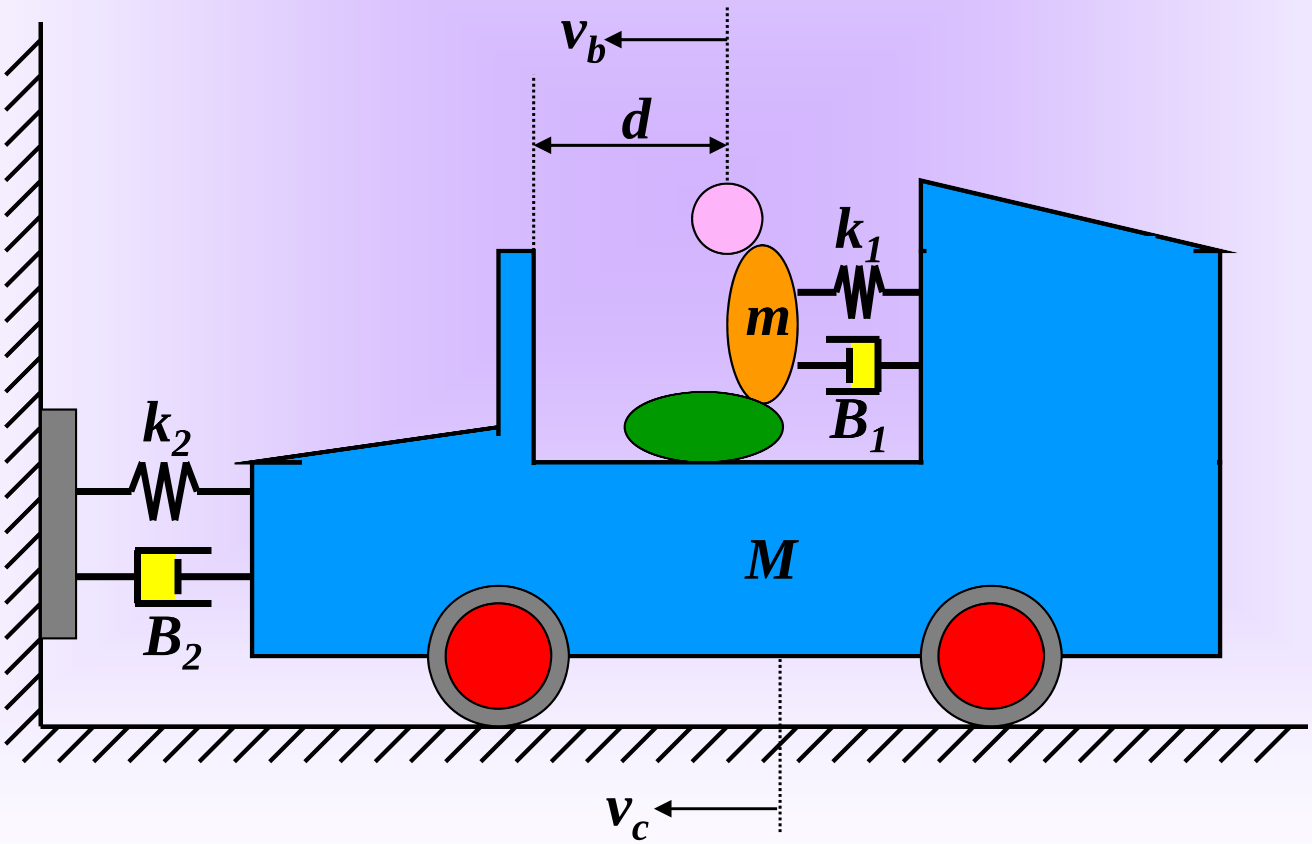
# 8th Homework - Solution

- In this homework, we shall model and simulate a mechanical system as well as exercise the state selection algorithm.
- We shall first model a car bumping into a wall using the 1D mechanical (translational) wrapped bond graph library.
- We shall then model the same car using a bond graph directly.
- Subsequently, we shall read out the model equations from the bond graph.
- Finally, we shall change one of the state variables using the state selection algorithm.

- Model description
- 1D mechanical wrapped bond graph model
- Direct bond graph model
- State selection

# Mechanical System

- We wish to analyze the following system:



# Mechanical System II

- Questions of interest:
  - Are the shock absorbers (***$k_2$,$B_2$***) and the safety belts (***$k_1$,$B_1$***) capable of preventing the driver from hitting his head on the front windshield if he drives with a velocity of ***40 km/h*** against a solid wall? What happens when the velocity at impact is ***80 km/h***?
  - How large is the maximal force that the driver experiences at these velocities?
  - How large is the critical velocity, below which the driver neither hits his head on the windshield, nor breaks his ribs?

