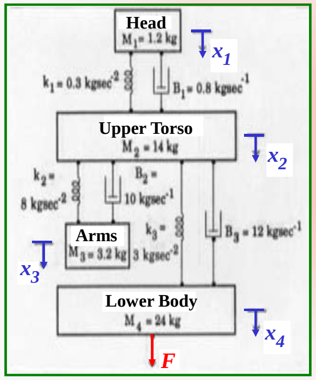
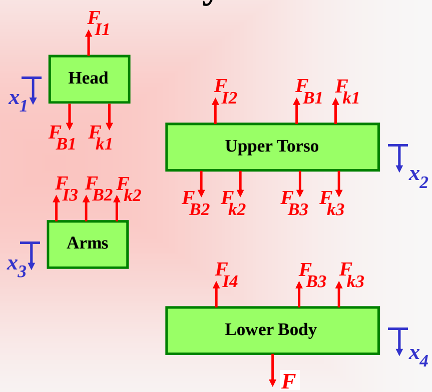
2nd Homework: Cervical Syndrome





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Mathematical Modeling of Physical Systems

1st Sub-problem

1. Derive a state-space model for this system. Since this is a linear time-invariant system, put it in linear state-space form and simulate the system in ***Matlab***.
2. Simulate the system during ***15 seconds***. Use a sinusoidal force ***(F)*** with a frequency of ***1.5 Hz***. As the system is linear, the amplitude of the input signal is irrelevant. 1.0 represents an excellent value. The output is the distance between the head and the shoulder. The initial conditions of all state variables may be assumed as 0.0. This is acceptable, since only the deviation of the output from the stationary position is of relevance.

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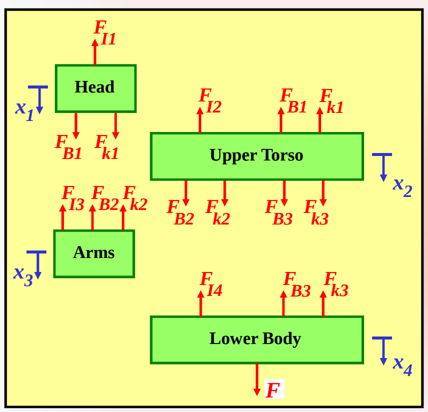

$F(t) = F_{I4} + F_{B3} + F_{k3}$

$0 = F_{I3} + F_{B2} + F_{k2}$

$F_{B2} + F_{k2} + F_{B3} + F_{k3} = F_{I2} + F_{B1} + F_{k1}$

$F_{B1} + F_{k1} = F_{I1}$

$F_{I1} = m_1 \cdot \frac{dv_1}{dt}$

$\frac{dx_1}{dt} = v_1$

$F_{I2} = m_2 \cdot \frac{dv_2}{dt}$

$\frac{dx_2}{dt} = v_2$

$F_{I3} = m_3 \cdot \frac{dv_3}{dt}$

$\frac{dx_3}{dt} = v_3$

$F_{I4} = m_4 \cdot \frac{dv_4}{dt}$

$\frac{dx_4}{dt} = v_4$

$F_{B1} = B_1 \cdot (v_2 - v_1)$

$F_{B2} = B_2 \cdot (v_3 - v_2)$

$F_{B3} = B_3 \cdot (v_4 - v_2)$

$F_{k1} = k_1 \cdot (x_2 - x_1)$

$F_{k2} = k_2 \cdot (x_3 - x_2)$

$F_{k3} = k_3 \cdot (x_4 - x_2)$

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$F(t) = F_{I4} + F_{B3} + F_{k3}$

