

Conversion to Controller-canonical Form

(Identical for continuous and discrete systems).

Algorithm:

(1) Compute the controllability matrix:

$$Q_c = [\underline{g}, \underline{F} \cdot \underline{g}, \underline{F}^2 \cdot \underline{g}, \dots, \underline{F}^{n-1} \cdot \underline{g}]$$

$$Q_c \in \mathbb{R}^{n \times n}$$

(2) Compute its inverse:

$$Q_c^{-1} = \text{inv}(Q_c)$$

(3) Extract the last row:

$$\underline{g}' = Q_c^{-1}(n, :)$$

(4) Build the transformation matrix:

$$T = \begin{bmatrix} \underline{q}' \\ \underline{q}' \cdot F \\ \underline{q}' \cdot F^2 \\ \vdots \\ \underline{q}' \cdot F^{n-1} \end{bmatrix} ; T \in \mathbb{R}^{n \times n}$$

(5) Perform a similarity transformation:

$$\left| \begin{array}{l} \hat{F} = T \cdot F \cdot T^{-1} \\ \hat{G} = T \cdot g \\ \hat{h}' = \underline{h}' \cdot T^{-1} \\ \hat{\lambda} = \lambda \end{array} \right|$$

The resulting system is in controller-canonical form.