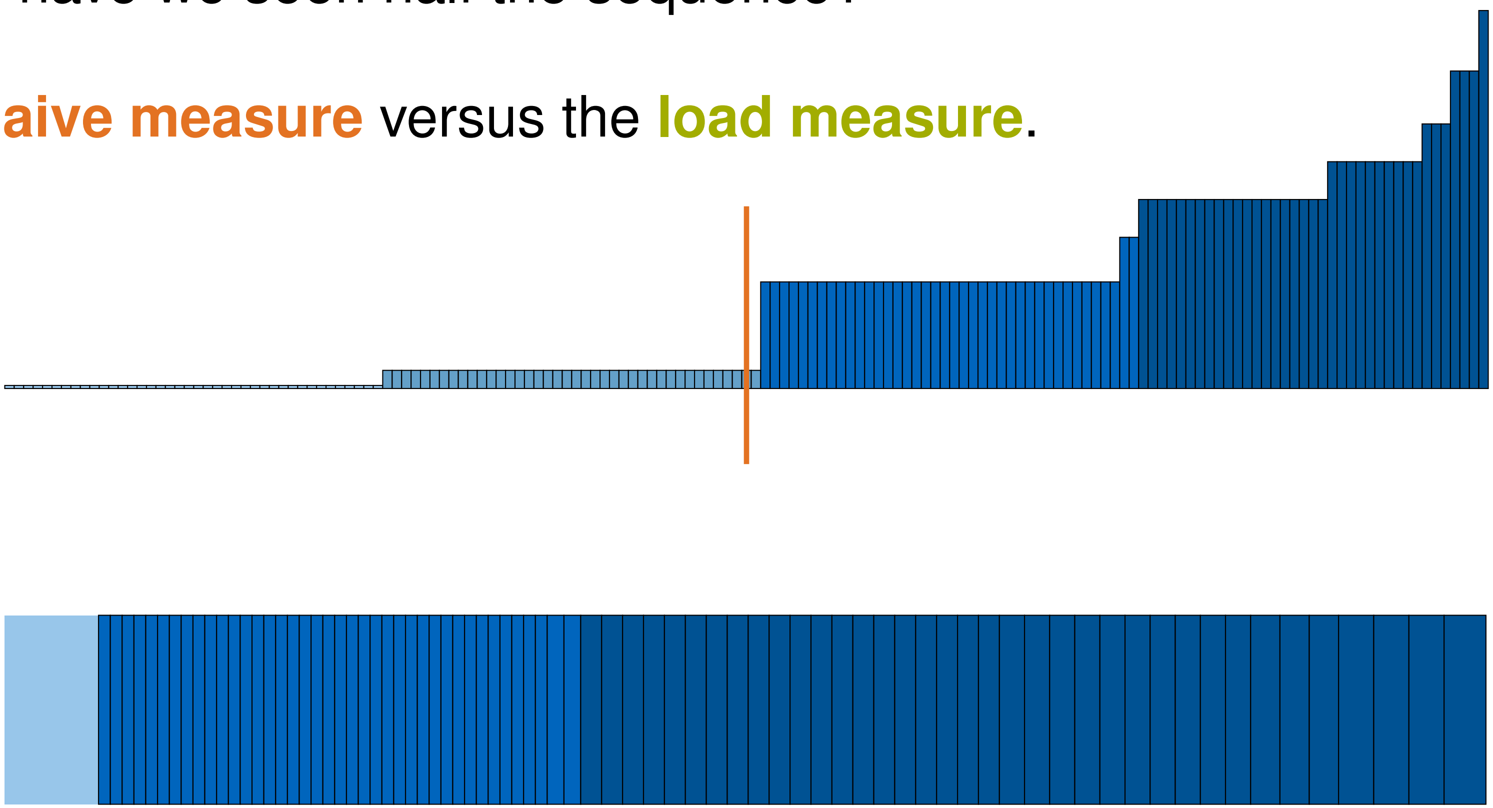
The **naive measure** versus the **load measure**.



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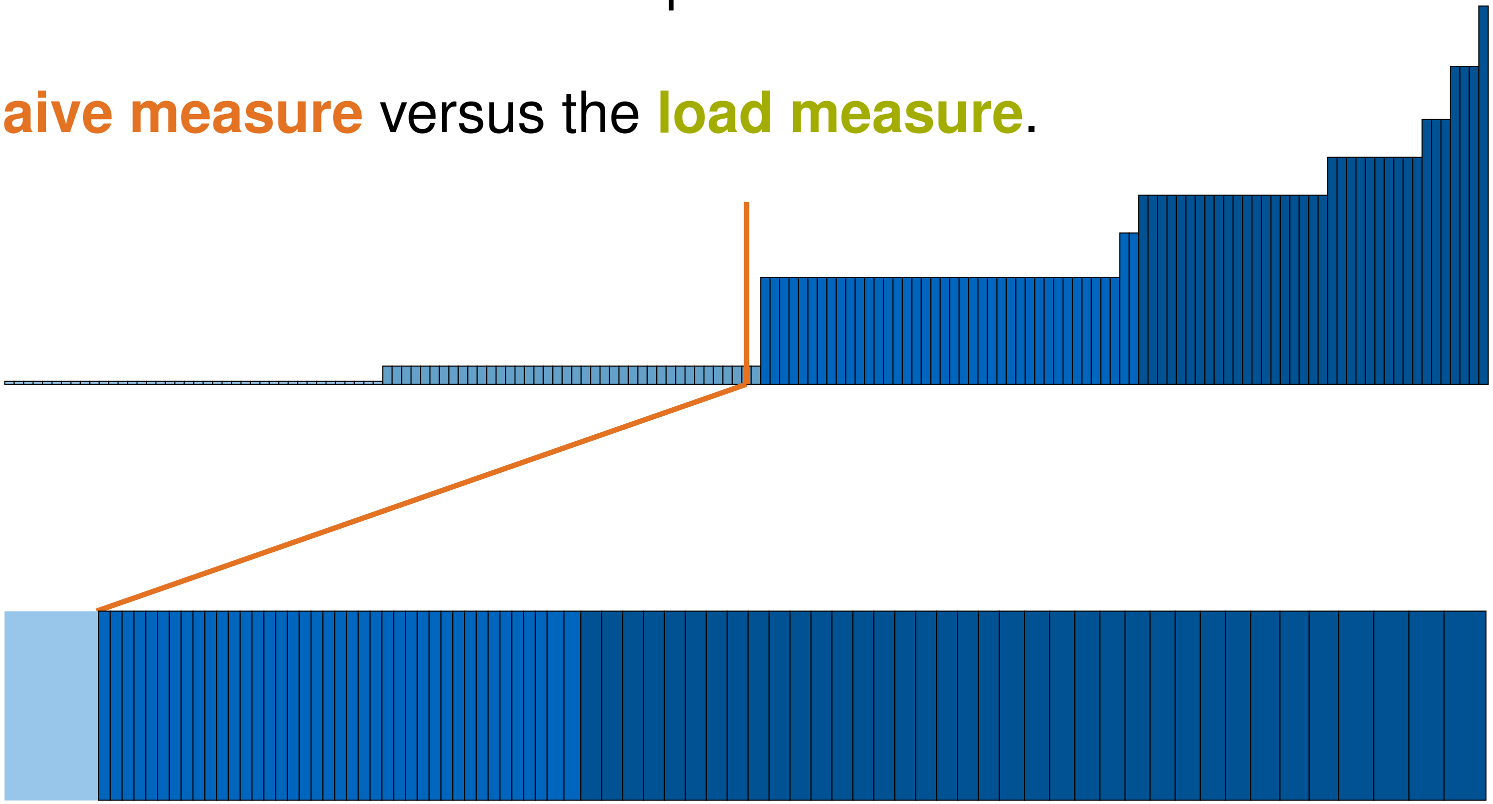

