6th Homework - Solution

- In this homework, we shall model and simulate a thermal system.
- We shall model heat conduction along a well-insulated copper rod.
- We shall furthermore study the effects of different spatial discretization schemes on the accuracy of the simulation results.

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Heat Conduction in a Copper Rod I

- A copper rod of length $l = 1\ m$ with a radius of $r = 1\ cm$ is initially in thermo-dynamical equilibrium at $T = 298\ K$.
- At $Time = 0$, the left end of the rod is brought in contact with a body that had been pre-heated to a temperature of $T_L = 390\ K$.
- We wish to model the rod using ***10 segments***, each with a length of $\Delta x = 10\ cm$. The boundary conditions are to be modeled such that the body to the left is replaced by a temperature source.
- It is assumed that no heat flows out at the right end of the rod, and that the rod is thermally so well insulated that no heat is lost anywhere along the rod.

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Heat Conduction in a Copper Rod II

- The density of copper is $\rho = 8960\ kg \cdot m^{-3}$. Its specific thermal conductivity is $\lambda = 401\ J \cdot m^{-1} \cdot s^{-1} \cdot K^{-1}$. Its specific heat capacity is $c = 386\ J \cdot kg^{-1} \cdot K^{-1}$.
- The heat conduction is modeled using the symmetric heat conduction element presented in class. This element is made available as part of the ***BondLib*** thermal sub-library.
- Simulate the system during ***5 hours***.

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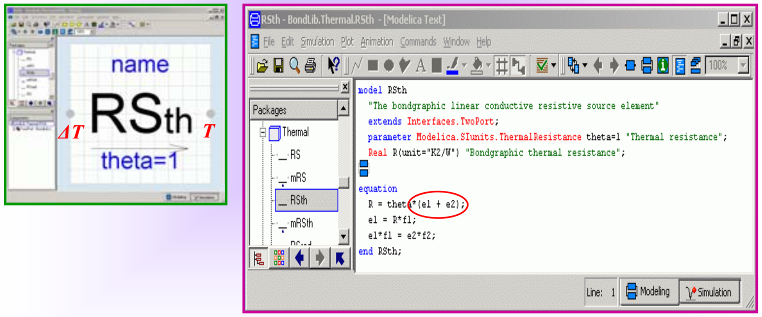
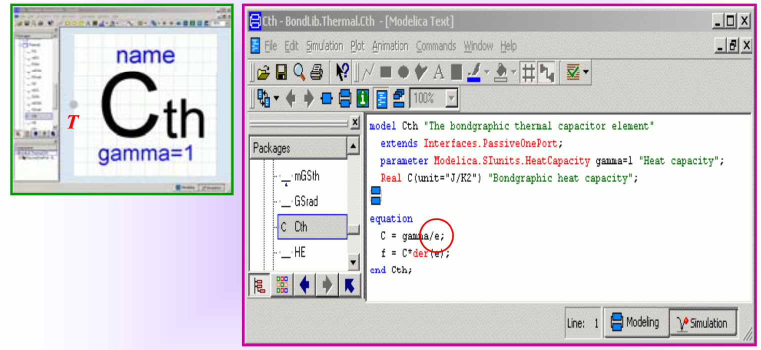
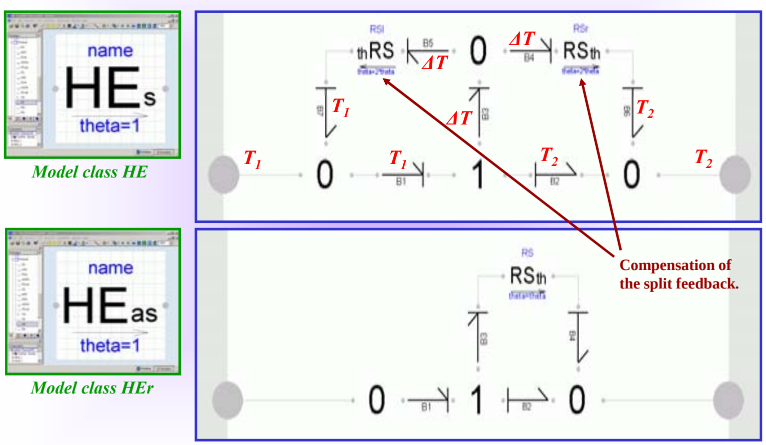
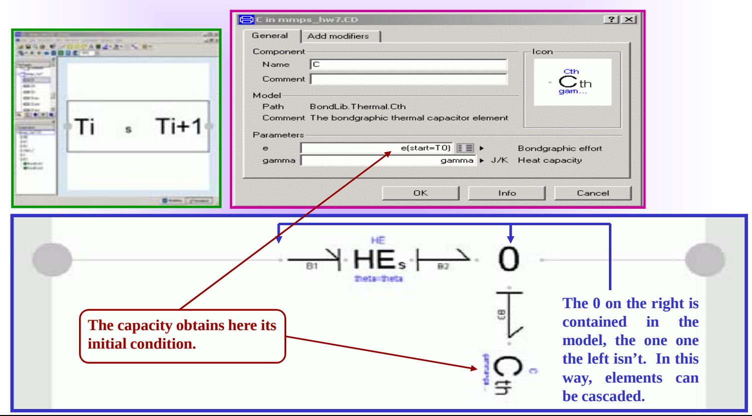
Let us start by looking at some thermal models in the ***Dymola BondLib library*** that we haven't used before. These are stored in its thermal sub-library.

