Bilinear Transformation:

We have seen that the following transformation:

$$W = \frac{z-1}{z+1} \; ; \quad z = \frac{1+w}{1-w}$$

maps the unity circle of the z-plane back onto the imaginary axis of the w-plane, and the inside of the unity circle into the LHP

$\Rightarrow$ when we discussed stability, this allowed us to use the more familiar Routh-criterion instead of the Jury-criterion

Unfortunately, as other points are not mapped back into corresponding points of the s-plane, the transformation was of little use for other purposes such as the determination of the margin of stability.

<u>However</u>: Frequency-domain techniques operate on values of $s=j\omega$ or $|z|=1$ only $\Longrightarrow$ here, the transformation may be more useful:

$$z = e^{j\omega T} = \cos\omega T + j\sin\omega T$$

$$\Rightarrow \quad W = \frac{z-1}{z+1} = \frac{\cos\omega T + j\sin\omega T - 1}{(\cos\omega T+1)+j\sin\omega T} \cdot \frac{(\cos\omega T+1)-j\sin\omega T}{(\cos\omega T+1)-j\sin\omega T}$$

$$= \frac{(\cos\omega T)^2 + \cos\omega T - \cos\omega T - 1 + (\sin\omega T)^2 + j\sin\omega T\cos\omega T}{(\cos\omega T+1)^2 + (\sin\omega T)^2}$$

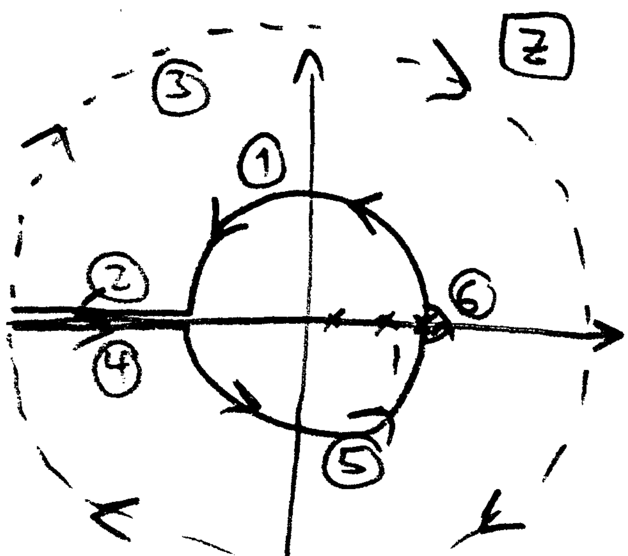
$$\underline{+j\sin\omega T - j\sin\omega T\cos\omega T + j\sin\omega T}$$

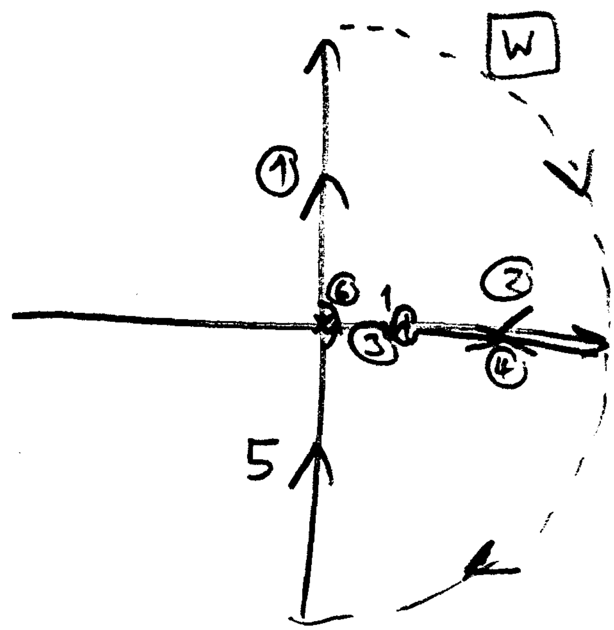
$$= \frac{1 - 1 + 2j\sin\omega T}{1 + 2\cos\omega T + 1}$$

$$= j\,\frac{\sin\omega T}{1 + \cos\omega T} = j \cdot tg\left(\frac{\omega T}{2}\right)$$

$$\Rightarrow \angle z \in [\emptyset, \pi] \Rightarrow w \in j[\emptyset, \infty)$$

$$\angle z \in [-\pi, \emptyset] \Rightarrow w \in j(-\infty, \emptyset]$$





We can use this:

$$w = \sigma_w + j\omega_w \quad [\neq s = \sigma + j\omega]$$

For $z = e^{j\omega T} \longrightarrow w = j\omega_w = j\,tg\left(\frac{\omega T}{2}\right)$

$$\Rightarrow \omega_w = tg\left(\frac{\omega T}{2}\right) = tg\left(\frac{\omega}{\Omega}\cdot\pi\right)$$

$$\Rightarrow \omega \in \left[\phi, \frac{\Omega}{2}\right] \Longrightarrow \omega_w \in \left[\phi, tg\left(\frac{\pi}{2}\right)\right]$$

$$\Rightarrow \omega_w \in [\phi, \infty)$$

$$\Rightarrow z = \frac{1+w}{1-w} = \frac{1+j\omega_w}{1-j\omega_w}$$

Replace in $G(z) \Longrightarrow G(j\omega_w)$

Then simply let ω_w walk from ϕ to infinity $\Longrightarrow$ regular Nyquist diagram.

Contrary to the previous application, this is now <u>very</u> practical.

In the end, the only thing that we still to do is to <u>rescale</u> the graph:

$$\omega = \frac{2}{T} \cdot tg^{-1}(\omega_W) = \frac{\Omega}{\pi} \cdot tg^{-1}(\omega_W)$$

<u>Bode-Diagram</u>:

The major nicety of the continuous Bode-Diagram was the fact that it was so easy to construct from straight lines.

Unfortunately, in the discrete case, this does not work so easily anymore.

However, the bilinear transformation saves our skin.

$E(j\omega_W)$ is a rational (complex) function in ω_W. The "path" is the same as in the continuous case

$\Longrightarrow$ The construction works just the same. In the end, the only thing we need to do is to assign values of ω to the real axis $\Rightarrow$ the real axis has a nonlinear distribution.