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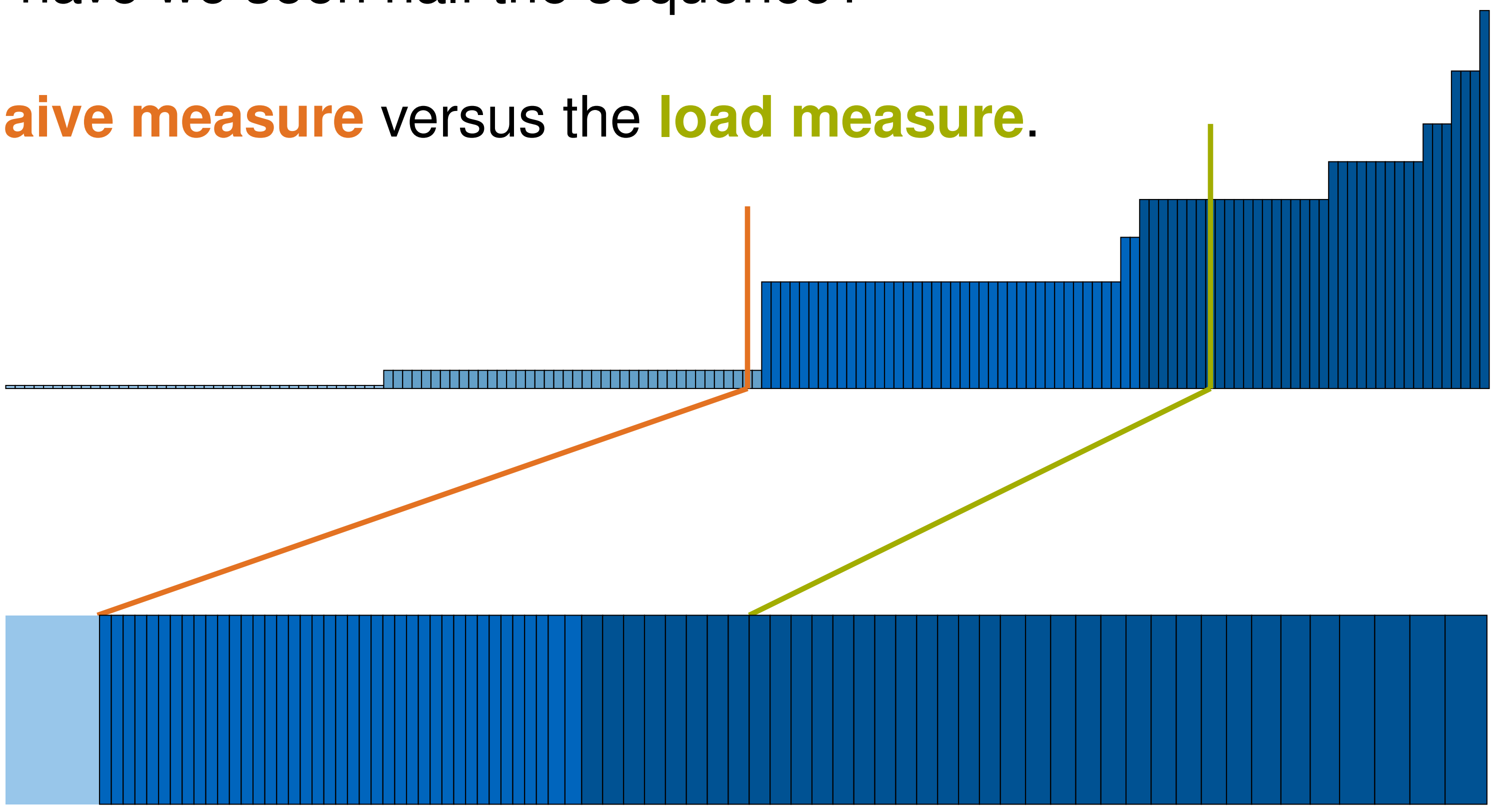