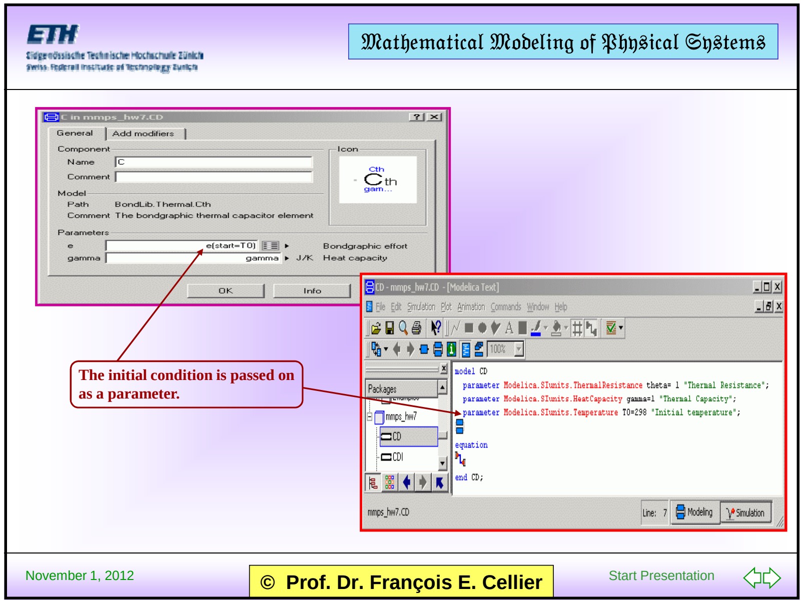






We can now start creating the individual chain links.

