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Mathematical Modeling of Physical Systems

## 10<sup>th</sup> Homework – Solution

- In this homework, we shall attempt the modeling of a planar mechanical system.
- We shall do so once by employing the planar mechanical sub-library of *MultiBondLib*, and once by working with a multi-bond graph directly.
- The version using the sub-library can be animated, whereas the direct version cannot be animated.

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- [Description of the problem](#)
- [The ElastoGap model](#)
- [Planar mechanical \(wrapped\) thread-pendulum model](#)
- [Multi-bond graph \(direct\) thread-pendulum model](#)

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## A Thread-Pendulum Model

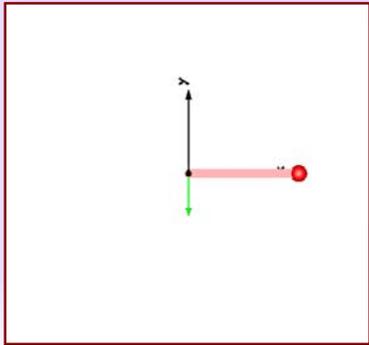
- In this homework problem, we shall be dealing with modeling and simulating a thread-pendulum.
- A thread-pendulum is a variant of the pendulum that we have been discussing previously. Here, the mass hangs on an infinitely thin mass-less thread instead of an infinitely thin rigid bar.
- Consequently, the mass has now two mechanical degrees of freedom rather than one.
- When the mass is at an elevation higher than the origin and is moving at a velocity that is too small, it will switch to a free-fall motion that lasts until the thread is fully extended again.

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## A Thread-Pendulum Model II



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### A Thread-Pendulum Model III

- The thread-pendulum can be described easily using the multi-body systems sub-library of the *Modelica standard library*:

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### A Thread-Pendulum Model IV

- We wish to simulate the pendulum using polar coordinates.
- Hence we start out with a revolute joint around the inertial frame. The *multi-body systems (MBS)* library makes use of the state selection algorithm to get *Dymola* to choose the relative angle and angular velocity as two state variables.
- The model then proceeds with a prismatic joint, adding the second mechanical degree of freedom. The MBS library uses the state selection algorithm to convince *Dymola* to use the relative position and velocity of the prismatic joint as the other two state variables.
- Hence we operate in polar coordinates around the origin (inertial frame).

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### A Thread-Pendulum Model V

- We still need to implement the constraint: the mass of the pendulum cannot move to a position that is farther away from the origin than the length of the thread.
- This constraint is tricky to implement. Let us consider for a moment a train engine impacting with a buffer-stop at a finite velocity.
- If the buffer-stop is infinitely rigid, the remaining kinetic energy of the train would have to be destroyed instantly, which requires an infinite force. This will either damage the locomotive, or the buffer-stop or both.
- Therefore, a real buffer-stop is flexible. It has a stiff spring and a damper built into the stop.

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### The ElastoGap Model

- We might be inclined to model the buffer-stop in the following fashion:

- When the train engine makes contact with the buffer-stop, the switch closes, and the spring/damper system becomes active.

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## The ElastoGap Model II

- Unfortunately, this approach fails.
- If the spring is modeled as a capacitor, then the capacitor only becomes active after contact. This means that the number of differential equations would increase by one at contact, which is something that *Dymola* currently doesn't support.
- Remember that all switches must be placed inside algebraic loops.
- If the spring is modeled as a modulated effort source, the spring won't have positional information available while the switch is open, and therefore cannot compute the spring force.

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## The ElastoGap Model III

- We wish to re-create the thread-pendulum model using the planar mechanical sub-library of *MultiBondLib*.
- Unfortunately, the 1D translational sub-library of *BondLib* doesn't offer an ElastoGap model yet.
- Hence your first task will be to create one.
- Of course, you could simply use the connectors of the 1D translational sub-library of *BondLib* and copy the equation model over from the corresponding sub-library of the *Modelica standard library*.
- However, that would be no fun. You are supposed to create a graphical ElastoGap model.

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## The ElastoGap Model IV

- In order for this to work, the spring/damper system must be continuously engaged. You cannot use a switch.
- One way how this task can be accomplished is by measuring not only the relative position between the two flanges, but also the force into the spring/damper system. You may then apply a pair of additional force sources at the two flanges that compensate the pair of forces (same magnitude, opposite sign) of the spring/damper system, whenever there is no contact, such that the total force at the two flanges adds up to zero.

