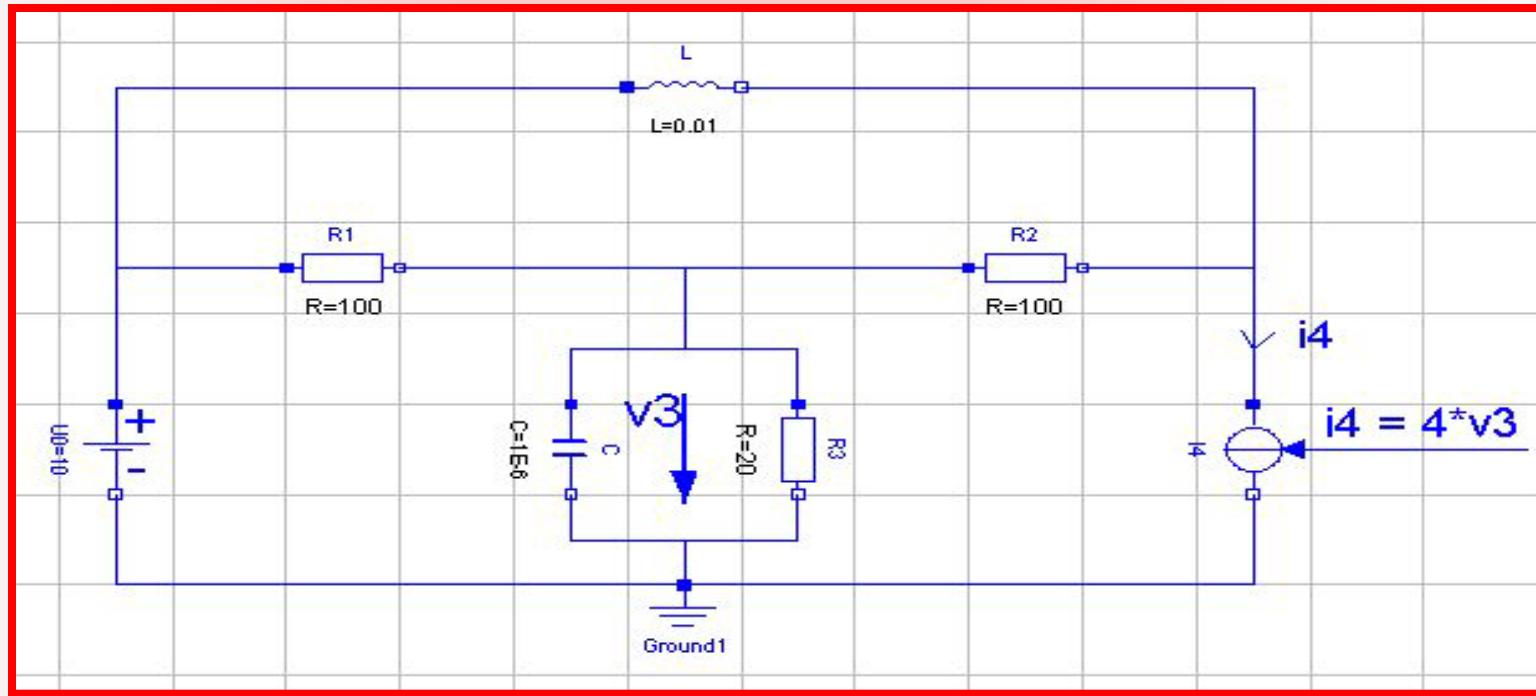


1st Homework Problem

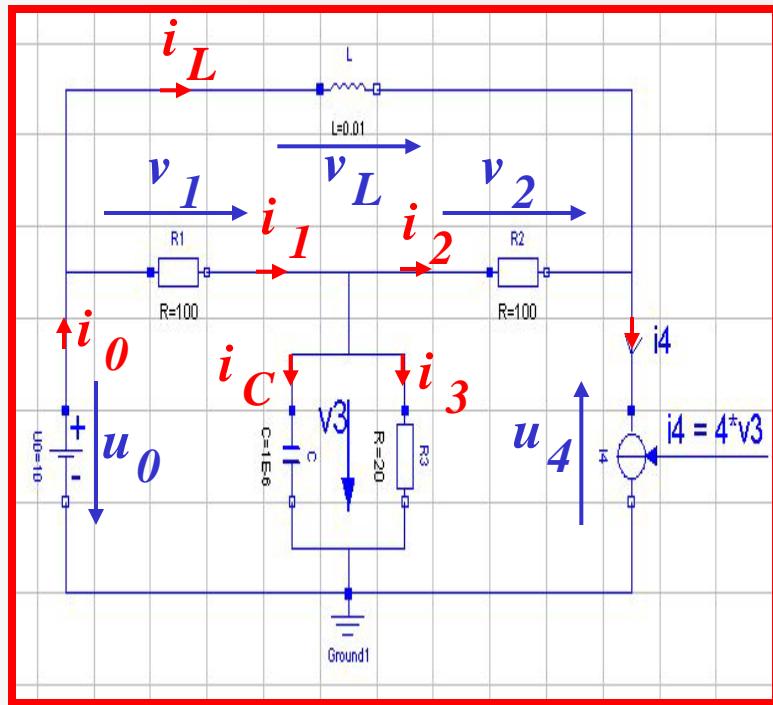
- We wish to analyze the following electrical circuit.