$0 = F_{I3} + F_{B2} + F_{k2}$

$F_{B2} + F_{k2} + F_{B3} + F_{k3} = F_{I2} + F_{B1} + F_{k1}$

$F_{B1} + F_{k1} = F_{I1}$

$F_{I1} = m_1 \cdot \frac{dv_1}{dt}$

$\frac{dx_1}{dt} = v_1$

$F_{I2} = m_2 \cdot \frac{dv_2}{dt}$

$\frac{dx_2}{dt} = v_2$

$F_{I3} = m_3 \cdot \frac{dv_3}{dt}$

$\frac{dx_3}{dt} = v_3$

$F_{I4} = m_4 \cdot \frac{dv_4}{dt}$

$\frac{dx_4}{dt} = v_4$

$F_{B1} = B_1 \cdot (v_2 - v_1)$

$F_{B2} = B_2 \cdot (v_3 - v_2)$

$F_{B3} = B_3 \cdot (v_4 - v_2)$

$F_{k1} = k_1 \cdot (x_2 - x_1)$

$F_{k2} = k_2 \cdot (x_3 - x_2)$

$F_{k3} = k_3 \cdot (x_4 - x_2)$

$\Rightarrow$

$F(t) = F_{I4} + F_{B3} + F_{k3}$

$0 = F_{I3} + F_{B2} + F_{k2}$

$F_{B2} + F_{k2} + F_{B3} + F_{k3} = F_{I2} + F_{B1} + F_{k1}$

$F_{B1} + F_{k1} = F_{I1}$

$F_{I1} = m_1 \cdot \frac{dv_1}{dt}$

$\frac{dx_1}{dt} = v_1$

$F_{I2} = m_2 \cdot \frac{dv_2}{dt}$

$\frac{dx_2}{dt} = v_2$

$F_{I3} = m_3 \cdot \frac{dv_3}{dt}$

$\frac{dx_3}{dt} = v_3$

$F_{I4} = m_4 \cdot \frac{dv_4}{dt}$

$\frac{dx_4}{dt} = v_4$

$F_{B1} = B_1 \cdot (v_2 - v_1)$

$F_{B2} = B_2 \cdot (v_3 - v_2)$

$F_{B3} = B_3 \cdot (v_4 - v_2)$

$F_{k1} = k_1 \cdot (x_2 - x_1)$

$F_{k2} = k_2 \cdot (x_3 - x_2)$

$F_{k3} = k_3 \cdot (x_4 - x_2)$

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$F(t) = F_{I4} + F_{B3} + F_{k3}$

$0 = F_{I3} + F_{B2} + F_{k2}$

$F_{B2} + F_{k2} + F_{B3} + F_{k3} = F_{I2} + F_{B1} + F_{k1}$

$F_{B1} + F_{k1} = F_{I1}$

$F_{I1} = m_1 \cdot \frac{dv_1}{dt}$	$F_{B1} = B_1 \cdot (v_2 - v_1)$
$\frac{dx_1}{dt} = v_1$	$F_{B2} = B_2 \cdot (v_3 - v_2)$
$F_{I2} = m_2 \cdot \frac{dv_2}{dt}$	$F_{B3} = B_3 \cdot (v_4 - v_2)$
$\frac{dx_2}{dt} = v_2$	$F_{k1} = k_1 \cdot (x_2 - x_1)$
$F_{I3} = m_3 \cdot \frac{dv_3}{dt}$	$F_{k2} = k_2 \cdot (x_3 - x_2)$
$\frac{dx_3}{dt} = v_3$	$F_{k3} = k_3 \cdot (x_4 - x_2)$
$F_{I4} = m_4 \cdot \frac{dv_4}{dt}$	
$\frac{dx_4}{dt} = v_4$	

$\Rightarrow$

$F(t) = F_{I4} + F_{B3} + F_{k3}$

$0 = F_{I3} + F_{B2} + F_{k2}$

$F_{B2} + F_{k2} + F_{B3} + F_{k3} = F_{I2} + F_{B1} + F_{k1}$

$F_{B1} + F_{k1} = F_{I1}$

$F_{I1} = m_1 \cdot \frac{dv_1}{dt}$	$F_{B1} = B_1 \cdot (v_2 - v_1)$
$\frac{dx_1}{dt} = v_1$	$F_{B2} = B_2 \cdot (v_3 - v_2)$
$F_{I2} = m_2 \cdot \frac{dv_2}{dt}$	$F_{B3} = B_3 \cdot (v_4 - v_2)$
$\frac{dx_2}{dt} = v_2$	$F_{k1} = k_1 \cdot (x_2 - x_1)$
$F_{I3} = m_3 \cdot \frac{dv_3}{dt}$	$F_{k2} = k_2 \cdot (x_3 - x_2)$
$\frac{dx_3}{dt} = v_3$	$F_{k3} = k_3 \cdot (x_4 - x_2)$
$F_{I4} = m_4 \cdot \frac{dv_4}{dt}$	
$\frac{dx_4}{dt} = v_4$	