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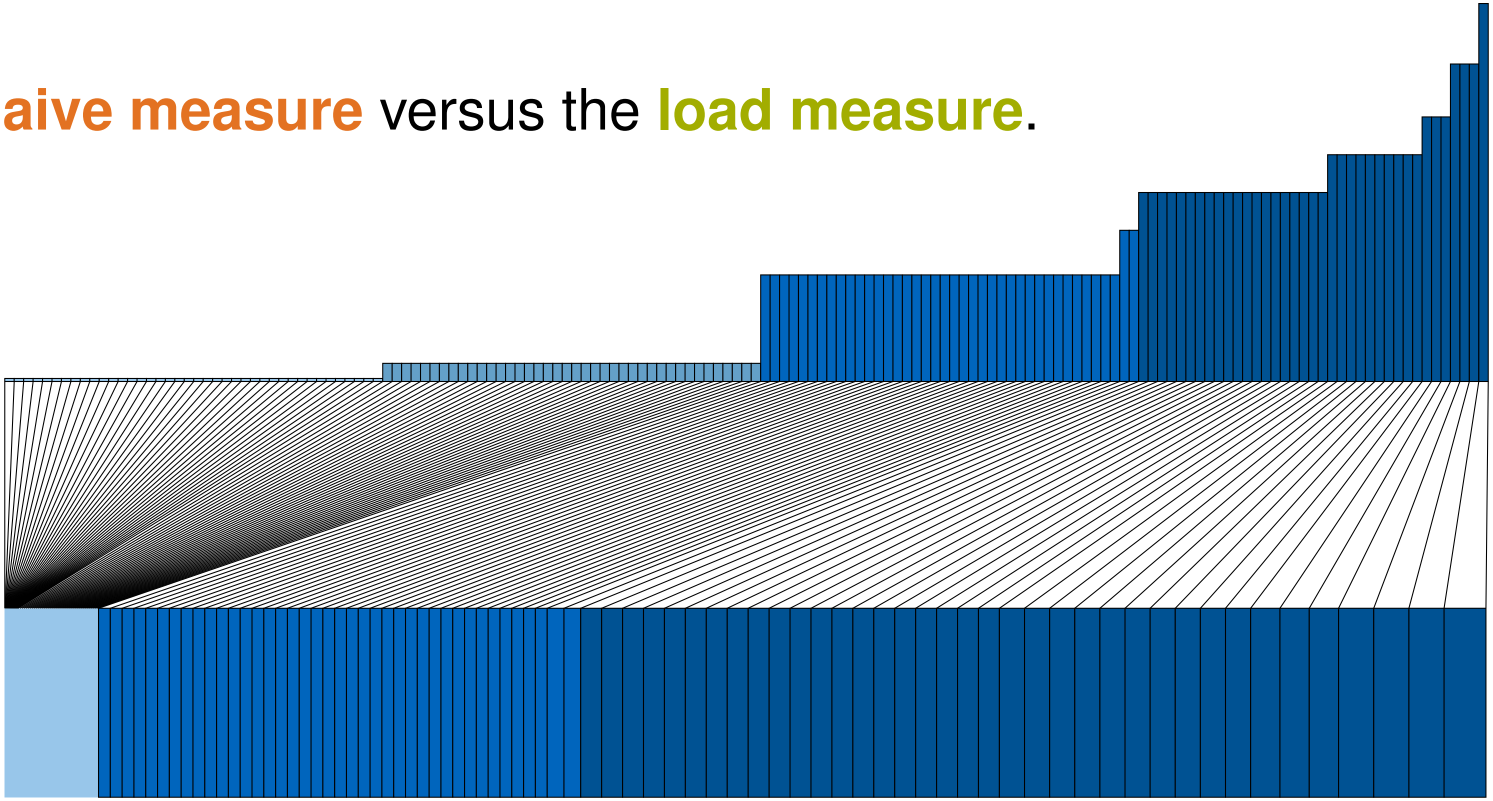
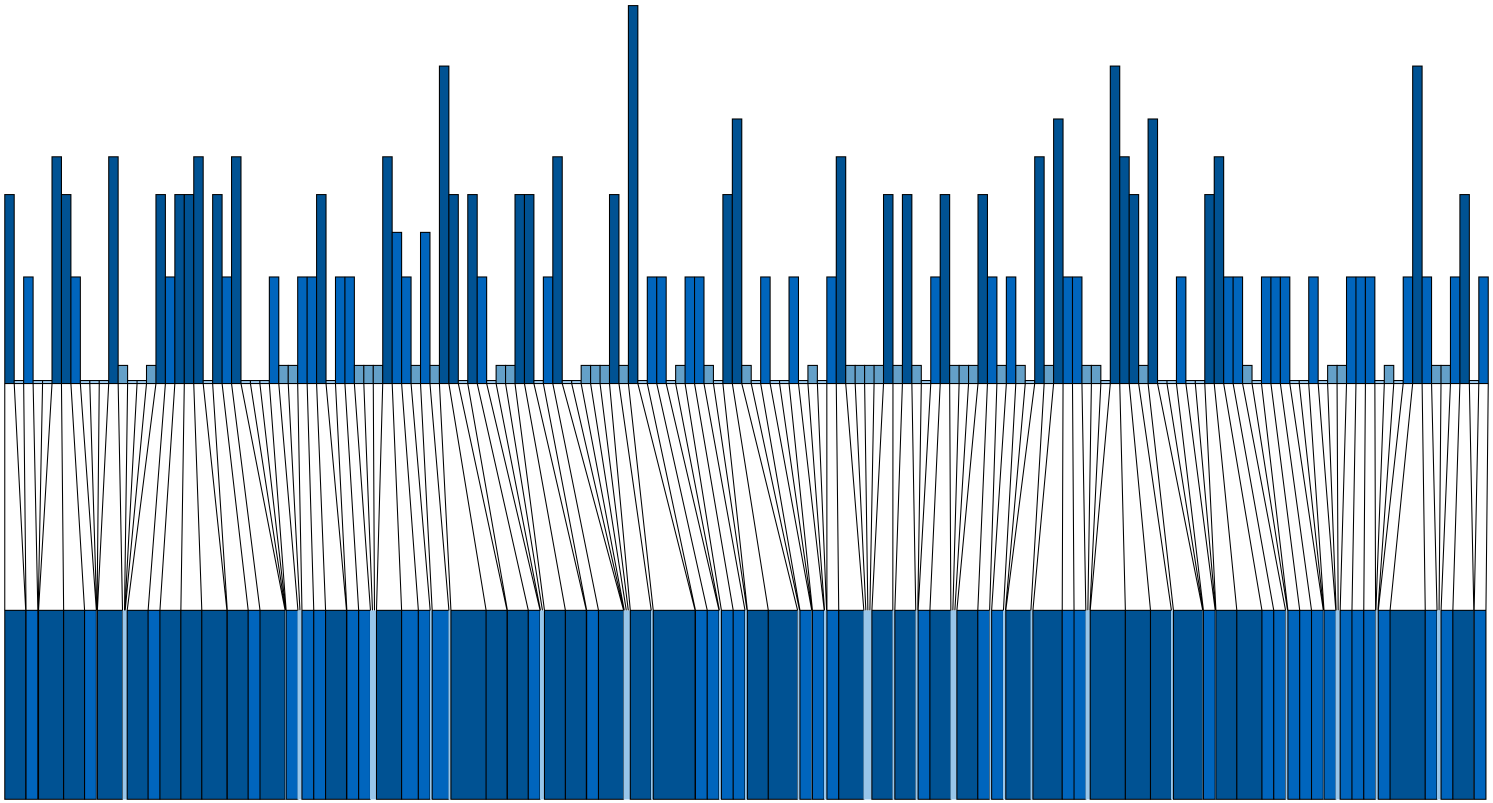


Different time measures.

How does random-order arrival impact the measures?



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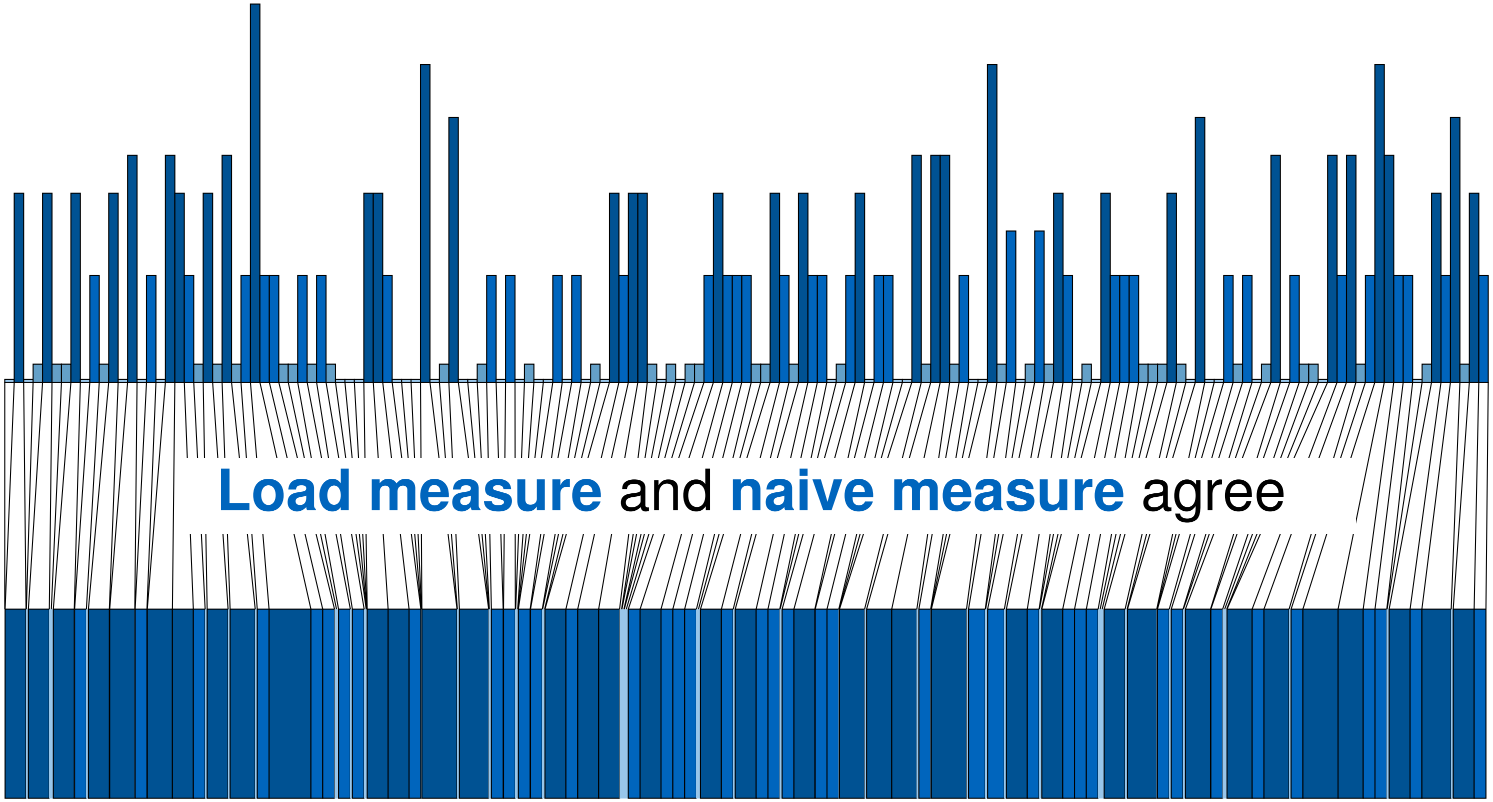