Problems I

1. Write down all of the equations that you need to describe this circuit mathematically. You may use either the technique involving potentials or that involving mesh equations.
2. Sort the equation system horizontally.
3. Sort the horizontally sorted equation system vertically.
4. Convert the horizontally and vertically sorted equation system to state-space form.





Component equations:

$$u_0 = f_I(t)$$

$$v_L = L \cdot \frac{di_L}{dt}$$

$$v_1 = R_1 \cdot i_1$$

$$i_C = C \cdot \frac{dv_3}{dt}$$

$$v_2 = R_2 \cdot i_2$$

$$i_4 = 4 \cdot v_3$$

Mesh equations:

$$u_0 = v_1 + v_3$$

$$v_2 = u_4 + v_3$$

$$v_L = v_1 + v_2$$

Node equations:

$$i_0 = i_1 + i_L$$

$$i_4 = i_L + i_2$$

$$i_1 = i_2 + i_3 + i_C$$

⇒ 13 equations in
13 unknowns



Problems II

1. Write down all of the equations that you need to describe this circuit mathematically. You may use either the technique involving potentials or that involving mesh equations.
2. Sort the equation system horizontally.
3. Sort the horizontally sorted equation system vertically.
4. Convert the horizontally and vertically sorted equation system to state-space form.



