

## 13<sup>th</sup> Homework

- In this homework, we shall model and simulate the population dynamics of *Rapa Nui*, an island in the South Pacific.

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## Rapa Nui (Easter Island)



Rapa Nui is a small island of 163.6 km<sup>2</sup> surface located in the South Pacific.

The island was discovered on Easter Sunday 1722 by the Dutch explorer [Jacob Roggeveen](#).

In September 1888, the island was annexed by Chile.

The current population is 3800 people.

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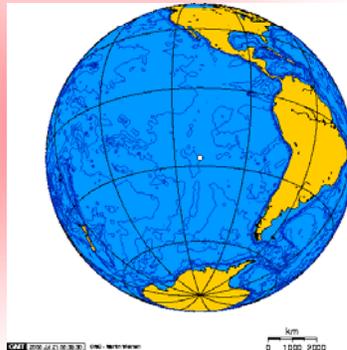
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## Rapa Nui (Easter Island) II

- Rapa Nui is one of the most isolated spots on this globe.
- The island is located 3600 km West of Chile and 2075 km East of the Pitcairn Island, where the descendents of the mutineers of the *Bounty* live, in itself a very isolated island.
- The island is in the Southern subtropical belt.



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## Rapa Nui (Easter Island) III



Rapa Nui is an island of volcanic origin.

When Roggeveen landed, he found a fairly desolate place with only a few thousand inhabitants of Polynesian descent living in poverty.

There were no trees left on the island.



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## Rapa Nui (Easter Island) IV

- Yet, the island seemed to have seen a much more prosperous past, as documented by its rich cultural inheritance.



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## Rapa Nui (Easter Island) V

- What had happened that doomed the once prosperous community?
- Anthropologists around the globe have been discussing this phenomenon for several decades.
- The most commonly offered explanation is the following. A relatively small number of people arrived to the island from Polynesia in catamarans around the year 400. They settled on this island.
- The population grew, and around the year 1200, started developing a rich culture, reflected in their famous stone heads cut out of lava stone.
- The population must have grown to somewhere around 10,000 people in the 16<sup>th</sup> century, but then both the population and the cultural activities started to decline rapidly.

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## Rapa Nui (Easter Island) VI

- The most common explanation for the Easter Island catastrophe is an ecological one.
- The inhabitants had used the local palm trees (*Jubaea chilensis*) to construct durable housing and also for making canoes that they needed for fishing.
- The inhabitants didn't stop cutting trees, until they had cut even the last remaining palm tree around 1600.
- At that time, they had lost the ability to fish efficiently and to construct durable dwellings.
- They had also lost the ability to ever get off their island again.

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## Problem Description

- In this homework problem, we shall attempt to model and simulate the previously outlined population dynamics scenario using the *System Dynamics* library of *Dymola*.
- There are two state variables: the population, and the number of trees.
- When the Polynesians arrived at the year 400, they numbered 24 people, and there were 40,000 trees on the island.
- We shall simulate the model using years as time units, starting in the year 400 and ending in the year 1900.

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## Birth Rate

- The birth rate is proportional to the population.
- We assume that 50% of the population are females.
- Since the life expectancy wasn't very high and these peoples started reproducing early in life, we assume that 75% of the females are of child-bearing age.
- We shall assume further that, on average, each woman gives birth to five children.
- We estimate the reproductive life span to be 26 years.
- This suffices to calculate the proportionality constant of the birth rate.

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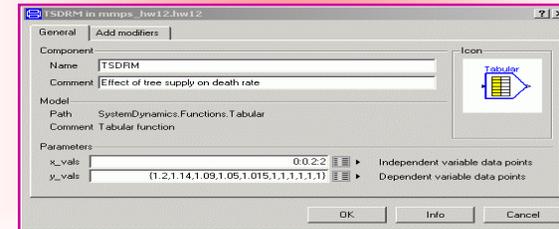
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## Death Rate

- The death rate is also proportional to the population.
- The normal annual death rate is assumed to be 67 people out of every 1000.
- However, the mortality rate increases with a shrinking tree ratio. This dependence is described using a tabular function.



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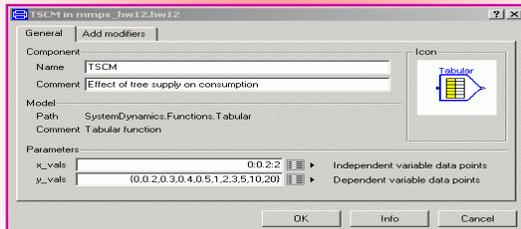
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## Tree Consumption

- We assume that tree reproduction is negligible. The tree population can only decrease.
- The tree consumption is proportional to the population.
- The consumption is however modulated by the available number of trees. The consumption decreases with a shrinking tree ratio.



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## Tree Ratio

- The tree ratio is determined as the quotient of the actual number of trees per person and the desired number of trees per person.
- The desired number of trees per person is set to be 0.5.
- The actual number of trees per person is the smaller of the available number of trees per person and the desired number of trees per person. You can model this using a non-linear two-input function.
- The available number of trees per person is the total number of trees divided by the current population.

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## Experiment Description

- Simulate the model and plot the total population as well as the total number of trees across simulated time.
- The model and experiment specification have been extracted from the book [Modeling Dynamic Systems: Lessons for a First Course, 2<sup>nd</sup> Edition](#) by Diana Fisher. Not every detail of the model description makes sense. For example, the two tables extend to 2.0, although the model logic prevents the independent variable to ever assume a value larger than 1.0.
- Another interesting description of the Easter Island ecological disaster can be found in the book [Sustainable Development for Engineers; A Handbook and Resource Guide](#) by Karel Moulder.

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## Newer Research Results

- A more recent research study questions some of the commonly made assumptions about the Easter Island dilemma.
- Recent archeological research suggests that the island has only been populated since the year 1200. The population expanded much more rapidly than previously assumed, and began almost immediately with the production of cultural monuments.
- Also, the decline of the tree population may not have been caused by human activity alone. The Polynesians brought with them populations of chicken and rats. The rat population exploded, and the rats gnawed on the roots of the palm trees, which may have been at least as much responsible for the decline in tree population as human logging.

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## Newer Research Results II

- Finally, the population was decimated not only by famine, but also by the arrival of the Europeans.
- The Europeans brought with them diseases that the Polynesians had no resistance to.
- Also around 1870, Peruvian slave ships carried away the entire remaining population of the island except for a little over 100 islanders who were able to hide from them.
- This study can be found in a recent edition of the [American Scientist](#) (Vol. 96, Nr. 5, September-October 2006).

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