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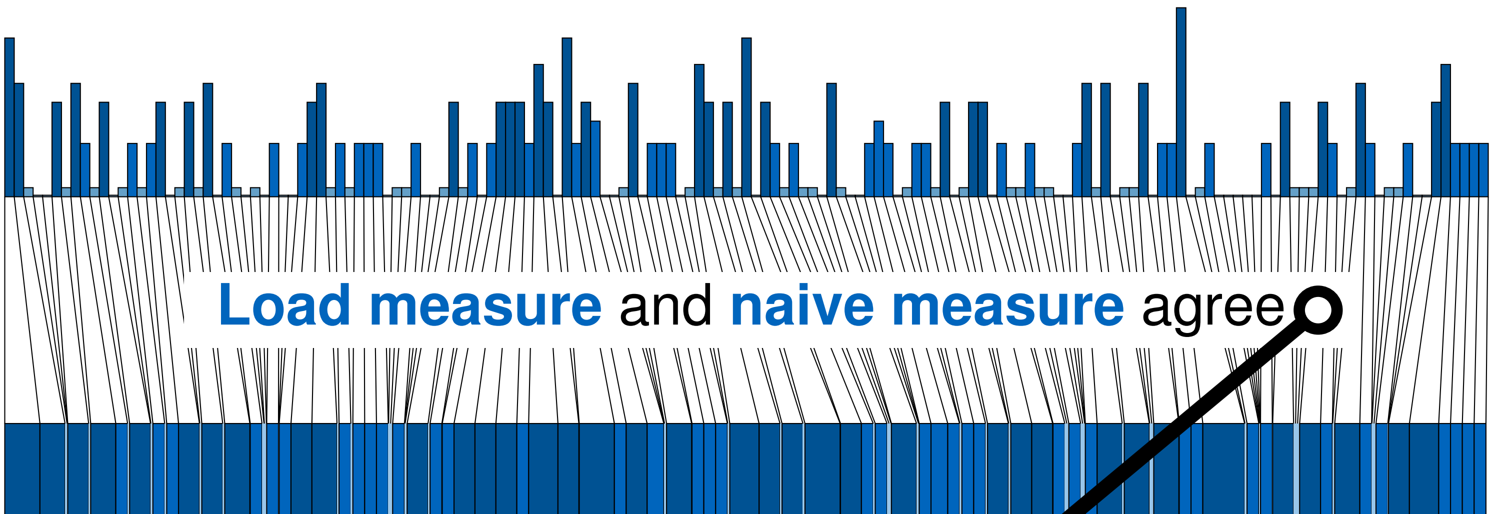


Load measure and **naive measure** agree

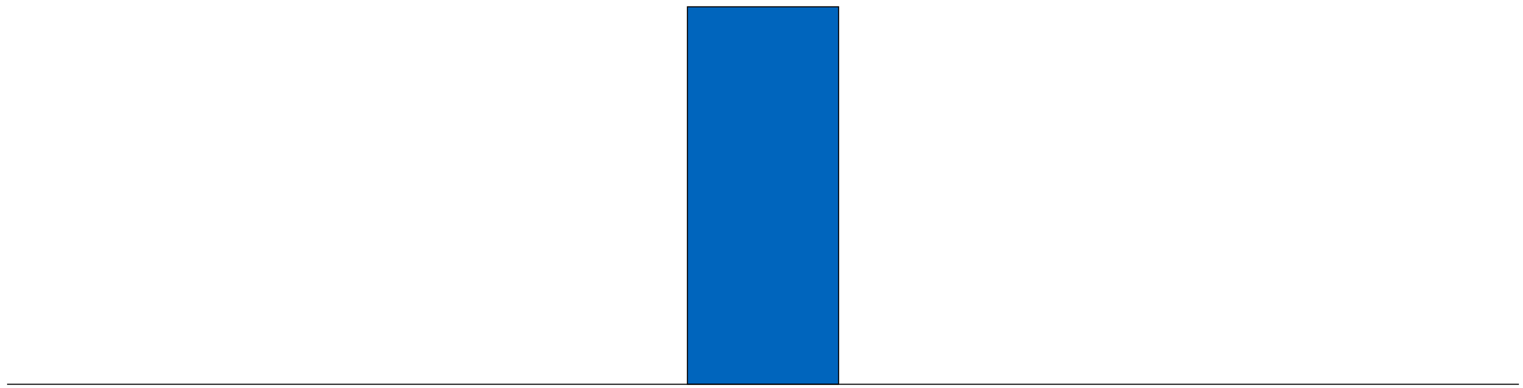
for **m** large

high probability

non-simple inputs

margin of error

How to foil reordering arguments.