Horizontal Sorting I

$$u_0 = f_I(t)$$

$$v_I = R_I \cdot i_I$$

$$v_2 = R_2 \cdot i_2$$

$$v_3 = R_3 \cdot i_3$$

$$v_L = L \cdot \frac{di_L}{dt}$$

$$i_C = C \cdot \frac{dv_3}{dt}$$

$$i_4 = 4 \cdot v_3$$

$$u_0 = v_I + v_3$$

$$v_L = v_I + v_2$$

$$v_2 = u_4 + v_3$$

$$i_0 = i_I + i_L$$

$$i_I = i_2 + i_3 + i_C$$

$$i_4 = i_L + i_2$$



$$u_0 = f_I(t)$$

$$v_I = R_I \cdot i_I$$

$$v_2 = R_2 \cdot i_2$$

$$v_3 = R_3 \cdot i_3$$

$$v_L = L \cdot \frac{di_L}{dt}$$

$$i_C = C \cdot \frac{dv_3}{dt}$$

$$i_4 = 4 \cdot v_3$$

$$u_0 = v_I + v_3$$

$$v_L = v_I + v_2$$

$$v_2 = u_4 + v_3$$

$$i_0 = i_I + i_L$$

$$i_I = i_2 + i_3 + i_C$$

$$i_4 = i_L + i_2$$

Horizontal Sorting II

$$u_0 = f_I(t)$$

$$v_I = R_I \cdot i_I$$

$$v_2 = R_2 \cdot i_2$$

$$v_3 = R_3 \cdot i_3$$

$$v_L = L \cdot \frac{di_L}{dt}$$

$$i_C = C \cdot \frac{dv_3}{dt}$$

$$i_4 = 4 \cdot v_3$$

$$u_0 = v_I + v_3$$

$$v_L = v_I + v_2$$

$$v_2 = u_4 + v_3$$

$$i_0 = i_I + i_L$$

$$i_I = i_2 + i_3 + i_C$$

$$i_4 = i_L + i_2$$



$$\textcolor{red}{u_0} = f_I(t)$$

$$v_I = R_I \cdot i_I$$

$$v_2 = R_2 \cdot i_2$$

$$v_3 = R_3 \cdot \textcolor{red}{i}_3$$

$$v_L = L \cdot \frac{di_L}{dt}$$

$$i_C = C \cdot \frac{dv_3}{dt}$$

$$\textcolor{red}{i}_4 = 4 \cdot v_3$$

$$u_0 = v_I + v_3$$

$$v_L = v_I + v_2$$

$$v_2 = u_4 + v_3$$

$$i_0 = i_I + i_L$$

$$i_I = i_2 + \textcolor{blue}{i}_3 + i_C$$

$$i_4 = \textcolor{blue}{i}_L + i_2$$



Horizontal Sorting III

$$\textcolor{red}{u}_0 = \textcolor{blue}{f}_I(t)$$

$$v_I = R_I \cdot i_1$$

$$v_2 = R_2 \cdot i_2$$

$$v_3 = R_3 \cdot \textcolor{red}{i}_3$$

$$v_L = L \cdot \frac{di_L}{dt}$$

$$i_C = C \cdot \frac{dv_3}{dt}$$

$$\textcolor{red}{i}_4 = 4 \cdot v_3$$

$$\textcolor{blue}{u}_0 = v_I + \textcolor{blue}{v}_3$$

$$v_L = v_I + v_2$$

$$v_2 = u_4 + \textcolor{blue}{v}_3$$

$$i_0 = i_1 + \textcolor{blue}{i}_L$$

$$i_I = i_2 + \textcolor{blue}{i}_3 + i_C$$

$$\textcolor{blue}{i}_4 = \textcolor{blue}{i}_L + i_2$$



$$\textcolor{red}{u}_0 = f_I(t)$$

$$\textcolor{blue}{v}_I = R_I \cdot i_1$$

$$v_2 = R_2 \cdot \textcolor{blue}{i}_2$$

$$v_3 = R_3 \cdot \textcolor{red}{i}_3$$

$$v_L = L \cdot \frac{di_L}{dt}$$

$$i_C = C \cdot \frac{dv_3}{dt}$$

$$\textcolor{red}{i}_4 = 4 \cdot v_3$$

$$\textcolor{blue}{u}_0 = \textcolor{red}{v}_I + v_3$$

$$v_L = \textcolor{blue}{v}_I + v_2$$

$$v_2 = u_4 + \textcolor{blue}{v}_3$$

$$i_0 = \textcolor{blue}{i}_1 + i_L$$

$$i_I = \textcolor{blue}{i}_2 + \textcolor{blue}{i}_3 + i_C$$

$$\textcolor{blue}{i}_4 = \textcolor{blue}{i}_L + \textcolor{red}{i}_2$$



Horizontal Sorting IV

$$\textcolor{red}{u}_0 = \textcolor{blue}{f}_I(t)$$

$$\textcolor{blue}{v}_I = \textcolor{black}{R}_I \cdot \textcolor{blue}{i}_I$$

$$\textcolor{black}{v}_2 = \textcolor{black}{R}_2 \cdot \textcolor{blue}{i}_2$$

$$\textcolor{blue}{v}_3 = \textcolor{black}{R}_3 \cdot \textcolor{red}{i}_3$$

$$\textcolor{black}{v}_L = L \cdot \frac{di_L}{dt}$$

$$\textcolor{black}{i}_C = C \cdot \frac{dv_3}{dt}$$

$$\textcolor{red}{i}_4 = 4 \cdot \textcolor{blue}{v}_3$$

$$\textcolor{blue}{u}_0 = \textcolor{red}{v}_I + \textcolor{blue}{v}_3$$

$$\textcolor{blue}{v}_L = \textcolor{blue}{v}_I + \textcolor{blue}{v}_2$$

$$\textcolor{black}{v}_2 = \textcolor{blue}{u}_4 + \textcolor{blue}{v}_3$$

$$\textcolor{blue}{i}_0 = \textcolor{blue}{i}_I + \textcolor{blue}{i}_L$$

$$\textcolor{blue}{i}_I = \textcolor{blue}{i}_2 + \textcolor{blue}{i}_3 + \textcolor{blue}{i}_C$$

$$\textcolor{blue}{i}_4 = \textcolor{blue}{i}_L + \textcolor{red}{i}_2$$



$$\textcolor{red}{u}_0 = \textcolor{blue}{f}_I(t)$$

$$\textcolor{blue}{v}_I = \textcolor{black}{R}_I \cdot \textcolor{red}{i}_I$$

$$\textcolor{red}{v}_2 = \textcolor{black}{R}_2 \cdot \textcolor{blue}{i}_2$$

$$\textcolor{blue}{v}_3 = \textcolor{black}{R}_3 \cdot \textcolor{red}{i}_3$$

$$\textcolor{black}{v}_L = L \cdot \frac{di_L}{dt}$$

$$\textcolor{black}{i}_C = C \cdot \frac{dv_3}{dt}$$

$$\textcolor{red}{i}_4 = 4 \cdot \textcolor{blue}{v}_3$$

$$\textcolor{blue}{u}_0 = \textcolor{red}{v}_I + \textcolor{blue}{v}_3$$

$$\textcolor{blue}{v}_L = \textcolor{blue}{v}_I + \textcolor{blue}{v}_2$$

$$\textcolor{black}{v}_2 = \textcolor{blue}{u}_4 + \textcolor{blue}{v}_3$$

$$\textcolor{red}{i}_0 = \textcolor{red}{i}_I + \textcolor{blue}{i}_L$$

$$\textcolor{blue}{i}_I = \textcolor{blue}{i}_2 + \textcolor{blue}{i}_3 + \textcolor{blue}{i}_C$$

$$\textcolor{blue}{i}_4 = \textcolor{blue}{i}_L + \textcolor{red}{i}_2$$



Horizontal Sorting V

$$\textcolor{red}{u}_0 = \textcolor{blue}{f}_I(t)$$

$$\textcolor{blue}{v}_I = \textcolor{red}{R}_I \cdot \textcolor{red}{i}_I$$

$$\textcolor{red}{v}_2 = \textcolor{red}{R}_2 \cdot \textcolor{blue}{i}_2$$

$$\textcolor{blue}{v}_3 = \textcolor{red}{R}_3 \cdot \textcolor{red}{i}_3$$

$$\textcolor{blue}{v}_L = L \cdot \frac{di_L}{dt}$$

$$\textcolor{blue}{i}_C = C \cdot \frac{dv_3}{dt}$$

$$\textcolor{red}{i}_4 = 4 \cdot \textcolor{blue}{v}_3$$

$$\textcolor{blue}{u}_0 = \textcolor{red}{v}_I + \textcolor{blue}{v}_3$$

$$\textcolor{blue}{v}_L = \textcolor{red}{v}_I + \textcolor{blue}{v}_2$$

$$\textcolor{blue}{v}_2 = \textcolor{red}{u}_4 + \textcolor{blue}{v}_3$$

$$\textcolor{red}{i}_0 = \textcolor{blue}{i}_I + \textcolor{blue}{i}_L$$

$$\textcolor{blue}{i}_I = \textcolor{blue}{i}_2 + \textcolor{blue}{i}_3 + \textcolor{blue}{i}_C$$