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$F(t) = F_{I4} + F_{B3} + F_{k3}$

$0 = F_{I3} + F_{B2} + F_{k2}$

$F_{B2} + F_{k2} + F_{B3} + F_{k3} = F_{I2} + F_{B1} + F_{k1}$

$F_{B1} + F_{k1} = F_{I1}$

$F_{I1} = m_1 \cdot \frac{dv_1}{dt}$	$F_{B1} = B_1 \cdot (v_2 - v_1)$
$\frac{dx_1}{dt} = v_1$	$F_{B2} = B_2 \cdot (v_3 - v_2)$
$F_{I2} = m_2 \cdot \frac{dv_2}{dt}$	$F_{B3} = B_3 \cdot (v_4 - v_2)$
$\frac{dx_2}{dt} = v_2$	$F_{k1} = k_1 \cdot (x_2 - x_1)$
$F_{I3} = m_3 \cdot \frac{dv_3}{dt}$	$F_{k2} = k_2 \cdot (x_3 - x_2)$
$\frac{dx_3}{dt} = v_3$	$F_{k3} = k_3 \cdot (x_4 - x_2)$
$F_{I4} = m_4 \cdot \frac{dv_4}{dt}$	
$\frac{dx_4}{dt} = v_4$	

$\Rightarrow$

$F_{I4} = F(t) - F_{B3} - F_{k3}$

$F_{I3} = - F_{B2} - F_{k2}$

$F_{I2} = F_{B2} + F_{k2} + F_{B3} + F_{k3} - F_{B1} - F_{k1}$

$F_{I1} = F_{B1} + F_{k1}$

$\frac{dv_1}{dt} = F_{I1} / m_1$	$F_{B1} = B_1 \cdot (v_2 - v_1)$
$\frac{dx_1}{dt} = v_1$	$F_{B2} = B_2 \cdot (v_3 - v_2)$
$\frac{dv_2}{dt} = F_{I2} / m_2$	$F_{B3} = B_3 \cdot (v_4 - v_2)$
$\frac{dx_2}{dt} = v_2$	$F_{k1} = k_1 \cdot (x_2 - x_1)$
$\frac{dv_3}{dt} = F_{I3} / m_3$	$F_{k2} = k_2 \cdot (x_3 - x_2)$
$\frac{dx_3}{dt} = v_3$	$F_{k3} = k_3 \cdot (x_4 - x_2)$
$\frac{dv_4}{dt} = F_{I4} / m_4$	
$\frac{dx_4}{dt} = v_4$	

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$$\frac{dv_1}{dt} = F_{I1} / m_1$$
$$= (F_{B1} + F_{k1}) / m_1$$
$$= (B_1 \cdot (v_2 - v_1) + k_1 \cdot (x_2 - x_1)) / m_1$$
$$= -(k_1/m_1) \cdot x_1 + (k_1/m_1) \cdot x_2 - (B_1/m_1) \cdot v_1 + (B_1/m_1) \cdot v_2$$

$$\frac{dv_2}{dt} = F_{I2} / m_2$$
$$= (F_{B2} + F_{k2} + F_{B3} + F_{k3} - F_{B1} - F_{k1}) / m_2$$
$$= (B_2 \cdot (v_3 - v_2) + k_2 \cdot (x_3 - x_2) + B_3 \cdot (v_4 - v_2) + k_3 \cdot (x_4 - x_2)$$
$$- B_1 \cdot (v_2 - v_1) - k_1 \cdot (x_2 - x_1)) / m_2$$
$$= (k_1/m_2) \cdot x_1 - ((k_1+k_2+k_3)/m_2) \cdot x_2 + (k_2/m_2) \cdot x_3 + (k_3/m_2) \cdot x_4$$
$$+ (B_1/m_2) \cdot v_1 - ((B_1+B_2+B_3)/m_2) \cdot v_2 + (B_2/m_2) \cdot v_3 + (B_3/m_2) \cdot v_4$$

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$$\frac{dv_3}{dt} = F_{I3} / m_3$$
$$= (- F_{B2} - F_{k2}) / m_3$$
$$= (- B_2 \cdot (v_3 - v_2) - k_2 \cdot (x_3 - x_2)) / m_3$$
$$= (k_2/m_3) \cdot x_2 - (k_2/m_3) \cdot x_3 + (B_2/m_3) \cdot v_2 - (B_2/m_3) \cdot v_3$$