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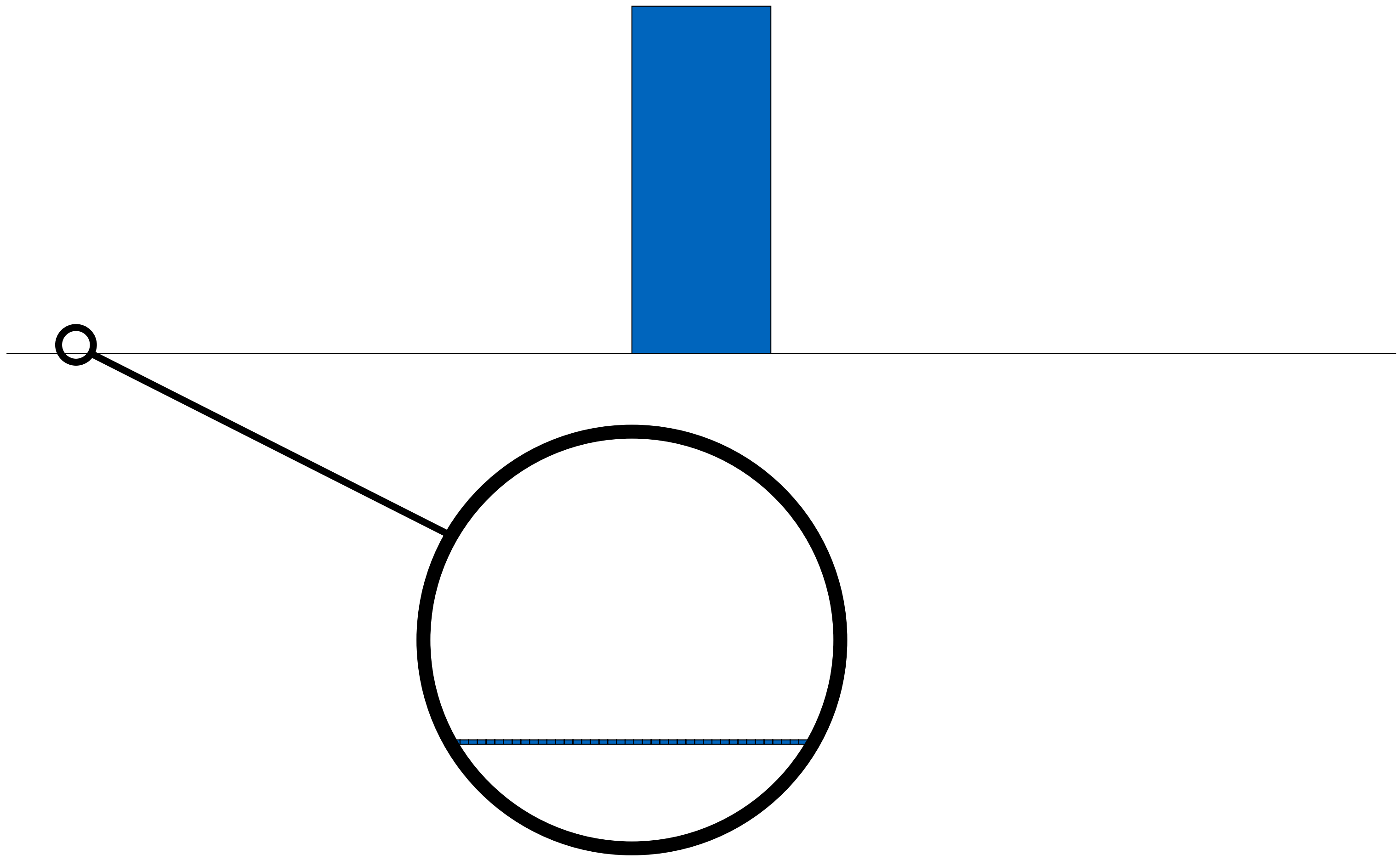

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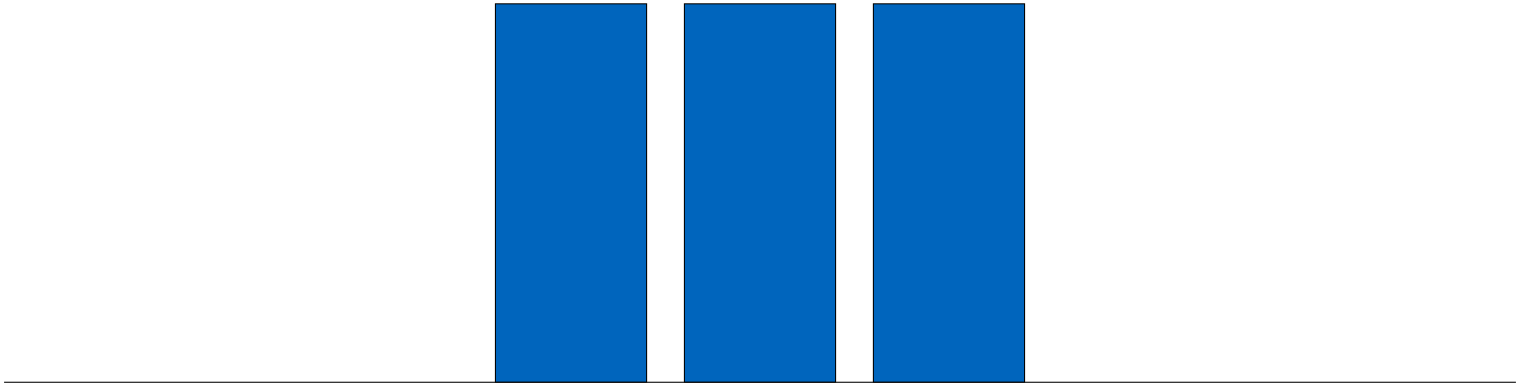
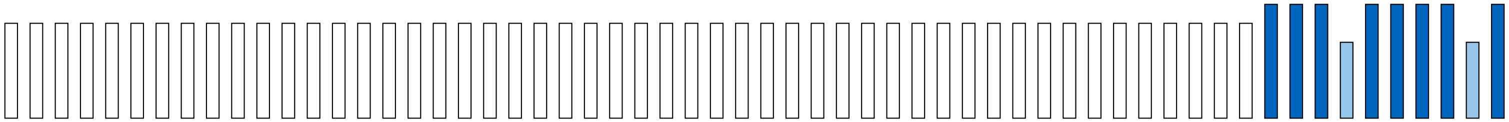


Large Job Magic: Large jobs for free

How to complete the sequence correctly?



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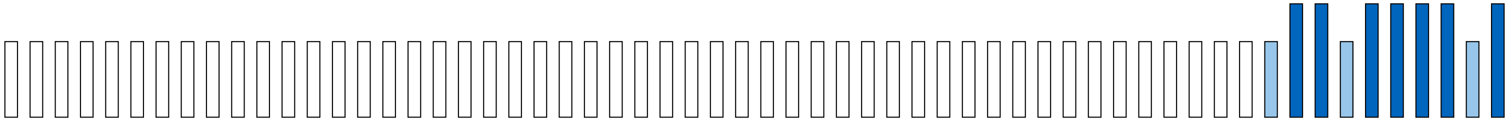
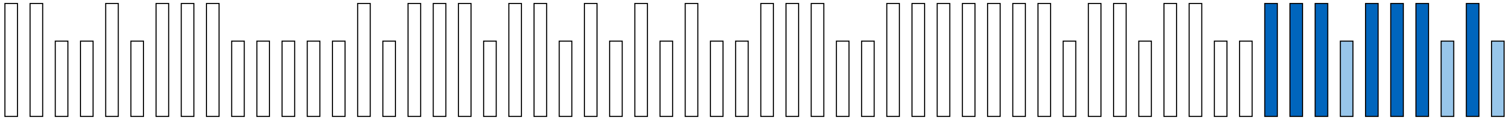


Large Job Magic: Large jobs for free

No high concentration of large jobs.