$$\textcolor{blue}{i}_4 = \textcolor{blue}{i}_L + \textcolor{red}{i}_2$$



$$\textcolor{red}{u}_0 = \textcolor{blue}{f}_I(t)$$

$$\textcolor{blue}{v}_I = \textcolor{red}{R}_I \cdot \textcolor{red}{i}_I$$

$$\textcolor{red}{v}_2 = \textcolor{red}{R}_2 \cdot \textcolor{blue}{i}_2$$

$$\textcolor{blue}{v}_3 = \textcolor{red}{R}_3 \cdot \textcolor{red}{i}_3$$

$$\textcolor{blue}{v}_L = L \cdot \frac{di_L}{dt}$$

$$\textcolor{blue}{i}_C = C \cdot \frac{dv_3}{dt}$$

$$\textcolor{red}{i}_4 = 4 \cdot \textcolor{blue}{v}_3$$

$$\textcolor{blue}{u}_0 = \textcolor{red}{v}_I + \textcolor{blue}{v}_3$$

$$\textcolor{red}{v}_L = \textcolor{blue}{v}_I + \textcolor{blue}{v}_2$$

$$\textcolor{blue}{v}_2 = \textcolor{red}{u}_4 + \textcolor{blue}{v}_3$$

$$\textcolor{red}{i}_0 = \textcolor{red}{i}_I + \textcolor{blue}{i}_L$$

$$\textcolor{blue}{i}_I = \textcolor{blue}{i}_2 + \textcolor{blue}{i}_3 + \textcolor{red}{i}_C$$

$$\textcolor{blue}{i}_4 = \textcolor{blue}{i}_L + \textcolor{red}{i}_2$$

Horizontal Sorting VI

$$\textcolor{red}{u}_0 = \textcolor{blue}{f}_I(t)$$

$$\textcolor{blue}{v}_I = \textcolor{red}{R}_I \cdot \textcolor{red}{i}_I$$

$$\textcolor{red}{v}_2 = \textcolor{red}{R}_2 \cdot \textcolor{blue}{i}_2$$

$$\textcolor{blue}{v}_3 = \textcolor{red}{R}_3 \cdot \textcolor{red}{i}_3$$

$$\textcolor{blue}{v}_L = L \cdot \frac{di_L}{dt}$$

$$\textcolor{blue}{i}_C = C \cdot \frac{dv_3}{dt}$$

$$\textcolor{red}{i}_4 = 4 \cdot \textcolor{blue}{v}_3$$

$$\textcolor{blue}{u}_0 = \textcolor{red}{v}_I + \textcolor{blue}{v}_3$$

$$\textcolor{red}{v}_L = \textcolor{blue}{v}_I + \textcolor{blue}{v}_2$$

$$\textcolor{blue}{v}_2 = \textcolor{red}{u}_4 + \textcolor{blue}{v}_3$$

$$\textcolor{red}{i}_0 = \textcolor{blue}{i}_I + \textcolor{blue}{i}_L$$

$$\textcolor{blue}{i}_I = \textcolor{blue}{i}_2 + \textcolor{blue}{i}_3 + \textcolor{red}{i}_C$$

$$\textcolor{blue}{i}_4 = \textcolor{blue}{i}_L + \textcolor{red}{i}_2$$



$$\textcolor{red}{u}_0 = \textcolor{blue}{f}_I(t)$$

$$\textcolor{blue}{v}_I = \textcolor{red}{R}_I \cdot \textcolor{red}{i}_I$$

$$\textcolor{red}{v}_2 = \textcolor{red}{R}_2 \cdot \textcolor{blue}{i}_2$$

$$\textcolor{blue}{v}_3 = \textcolor{red}{R}_3 \cdot \textcolor{red}{i}_3$$

$$\textcolor{blue}{v}_L = L \cdot \frac{di_L}{dt}$$

$$\textcolor{blue}{i}_C = C \cdot \frac{dv_3}{dt}$$

$$\textcolor{red}{i}_4 = 4 \cdot \textcolor{blue}{v}_3$$

$$\textcolor{blue}{u}_0 = \textcolor{red}{v}_I + \textcolor{blue}{v}_3$$

$$\textcolor{red}{v}_L = \textcolor{blue}{v}_I + \textcolor{blue}{v}_2$$

$$\textcolor{blue}{v}_2 = \textcolor{red}{u}_4 + \textcolor{blue}{v}_3$$

$$\textcolor{red}{i}_0 = \textcolor{red}{i}_I + \textcolor{blue}{i}_L$$

$$\textcolor{blue}{i}_I = \textcolor{blue}{i}_2 + \textcolor{blue}{i}_3 + \textcolor{red}{i}_C$$