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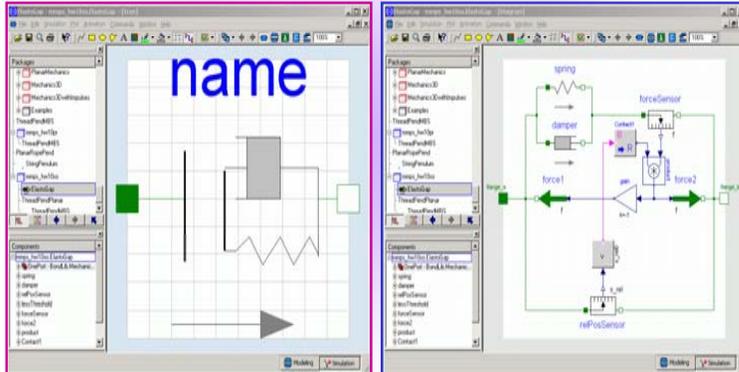
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## The ElastoGap Model V

The diagram illustrates a control system for two flanges, flange\_a and flange\_b. A central block labeled 'Some Logic' is connected to both flanges. On the left, 'force\_a' is applied to flange\_a, and on the right, 'force\_b' is applied to flange\_b. A 'forceSensor' is connected to the 'Some Logic' block, receiving a signal 'f'. A 'relPositionSensor' is also connected to the 'Some Logic' block, receiving a signal 's\_rel'. The 'Some Logic' block outputs 'force\_a' and 'force\_b' to the flanges. A spring/damper system is connected between the two flanges, with a 'spring' and a 'damper' component. The spring/damper system outputs a 'force' signal to the 'Some Logic' block.

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## The ElastoGap Model VI



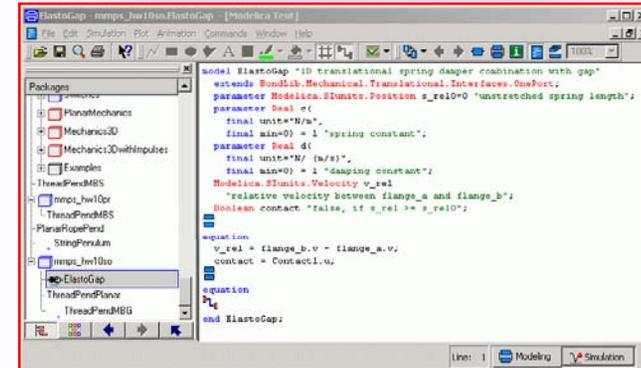
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## The ElastoGap Model VII



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## The Planar Thread-Pendulum Model

- For the thread-pendulum model, we need two ElastoGap models, because in every direction there are always two constraints, i.e., two positions, where the thread is completely stretched.
- You are now to re-create the thread-pendulum model from elements of the planar mechanical sub-library of *MultiBondLib*, and the previously created ElastoGap model.
- You have access to the 3D mechanical model using the MBS library. You can read all of the necessary parameter values out from it.

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## The Planar Thread-Pendulum Model II

- Duplicate the animated planar model and simulate it.
- The MBS library normalizes the angles differently from the planar library, i.e., you'll need to modify two parameters of the model in order to avoid getting a motion that is mirrored at the vertical axis.
- Compare the number of equations before and after optimization obtained by the two models as well as their execution efficiencies.

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## The Planar Thread-Pendulum Model III

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## The Planar Thread-Pendulum Model IV

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The planar solution has initially more equations due to the bond graphs. The compiled model still has more equations due to the graphical *ElastoGap* model.

Standard MBS 3D Library

MBG Mechanical 2D Library

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## The Planar Thread-Pendulum Model V

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The execution speeds of the two models cannot be distinguished.

Standard MBS 3D Library

MBG Mechanical 2D Library

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## The Planar Thread-Pendulum Model VI

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Standard MBS 3D Library

MBG Mechanical 2D Library

## The Multi-bond Graph Thread-Pendulum Model

- You are now supposed to create a third model of the thread-pendulum, this time using a direct multi-bond graph approach, i.e., without wrapping.
- You may formulate the two constraints in the equation window, i.e., you don't need to create a direct bond-graph version of the ElastoGap model.
- This model cannot be animated. Plot the vertical motion of the pendulum mass against its horizontal motion.
- Compare the number of equations and execution speed with the previous two solutions.