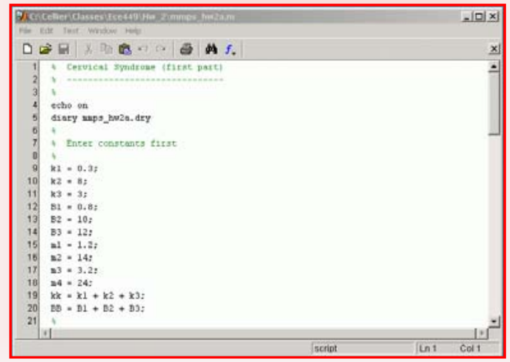
$$\frac{dv_4}{dt} = F_{I4} / m_4$$
$$= (F(t) - F_{B3} - F_{k3}) / m_4$$
$$= (F(t) - B_3 \cdot (v_4 - v_2) - k_3 \cdot (x_4 - x_2)) / m_4$$
$$= F(t) / m_4 + (k_3/m_4) \cdot x_2 - (k_3/m_4) \cdot x_4 + (B_3/m_4) \cdot v_2 - (B_3/m_4) \cdot v_4$$

$$y = x_1 - x_2$$

1st Sub-problem

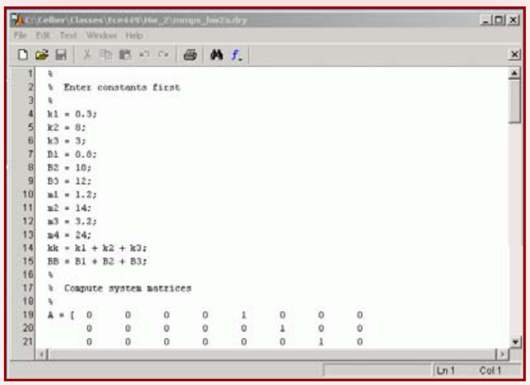
1. Derive a state-space model for this system. Since this is a linear time-invariant system, put it in linear state-space form and simulate the system in ***Matlab***.
2. Simulate the system during ***15 seconds***. Use a sinusoidal force ***(F)*** with a frequency of ***1.5 Hz***. As the system is linear, the amplitude of the input signal is irrelevant. 1.0 represents an excellent value. The output is the distance between the head and the shoulder. The initial conditions of all state variables may be assumed as 0.0. This is acceptable, since only the deviation of the output from the stationary position is of relevance.

Matlab Code (1st Part)



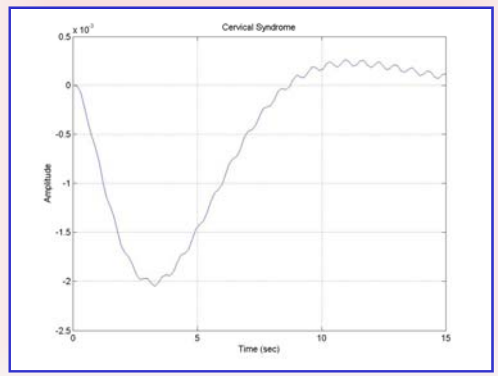
```
%  Cervical Syndrome (first part)
%  ------------------------------
%
echo on
diary mmps_hw2a.dry
%
%  Enter constants first
%
k1 = 0.3;
k2 = 8;
k3 = 3;
B1 = 0.8;
B2 = 10;
B3 = 12;
m1 = 1.2;
m2 = 14;
m3 = 3.2;
m4 = 24;
kk = k1 + k2 + k3;
BB = B1 + B2 + B3;
%
```


Matlab Diary File (1st Part)



```
%
%  Enter constants first
%
k1 = 0.3;
k2 = 8;
k3 = 3;
B1 = 0.8;
B2 = 10;
B3 = 12;
m1 = 1.2;
m2 = 14;
m3 = 3.2;
m4 = 24;
kk = k1 + k2 + k3;
BB = B1 + B2 + B3;
%
%  Compute system matrices
%
A = [  0   0   0   0   1   0   0   0
       0   0   0   0   0   1   0   0
       0   0   0   0   0   0   1   0
```