$$\textcolor{red}{i}_4 = \textcolor{blue}{i}_L + \textcolor{red}{i}_2$$

Horizontal Sorting VII

$$\textcolor{red}{u}_0 = \textcolor{blue}{f}_I(t)$$

$$\textcolor{blue}{v}_I = \textcolor{black}{R}_I \cdot \textcolor{red}{i}_I$$

$$\textcolor{red}{v}_2 = \textcolor{black}{R}_2 \cdot \textcolor{blue}{i}_2$$

$$\textcolor{blue}{v}_3 = \textcolor{black}{R}_3 \cdot \textcolor{red}{i}_3$$

$$\textcolor{blue}{v}_L = L \cdot \frac{di_L}{dt}$$

$$\textcolor{blue}{i}_C = C \cdot \frac{dv_3}{dt}$$

$$\textcolor{red}{i}_4 = 4 \cdot \textcolor{blue}{v}_3$$

$$\textcolor{blue}{u}_0 = \textcolor{red}{v}_I + \textcolor{blue}{v}_3$$

$$\textcolor{red}{v}_L = \textcolor{blue}{v}_I + \textcolor{blue}{v}_2$$

$$\textcolor{blue}{v}_2 = \textcolor{red}{u}_4 + \textcolor{blue}{v}_3$$

$$\textcolor{red}{i}_0 = \textcolor{blue}{i}_I + \textcolor{blue}{i}_L$$

$$\textcolor{blue}{i}_I = \textcolor{blue}{i}_2 + \textcolor{blue}{i}_3 + \textcolor{red}{i}_C$$

$$\textcolor{blue}{i}_4 = \textcolor{blue}{i}_L + \textcolor{red}{i}_2$$



$$\textcolor{red}{u}_0 = \textcolor{blue}{f}_I(t)$$

$$\textcolor{red}{i}_I = \textcolor{blue}{v}_I / \textcolor{black}{R}_I$$

$$\textcolor{red}{v}_2 = \textcolor{black}{R}_2 \cdot \textcolor{blue}{i}_2$$

$$\textcolor{red}{i}_3 = \textcolor{blue}{v}_3 / \textcolor{black}{R}_3$$

$$\frac{di_L}{dt} = \textcolor{blue}{v}_L / L$$

$$\frac{dv_3}{dt} = \textcolor{blue}{i}_C / C$$

$$\textcolor{red}{i}_4 = 4 \cdot \textcolor{blue}{v}_3$$

$$\textcolor{red}{v}_I = \textcolor{blue}{u}_0 - \textcolor{blue}{v}_3$$

$$\textcolor{red}{v}_L = \textcolor{blue}{v}_I + \textcolor{blue}{v}_2$$

$$\textcolor{red}{u}_4 = \textcolor{blue}{v}_2 - \textcolor{blue}{v}_3$$

$$\textcolor{red}{i}_0 = \textcolor{blue}{i}_I + \textcolor{blue}{i}_L$$

$$\textcolor{red}{i}_C = \textcolor{blue}{i}_I - \textcolor{blue}{i}_2 - \textcolor{blue}{i}_3$$

$$\textcolor{red}{i}_2 = \textcolor{blue}{i}_4 - \textcolor{blue}{i}_L$$

Problems III

1. Write down all of the equations that you need to describe this circuit mathematically. You may use either the technique involving potentials or that involving mesh equations.
2. Sort the equation system horizontally.
3. Sort the horizontally sorted equation system vertically.
4. Convert the horizontally and vertically sorted equation system to state-space form.



Vertical Sorting I

$u_0 = f_I(t)$	$v_I = u_0 - v_3$
$i_I = v_I / R_I$	$v_L = v_I + v_2$
$v_2 = R_2 \cdot i_2$	$u_4 = v_2 - v_3$
$i_3 = v_3 / R_3$	$i_0 = i_I + i_L$
$\frac{di_L}{dt} = v_L / L$	$i_C = i_I - i_2 - i_3$
$\frac{dv_3}{dt} = i_C / C$	$i_2 = i_4 - i_L$
$i_4 = 4 \cdot v_3$	



$u_0 = f_I(t)$	$v_I = u_0 - v_3$
$i_3 = v_3 / R_3$	$v_L = v_I + v_2$
$i_4 = 4 \cdot v_3$	$u_4 = v_2 - v_3$
$i_I = v_I / R_I$	$i_0 = i_I + i_L$
$v_2 = R_2 \cdot i_2$	$i_C = i_I - i_2 - i_3$
$\frac{di_L}{dt} = v_L / L$	$i_2 = i_4 - i_L$
$\frac{dv_3}{dt} = i_C / C$	