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The initial condition is passed on as a parameter.
equation
end CD;
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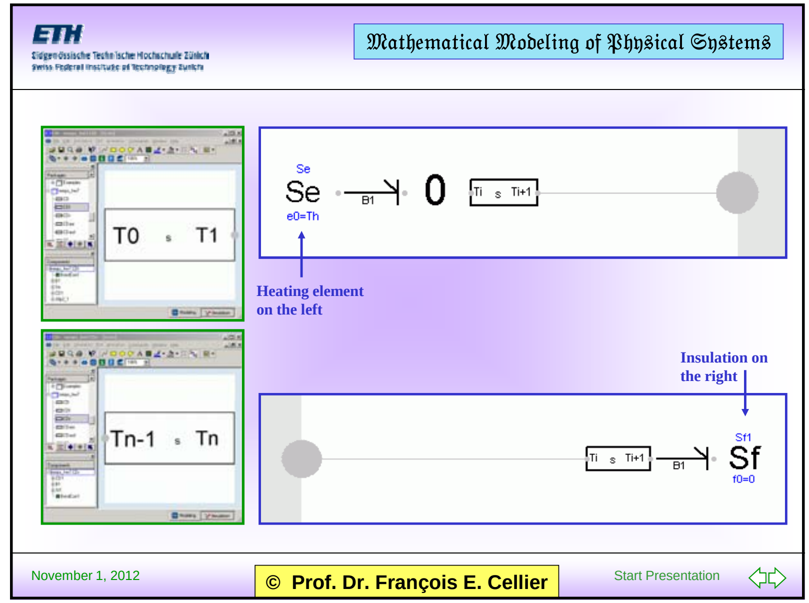
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T0
T1
Tn-1
Tn
Se
e0=Th
B1
0
Ti
Ti+1
Heating element on the left
Insulation on the right
Sf
f0=0
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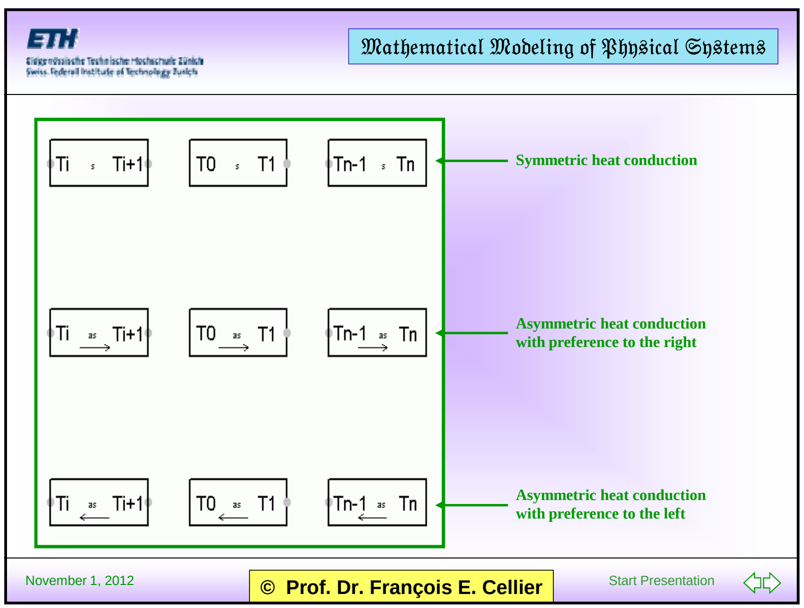
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Ti
Ti+1
T0
T1
Tn-1
Tn
Symmetric heat conduction
Asymmetric heat conduction with preference to the right
Asymmetric heat conduction with preference to the left
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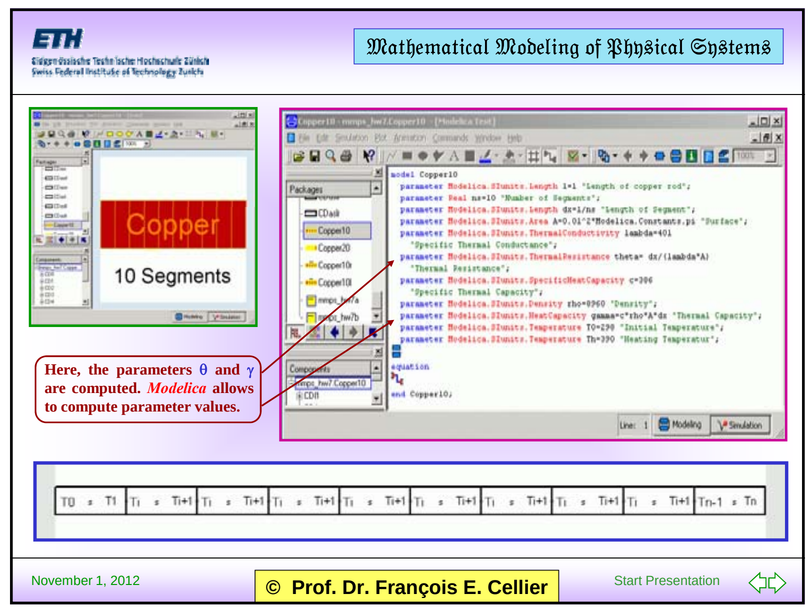
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Copper
10 Segments
Here, the parameters θ and γ are computed. *Modelica* allows to compute parameter values.
T0
T1
Ti
Ti+1
Tn-1
Tn
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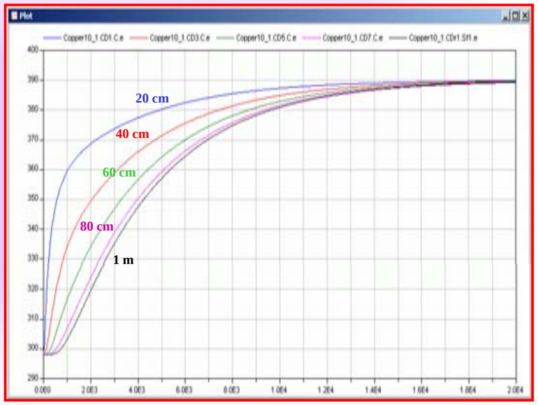
6th Homework

1st Part

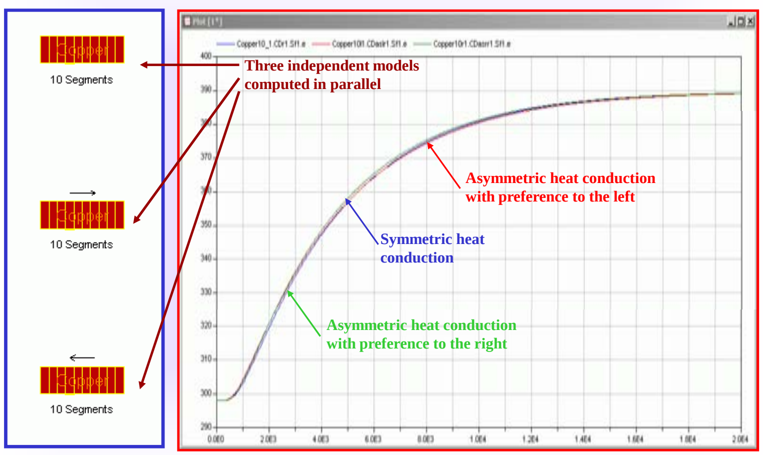
Thermal System

Temperature values as functions of time and space



Influence of Asymmetric Entropy Feed

- Replace the symmetric heat conduction element by two asymmetric elements; one, in which the generated entropy is fed only to the right, the other, in which it is fed exclusively to the left.
- The ***BondLib*** library offers such an element as well.
- Simulate the so modified model, and present, on a single plot, the results of the three simulation models.
- You may either calculate the three models sequentially while preserving the results from one to the next, or you may simulate the three models in parallel.



Influence of Discretization

- We return to using the symmetric model. However this time, we wish to model the system using ***20 segments***, each with a length of $\Delta x = 5\ cm$.
- Simulate the so modified model, and present the results obtained in this way graphically together with the original simulation results.

6th Homework

2nd Part

Thermal System

10 Segments

20 Segments