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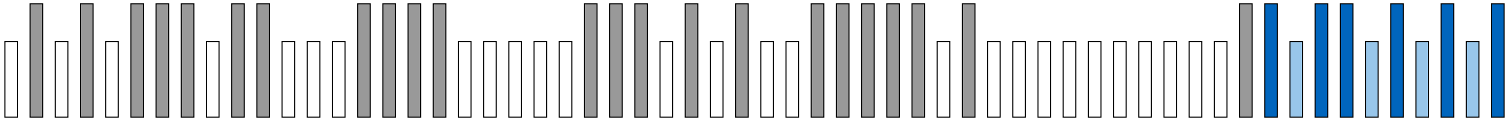
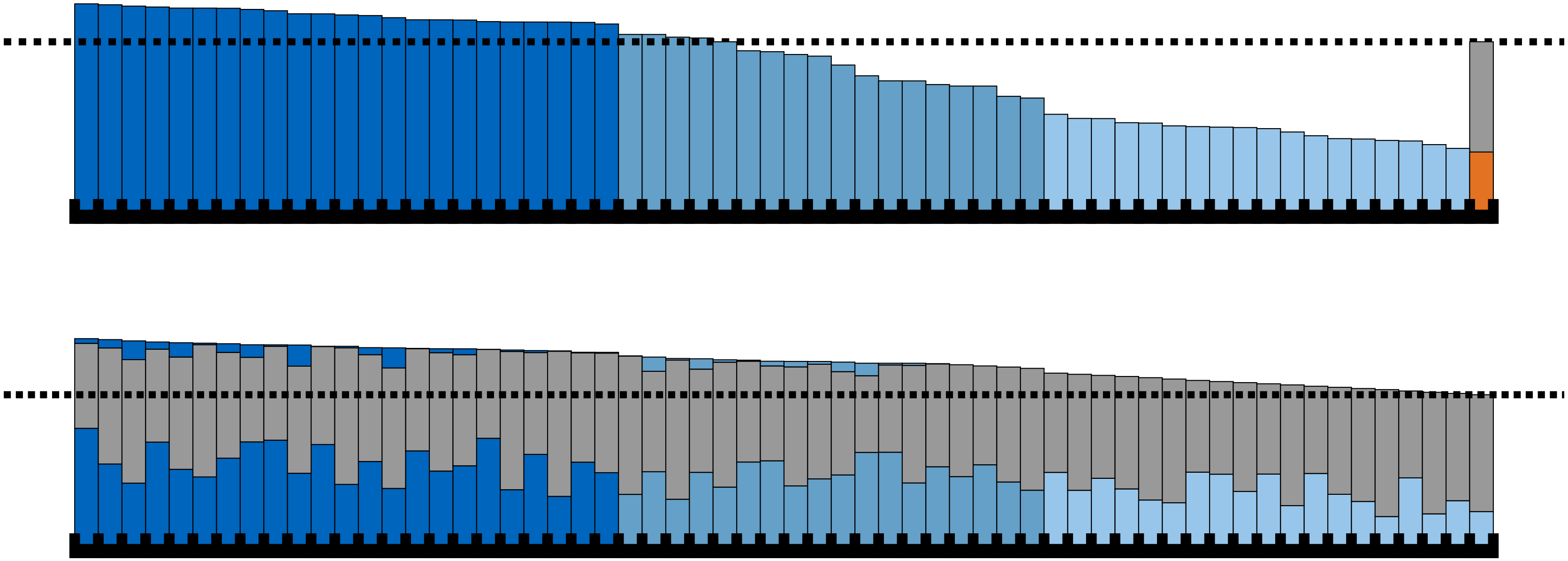
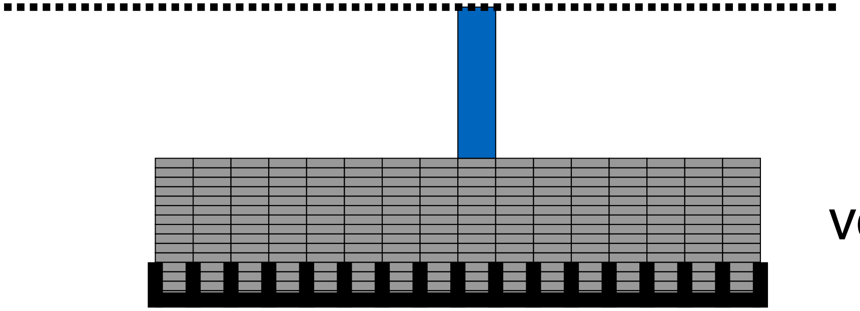


We call it **Large Job Magic**.



Oracle-like properties

A difficult sequence has **large jobs** close to the **end**.



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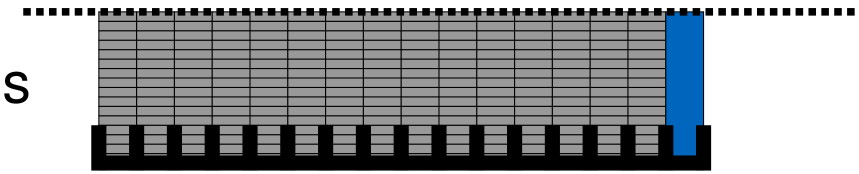

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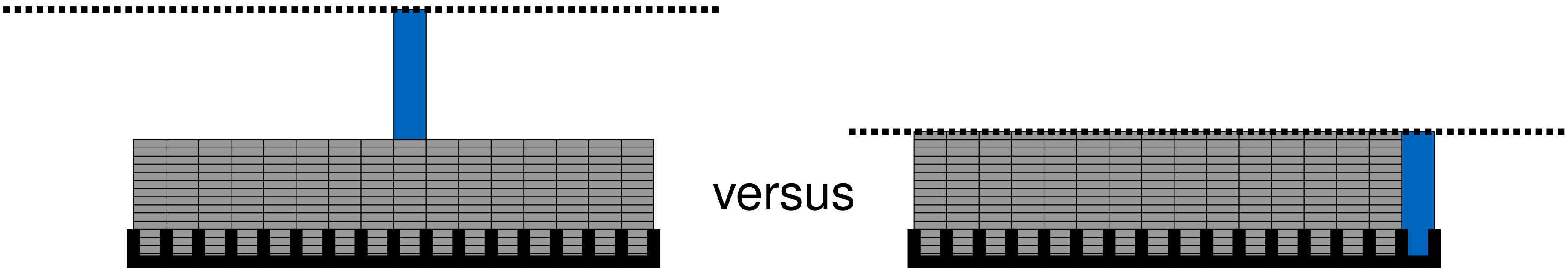

[Osborn and Torng, 2008] Highly probable if there are **many large jobs**.