Vertical Sorting II

$$\textcolor{red}{u_0} = f_I(t)$$

$$\textcolor{red}{i_3} = \textcolor{blue}{v_3}/R_3$$

$$\textcolor{red}{i_4} = 4 \cdot \textcolor{blue}{v_3}$$

$$i_1 = v_I/R_1$$

$$v_2 = R_2 \cdot i_2$$

$$\frac{di_L}{dt} = v_L/L$$

$$\frac{dv_3}{dt} = i_C/C$$

$$v_I = \textcolor{blue}{u}_0 - \textcolor{blue}{v}_3$$

$$v_L = v_I + v_2$$

$$u_4 = v_2 - \textcolor{blue}{v}_3$$

$$i_0 = i_1 + \textcolor{blue}{i}_L$$

$$i_C = i_1 - i_2 - \textcolor{blue}{i}_3$$

$$i_2 = \textcolor{blue}{i}_4 - i_L$$



$$\textcolor{red}{u_0} = f_I(t)$$

$$\textcolor{red}{i_3} = \textcolor{blue}{v}_3/R_3$$

$$\textcolor{red}{i_4} = 4 \cdot \textcolor{blue}{v}_3$$

$$\textcolor{red}{v}_I = \textcolor{blue}{u}_0 - v_3$$

$$\textcolor{red}{i}_2 = \textcolor{blue}{i}_4 - i_L$$

$$i_1 = \textcolor{blue}{v}_I/R_1$$

$$v_2 = R_2 \cdot i_2$$

$$\frac{di_L}{dt} = v_L/L$$

$$\frac{dv_3}{dt} = i_C/C$$

$$v_L = \textcolor{blue}{v}_I + v_2$$

$$u_4 = v_2 - \textcolor{blue}{v}_3$$

$$i_0 = i_1 + \textcolor{blue}{i}_L$$

$$i_C = i_1 - i_2 - \textcolor{blue}{i}_3$$



Vertical Sorting III

$$\textcolor{red}{u}_0 = f_I(t)$$

$$\textcolor{red}{i}_3 = \textcolor{blue}{v}_3 / R_3$$

$$\textcolor{red}{i}_4 = 4 \cdot \textcolor{blue}{v}_3$$

$$\textcolor{red}{v}_I = \textcolor{blue}{u}_0 - \textcolor{blue}{v}_3$$

$$\textcolor{red}{i}_2 = \textcolor{blue}{i}_4 - \textcolor{blue}{i}_L$$

$$\textcolor{blue}{i}_1 = \textcolor{blue}{v}_I / R_I$$

$$\textcolor{blue}{v}_2 = R_2 \cdot \textcolor{blue}{i}_2$$

$$\frac{di_L}{dt} = v_L / L$$

$$\frac{dv_3}{dt} = i_C / C$$

$$v_L = \textcolor{blue}{v}_I + \textcolor{blue}{v}_2$$

$$u_4 = \textcolor{blue}{v}_2 - \textcolor{blue}{v}_3$$

$$i_0 = \textcolor{blue}{i}_1 + \textcolor{blue}{i}_L$$

$$i_C = \textcolor{blue}{i}_1 - \textcolor{blue}{i}_2 - \textcolor{blue}{i}_3$$



$$\textcolor{red}{u}_0 = f_I(t)$$

$$\textcolor{red}{i}_3 = \textcolor{blue}{v}_3 / R_3$$

$$\textcolor{red}{i}_4 = 4 \cdot \textcolor{blue}{v}_3$$

$$\textcolor{red}{v}_I = \textcolor{blue}{u}_0 - \textcolor{blue}{v}_3$$

$$\textcolor{red}{i}_2 = \textcolor{blue}{i}_4 - \textcolor{blue}{i}_L$$

$$\textcolor{blue}{i}_1 = \textcolor{blue}{v}_I / R_I$$

$$\textcolor{red}{v}_2 = R_2 \cdot \textcolor{blue}{i}_2$$

$$\frac{di_L}{dt} = v_L / L$$

$$\frac{dv_3}{dt} = i_C / C$$

$$v_L = \textcolor{blue}{v}_I + \textcolor{blue}{v}_2$$

$$u_4 = \textcolor{blue}{v}_2 - \textcolor{blue}{v}_3$$

$$i_0 = \textcolor{blue}{i}_1 + \textcolor{blue}{i}_L$$

$$i_C = \textcolor{blue}{i}_1 - \textcolor{blue}{i}_2 - \textcolor{blue}{i}_3$$



Vertical Sorting IV

$$\textcolor{red}{u}_0 = f_I(t)$$

$$\textcolor{red}{i}_3 = \textcolor{blue}{v}_3 / R_3$$

$$\textcolor{red}{i}_4 = 4 \cdot \textcolor{blue}{v}_3$$

$$\textcolor{red}{v}_I = u_0 - v_3$$

$$\textcolor{red}{i}_2 = i_4 - i_L$$

$$\textcolor{red}{i}_I = \textcolor{blue}{v}_I / R_I$$

$$\textcolor{red}{v}_2 = R_2 \cdot i_2$$

$$\frac{di_L}{dt} = v_L / L$$

$$\frac{dv_3}{dt} = i_C / C$$

$$v_L = \textcolor{blue}{v}_I + v_2$$

$$u_4 = \textcolor{blue}{v}_2 - v_3$$

$$i_0 = \textcolor{blue}{i}_I + i_L$$

$$i_C = \textcolor{blue}{i}_I - i_2 - i_3$$



$$\textcolor{red}{u}_0 = f_I(t)$$

$$\textcolor{red}{i}_3 = \textcolor{blue}{v}_3 / R_3$$

$$\textcolor{red}{i}_4 = 4 \cdot \textcolor{blue}{v}_3$$

$$\textcolor{red}{v}_I = u_0 - v_3$$

$$\textcolor{red}{i}_2 = i_4 - i_L$$

$$\textcolor{red}{i}_I = \textcolor{blue}{v}_I / R_I$$

$$\textcolor{red}{v}_2 = R_2 \cdot i_2$$

$$\textcolor{red}{v}_L = \textcolor{blue}{v}_I + v_2$$