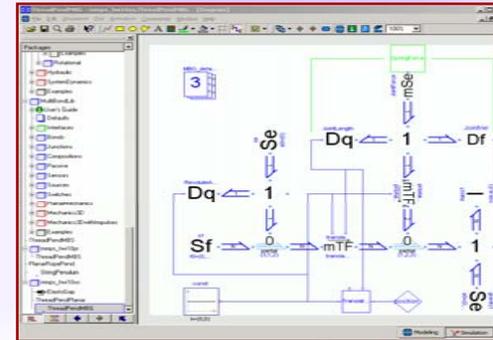
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## The Multi-bond Graph Thread-Pendulum Model II



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## The Multi-bond Graph Thread-Pendulum Model III

```

model ThreadPendMBG "Thread-Pendulum Modeled Using Multi-bond Graph"
  parameter Modelica.SIunits.Acceleration g = 9.81;
  parameter Modelica.SIunits.Mass m = 0.3;
  parameter Modelica.SIunits.Length s = 0.65 "length of string";
  parameter Real c(tunic="N/m") = 5000 "spring coefficient";
  parameter Real d(tunic="N.s/m") = 500 "spring coefficient";
  Modelica.SIunits.Force f1;
  Modelica.SIunits.Force f2;

equation
  f1 = if JointLength.q[1] > s then c*(s-JointLength.q[1]) - d*JointVel.f_out[1] else 0;
  f2 = if JointLength.q[1] < -s then c*(s+JointLength.q[1]) - d*JointVel.f_out[1] else 0;
  JointForce.s[1] = f1 + f2;

equation
  ThreadPendMBG;
end ThreadPendMBG;
  
```

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## The Multi-bond Graph Thread-Pendulum Model IV

Statistics for Standard MBS 3D Library:

- Original Model: 64 components, 773 variables, 302 constraints
- Translated Model: 1128 constraints, 105 free parameters, 122 parameters, 188 outputs
- DAE having 1507 scale equations

Standard MBS 3D Library

Statistics for MBG Library:

- Original Model: 34 components, 302 variables, 105 constraints
- Translated Model: 217 constraints, 12 free parameters, 122 parameters, 188 outputs
- DAE having 290 scale equations

MBG Library (1)

The direct solution contains far fewer equations initially, in part because the wrapping has been avoided, but mostly because of the missing animation. Also the compiled model contains less equations due to the missing animation equations.

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# The Multi-bond Graph Thread-Pendulum Model V

```

System Error | Translation | Debug Error | Simulation
-----
[Log] File of program: /opt/mbs3d/.../DynaSim
(Generated: Mon Nov 24 18:33:41 2007)

DynaSim started
... "data.txt" loading (DynaSim input file)
... "ThreadPendMBG.mat" creating (simulation result file)
Integration started at T = 0 using integration method DASSL
(DAS multi-step solver (dassl/dassl) of SUNDIALS modified by Dynasim)
Integration terminated successfully at T = 0.000000
CPU-time for integration: 0.00 seconds
CPU-time for one GBG iteration: 0.000000 seconds
Number of event points: 111
Number of GBG points: 501
Number of successful steps: 362
Number of F-evaluations: 1008
Number of H-evaluations: 929
Number of function-evaluations: 78
Number of (model) time events: 0
Number of (D) time events: 0
Number of state events: 8
Number of step events: 0
Maximum integration stepsize: 7.48e-008
Maximum integration stepsize: 0.0739
Maximum integration order: 8
Calling terminal section
... "data.txt" creating (final states)
    
```

```

System Error | Translation | Debug Error | Simulation
-----
[Log] File of program: /opt/mbs3d/.../DynaSim
(Generated: Mon Nov 24 18:41:39 2007)

DynaSim started
... "data.txt" loading (DynaSim input file)
... "ThreadPendMBG.mat" creating (simulation result file)
Integration started at T = 0 using integration method DASSL
(DAS multi-step solver (dassl/dassl) of SUNDIALS modified by Dynasim)
Integration terminated successfully at T = 0.000000
CPU-time for integration: 0.00 seconds
CPU-time for one GBG iteration: 0.000000 seconds
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Calling terminal section
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```

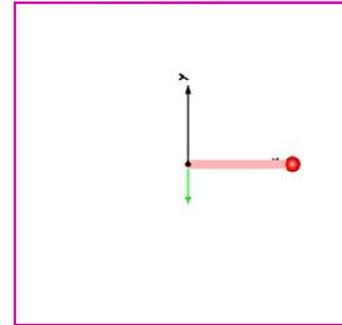
The execution speeds of the three models cannot be distinguished.

Standard MBS 3D Library

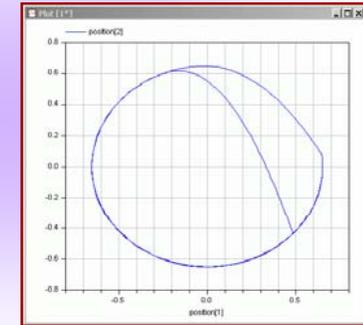
MBG Library (Direct)



# The Multi-bond Graph Thread-Pendulum Model VI



Standard MBS 3D Library



MBG Library (Direct)