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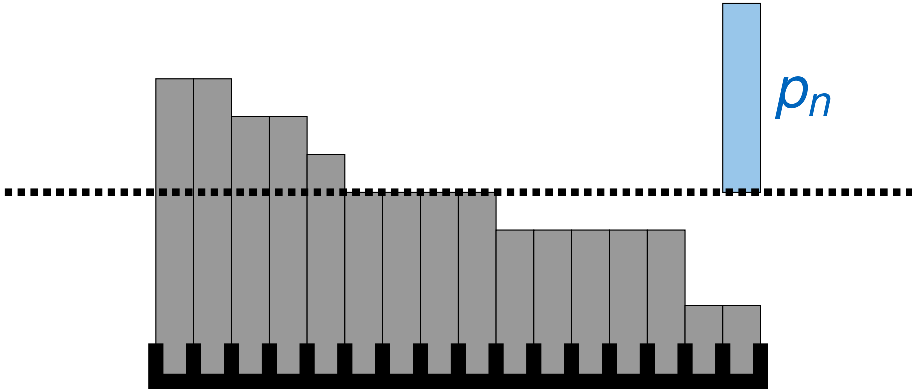

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Many large jobs work as an oracle for OPT.

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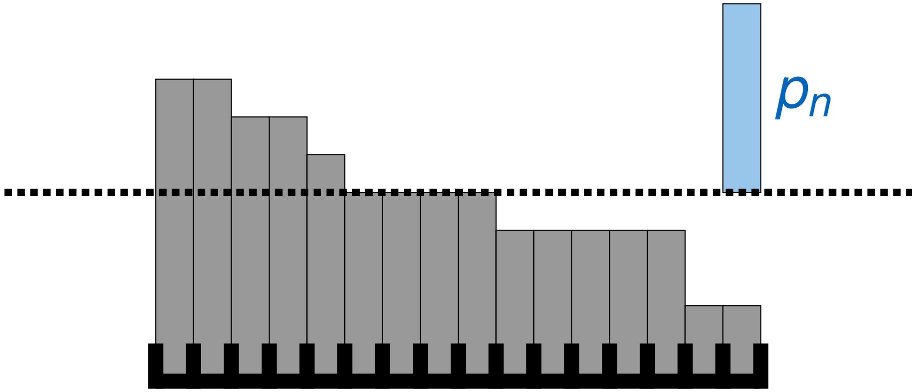

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A competitive ratio of about 1.89 in expectation.

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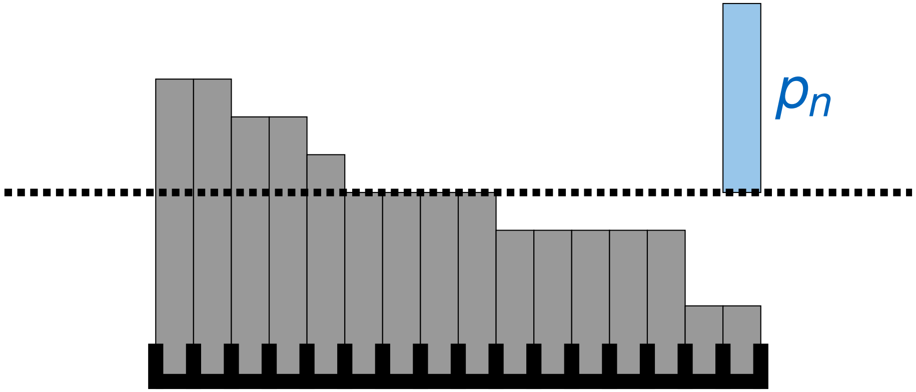

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The difficult part is obtaining such improvement with high probability

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Idea: Make the algorithm robust towards few large jobs.

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Problem: This already requires the **’oracle’** .

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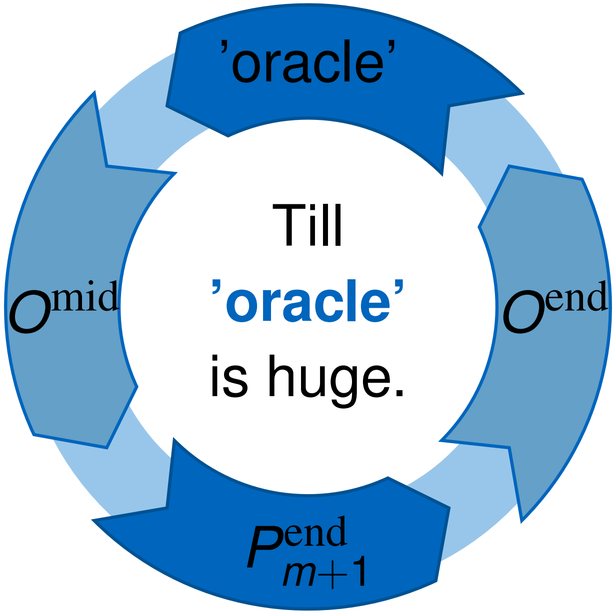
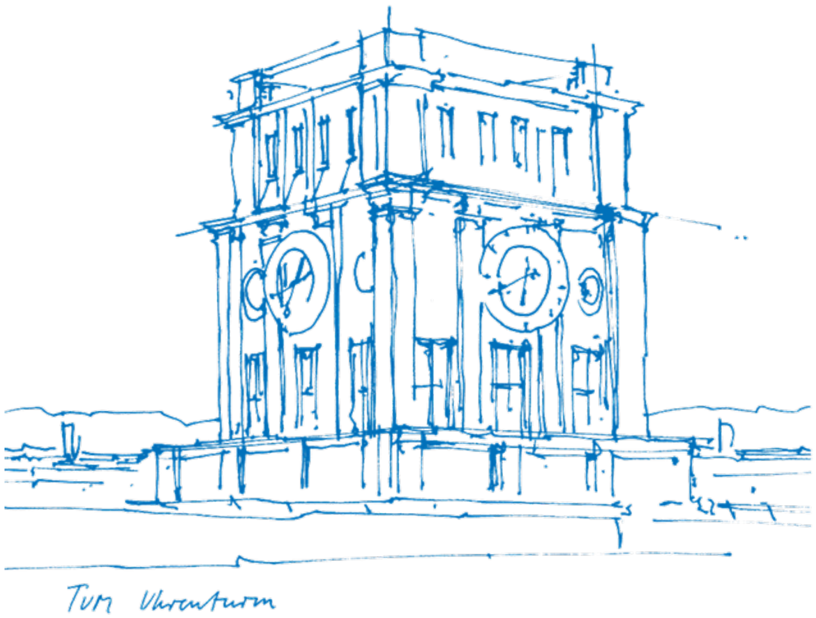
Idea: Make the algorithm robust towards few large jobs.

Problem: This already requires the **’oracle’** .