$$\textcolor{red}{u}_4 = v_2 - v_3$$

$$\textcolor{red}{i}_0 = \textcolor{blue}{i}_I + i_L$$

$$\textcolor{red}{i}_C = \textcolor{blue}{i}_I - i_2 - i_3$$

$$\frac{di_L}{dt} = v_L / L$$

$$\frac{dv_3}{dt} = i_C / C$$



Vertical Sorting V

$$\textcolor{red}{u}_0 = f_1(t)$$

$$\textcolor{red}{i}_3 = \textcolor{blue}{v}_3 / R_3$$

$$\textcolor{red}{i}_4 = 4 \cdot \textcolor{blue}{v}_3$$

$$\textcolor{red}{v}_1 = u_0 - \textcolor{blue}{v}_3$$

$$\textcolor{red}{i}_2 = i_4 - i_L$$

$$\textcolor{red}{i}_1 = \textcolor{blue}{v}_1 / R_1$$

$$\textcolor{red}{v}_2 = R_2 \cdot \textcolor{blue}{i}_2$$

$$\textcolor{red}{v}_L = \textcolor{blue}{v}_1 + \textcolor{blue}{v}_2$$

$$\textcolor{red}{u}_4 = \textcolor{blue}{v}_2 - \textcolor{blue}{v}_3$$

$$\textcolor{red}{i}_0 = i_1 + i_L$$

$$\textcolor{red}{i}_C = i_1 - i_2 - i_3$$

$$\frac{di_L}{dt} = \textcolor{blue}{v}_L / L$$

$$\frac{dv_3}{dt} = \textcolor{blue}{i}_C / C$$



$$\textcolor{red}{u}_0 = f_1(t)$$

$$\textcolor{red}{i}_3 = \textcolor{blue}{v}_3 / R_3$$

$$\textcolor{red}{i}_4 = 4 \cdot \textcolor{blue}{v}_3$$

$$\textcolor{red}{v}_1 = u_0 - \textcolor{blue}{v}_3$$

$$\textcolor{red}{i}_2 = i_4 - i_L$$

$$\textcolor{red}{i}_1 = \textcolor{blue}{v}_1 / R_1$$

$$\textcolor{red}{v}_2 = R_2 \cdot \textcolor{blue}{i}_2$$

$$\textcolor{red}{v}_L = \textcolor{blue}{v}_1 + \textcolor{blue}{v}_2$$

$$\textcolor{red}{u}_4 = \textcolor{blue}{v}_2 - \textcolor{blue}{v}_3$$

$$\textcolor{red}{i}_0 = i_1 + i_L$$

$$\textcolor{red}{i}_C = i_1 - i_2 - i_3$$

$$\frac{di_L}{dt} = \textcolor{blue}{v}_L / L$$

$$\frac{dv_3}{dt} = \textcolor{blue}{i}_C / C$$



Problems IV

1. Write down all of the equations that you need to describe this circuit mathematically. You may use either the technique involving potentials or that involving mesh equations.
2. Sort the equation system horizontally.
3. Sort the horizontally sorted equation system vertically.
4. Convert the horizontally and vertically sorted equation system to state-space form.



State-space Description I

$$\textcolor{red}{u}_0 = f_I(t)$$

$$\textcolor{red}{i}_3 = \textcolor{blue}{v}_3 / R_3$$

$$\textcolor{red}{i}_4 = 4 \cdot \textcolor{blue}{v}_3$$

$$\textcolor{red}{v}_1 = u_0 - \textcolor{blue}{v}_3$$

$$\textcolor{red}{i}_2 = i_4 - i_L$$

$$\textcolor{red}{i}_1 = \textcolor{blue}{v}_1 / R_1$$

$$\textcolor{red}{v}_2 = R_2 \cdot i_2$$

$$\textcolor{red}{v}_L = \textcolor{blue}{v}_I + \textcolor{blue}{v}_2$$

$$\textcolor{red}{u}_4 = \textcolor{blue}{v}_2 - \textcolor{blue}{v}_3$$

$$\textcolor{red}{i}_0 = \textcolor{blue}{i}_I + \textcolor{blue}{i}_L$$

$$\textcolor{red}{i}_C = \textcolor{blue}{i}_1 - \textcolor{blue}{i}_2 - \textcolor{blue}{i}_3$$

$$\frac{di_L}{dt} = \textcolor{blue}{v}_L / L$$

$$\frac{dv_3}{dt} = \textcolor{blue}{i}_C / C$$

$$\begin{aligned}
 \frac{di_L}{dt} &= \textcolor{blue}{v}_L / L \\
 &= (\textcolor{blue}{v}_I + \textcolor{blue}{v}_2) / L \\
 &= \textcolor{blue}{v}_I / L + \textcolor{blue}{v}_2 / L \\
 &= (\textcolor{blue}{u}_0 - \textcolor{blue}{v}_3) / L + (R_2 / L) \cdot \textcolor{blue}{i}_2 \\
 &= \textcolor{blue}{u}_0 / L - \textcolor{blue}{v}_3 / L + (R_2 / L) \cdot (\textcolor{blue}{i}_4 - \textcolor{blue}{i}_L) \\
 &= \textcolor{blue}{u}_0 / L - \textcolor{blue}{v}_3 / L + (4R_2 / L) \cdot \textcolor{blue}{v}_3 \\
 &\quad - (R_2 / L) \cdot \textcolor{blue}{i}_L \\
 &= -(R_2 / L) \cdot \textcolor{blue}{i}_L + (4R_2 / L - 1 / L) \cdot \textcolor{blue}{v}_3 \\
 &\quad + (1/L) \cdot \textcolor{blue}{u}_0
 \end{aligned}$$