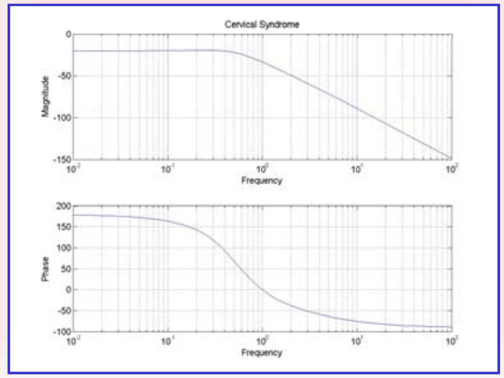

Matlab Simulation (1st Part)



2nd Sub-problem

3. In order to be able to better analyze the ***resonance phenomena***, we wish to obtain a ***Bode diagram*** of the system. To this end, we generate a logarithmic base of frequency values in the range from 0.01 Hz to 100 Hz by means of Matlab's ***logspace*** function. The ***Bode*** function may now be used to compute the Bode diagram. The amplitude needs to be converted to decibels. Using the functions ***subplot***, ***semilogx***, ***grid***, ***title***, ***xlabel***, and ***ylabel***, the Bode diagram shall now be displayed on two separate graphs on the same page.

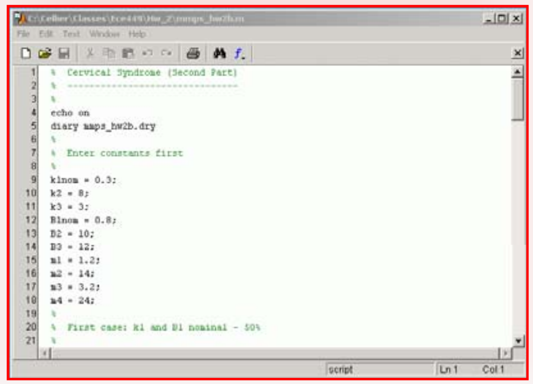
Matlab Bode Diagram (1st Part)



3rd Sub-problem

4. Finally, we wish to perform a ***sensitivity analysis***. We want to study the variability of the spring constant and the damper between head and upper torso. For this purpose, we assume a variability of these two parameters of ±50%.
5. Repeat the frequency analysis for the four worst-case combinations of the two parameters.
6. Plot the maxima and minima of the amplitude and phase curves as a sensitivity Bode diagram.

Matlab Code (2nd Part)



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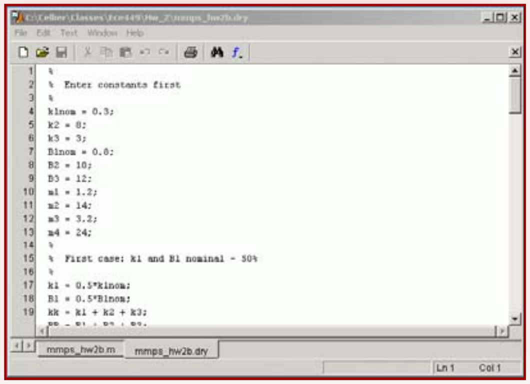
Matlab Diary File (2^{nd} Part)



```
%
%  Enter constants first
%
k1nom = 0.3;
k2 = 8;
k3 = 3;
B1nom = 0.8;
B2 = 10;
B3 = 12;
m1 = 1.2;
m2 = 14;
m3 = 3.2;
m4 = 24;
%
%  First case: k1 and B1 nominal - 50%
%
k1 = 0.5*k1nom;
B1 = 0.5*B1nom;
kk = k1 + k2 + k3;
```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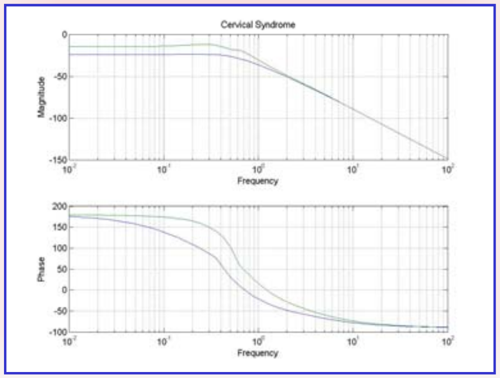
Matlab Bode Diagram (2^{nd} Part)



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