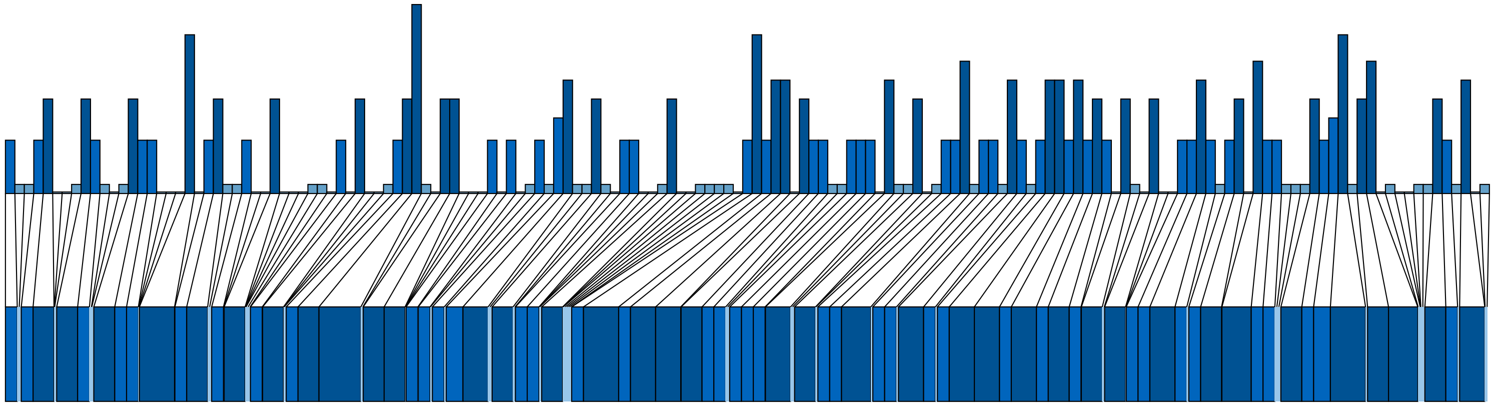


Take-away



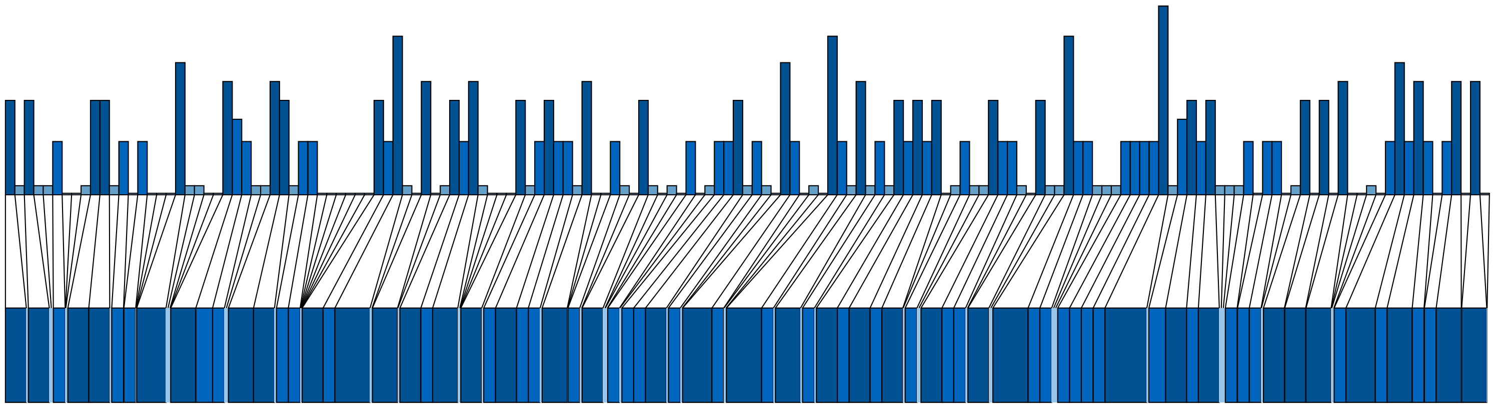
Load Lemma:

Relate algorithmic and probabilistic properties.

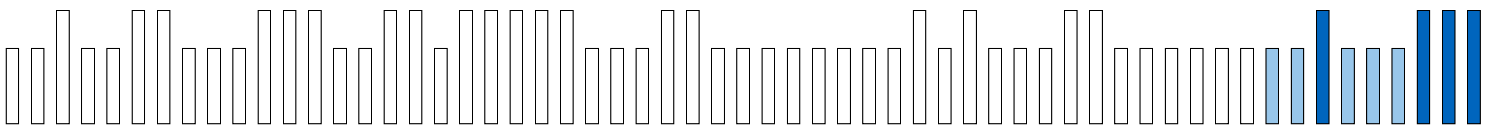


Load Lemma:

Relate algorithmic and probabilistic properties.

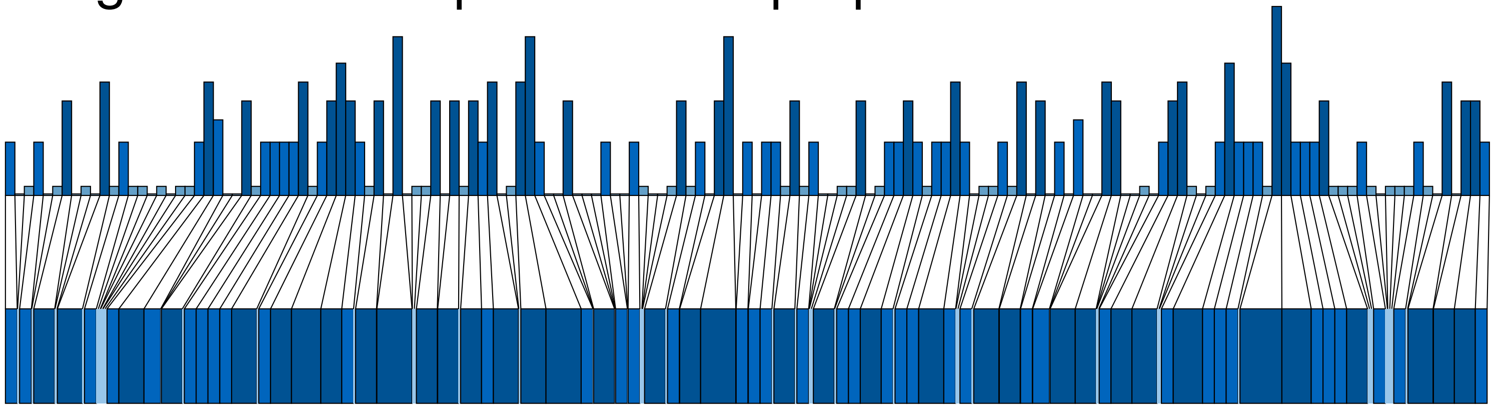


Large Job Magic: Get large jobs for free.

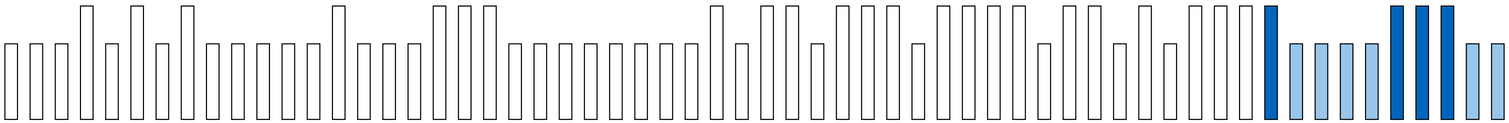


Load Lemma:

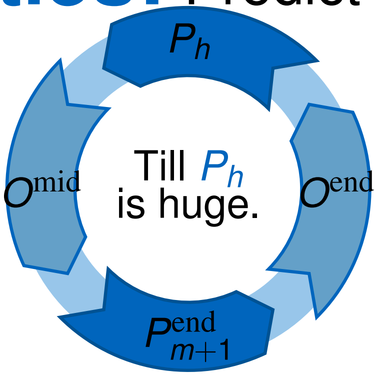
Relate algorithmic and probabilistic properties.



Large Job Magic: Get large jobs for free.



Oracle-like properties: Predict the future.



Thank you!

TUM Uhrenturm