State-space Description II

$$\textcolor{red}{u}_0 = f_I(t)$$

$$\textcolor{red}{i}_3 = \textcolor{blue}{v}_3 / R_3$$

$$\textcolor{red}{i}_4 = 4 \cdot \textcolor{blue}{v}_3$$

$$\textcolor{red}{v}_I = u_0 - v_3$$

$$\textcolor{red}{i}_2 = i_4 - i_L$$

$$\textcolor{red}{i}_I = \textcolor{blue}{v}_I / R_I$$

$$\textcolor{red}{v}_2 = R_2 \cdot i_2$$

$$\textcolor{red}{v}_L = \textcolor{blue}{v}_I + v_2$$

$$\textcolor{red}{u}_4 = \textcolor{blue}{v}_2 - v_3$$

$$\textcolor{red}{i}_0 = i_I + i_L$$

$$\textcolor{red}{i}_C = i_I - i_2 - i_3$$

$$\frac{di_L}{dt} = \textcolor{blue}{v}_L / L$$

$$\frac{dv_3}{dt} = \textcolor{blue}{i}_C / C$$

$$\begin{aligned}
\frac{dv_3}{dt} &= \textcolor{blue}{i}_C / C \\
&= (\textcolor{blue}{i}_I - \textcolor{blue}{i}_2 - \textcolor{blue}{i}_3) / C \\
&= \textcolor{blue}{v}_I / (R_I \cdot C) - (\textcolor{blue}{i}_4 - \textcolor{blue}{i}_L) / C \\
&\quad - \textcolor{blue}{v}_3 / (R_3 \cdot C) \\
&= (\textcolor{blue}{u}_0 - \textcolor{blue}{v}_3) / (R_I \cdot C) - (4/C) \cdot \textcolor{blue}{v}_3 \\
&\quad + (1/C) \cdot \textcolor{blue}{i}_L - \textcolor{blue}{v}_3 / (R_3 \cdot C) \\
&= (1/C) \cdot \textcolor{blue}{i}_L \\
&\quad - (1/(R_I \cdot C) + 1/(R_3 \cdot C) + 4/C) \cdot \textcolor{blue}{v}_3 \\
&\quad + 1/(R_I \cdot C) \cdot \textcolor{blue}{u}_0
\end{aligned}$$



State-space Description III

$$\frac{di_L}{dt} = -(R_2/L) \cdot i_L + (4R_2/L - 1/L) \cdot v_3 + (1/L) \cdot u_0$$

$$\frac{dv_3}{dt} = (1/C) \cdot i_L - (1/(R_I \cdot C) + 1/(R_3 \cdot C) + 4/C) \cdot v_3 + 1/(R_I \cdot C) \cdot u_0$$

$$\begin{bmatrix} \frac{di_L}{dt} \\ \frac{dv_3}{dt} \end{bmatrix} = \begin{bmatrix} -\frac{R_2}{L} & \left[\frac{4R_2}{L} - \frac{1}{L} \right] \\ \frac{1}{C} & -\left[\frac{1}{R_I \cdot C} + \frac{1}{R_3 \cdot C} + \frac{4}{C} \right] \end{bmatrix} \cdot \begin{bmatrix} i_L \\ v_3 \end{bmatrix} + \begin{bmatrix} \frac{1}{L} \\ \frac{1}{R_I \cdot C} \end{bmatrix} \cdot u_0$$



State-space Description IV

$$\begin{bmatrix} \frac{di_L}{dt} \\ \frac{dv_3}{dt} \end{bmatrix} = \begin{bmatrix} -\frac{R_2}{L} & \left[\frac{4R_2}{L} - \frac{1}{L} \right] \\ \frac{1}{C} & -\left[\frac{1}{R_I \cdot C} + \frac{1}{R_3 \cdot C} + \frac{4}{C} \right] \end{bmatrix} \cdot \begin{bmatrix} i_L \\ v_3 \end{bmatrix} + \begin{bmatrix} \frac{1}{L} \\ \frac{1}{R_I \cdot C} \end{bmatrix} \cdot u_0$$
$$v_3 = \begin{bmatrix} 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} i_L \\ v_3 \end{bmatrix}$$



Problems V

5. Program the state-space model in Matlab.
6. Simulate the circuit across **50 μ sec** of simulated time, und plot the variable v_3 as a function of time. You may assume that the initial Voltage across the capacitance and the initial current through the inductivity are equal to **0.0**.



C:\Cellier\Classes\Ece449\Hw_1\mmmps_hw1.m

```
File Edit Text Window Help
script Ln 1 Col 1
```

```
1 R1 = 100; R2 = 100; R3 = 20;
2 C = 1e-6; L = 0.01;
3 R1C = 1/(R1*C); R3C = 1/(R3*C);
4 Linv = 1/L; Cinv = 1/C;
5 all = -R2*Linv;
6 a12 = (4*R2 - 1)*Linv;
7 a21 = Cinv;
8 a22 = -(R1C + R3C + 4*Cinv);
9 A = [ all , a12 ; a21 , a22 ];
10 b = [ Linv ; R1C ];
11 c = [ 0 , 1 ];
12 d = 0;
13 S = ss(A,b,c,d);
14 t = [ 0 : 5e-8 : 5e-5 ];
15 u = 10*ones(size(t));
16 x0 = zeros(2,1);
17 y = lsim(S,u,t,x0);
18 plot(t,y)
19 grid on
20 return
21
```