# Mechanical System III

- Data:

  Mass of vehicle ($M$) = 1500 kg
  Mass of driver ($m$) = 100 kg
  Stiffness of safety belt ($k_1$) = 10’000 N/m
  Stiffness of shock absorber ($k_2$) = 300’000 N/m
  Damping of safety belt ($B_1$) = 500 Ns/m
  Damping of shock absorber ($B_2$) = 80’000 Ns/m

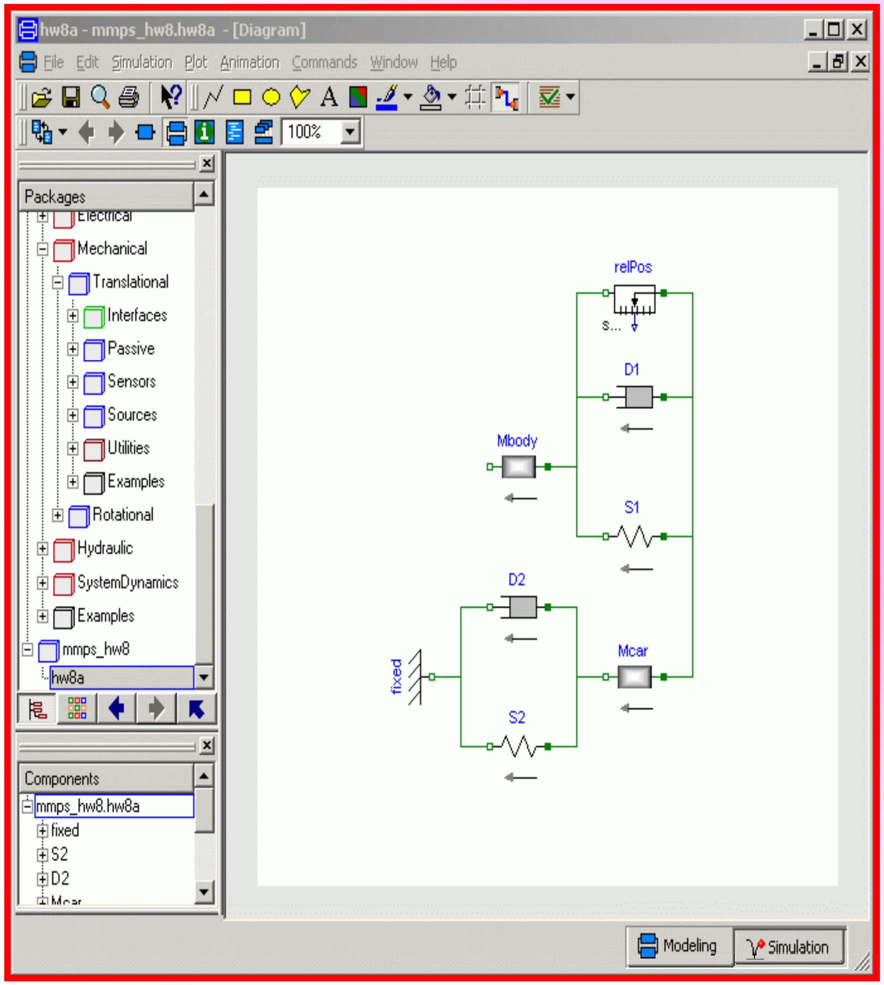
- Limit values:

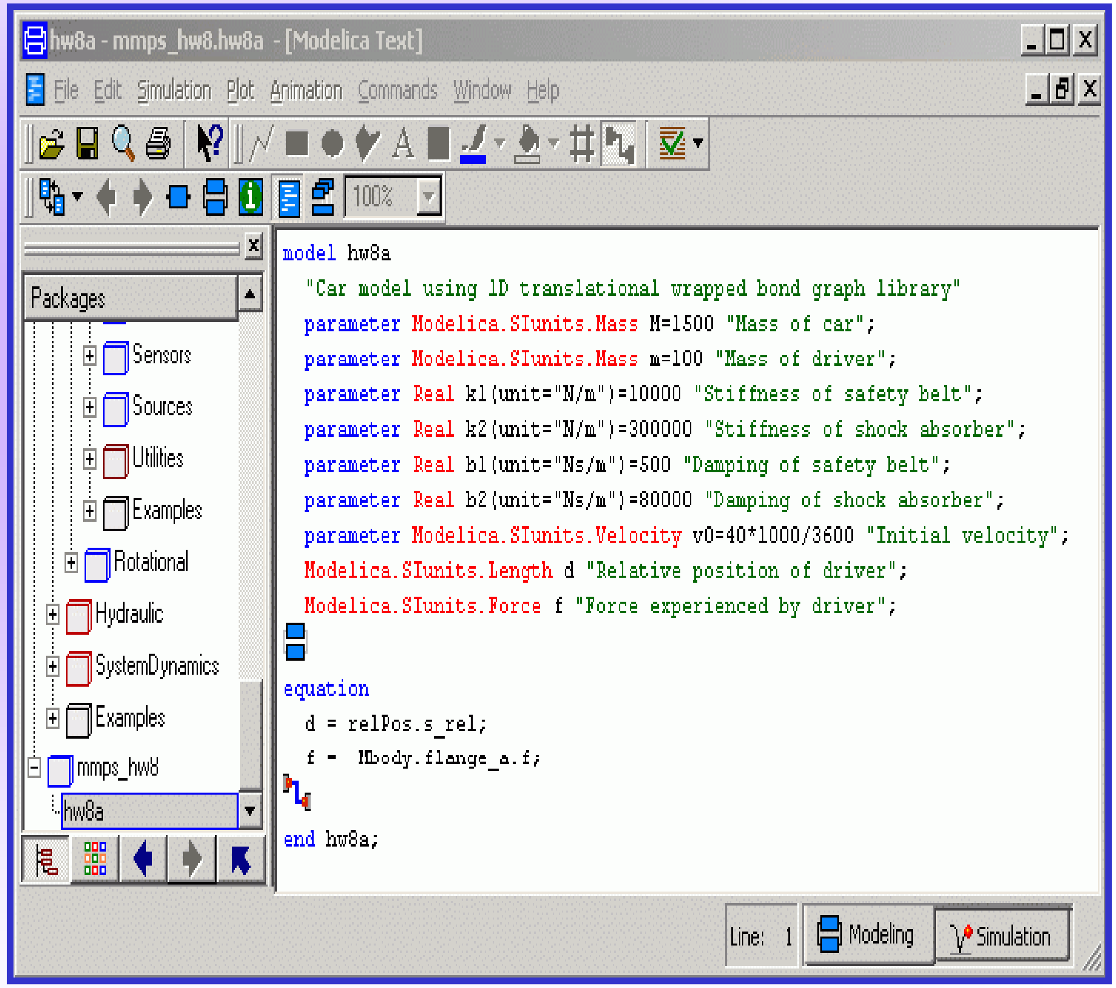
  Safety belt tested up to ($F_1$) < 13’340 N
  Ribs break beyond ($F_2$) > 6670 N
  Distance to windshield ($d$) = 0.5 m

# Mechanical System IV

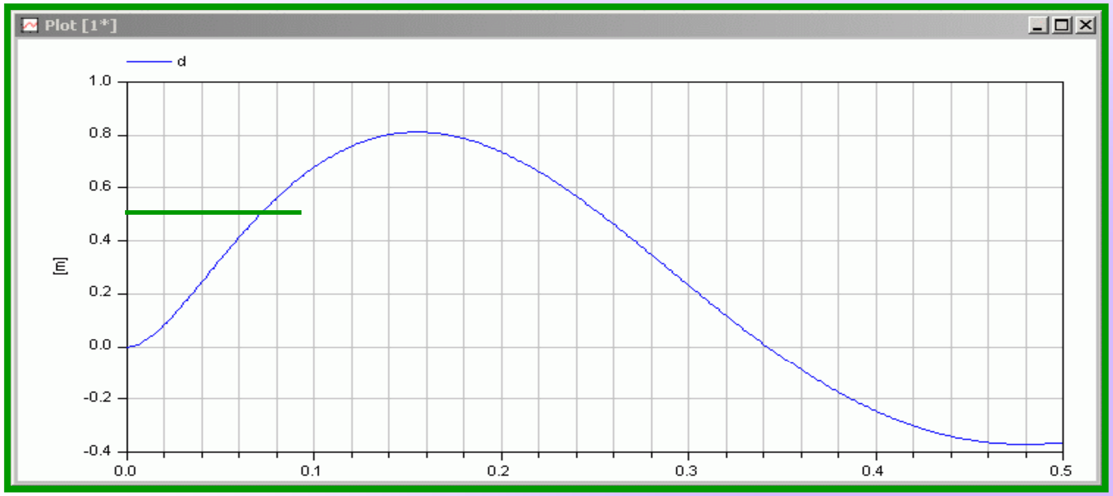
- Model the car and the driver using two sliding masses of the translational sub-library of the mechanical sub-library of ***BondLib***.
- Simulate the system across ***0.5 sec*** of simulated time, and answer the questions that were raised before.

# Mechanical System V



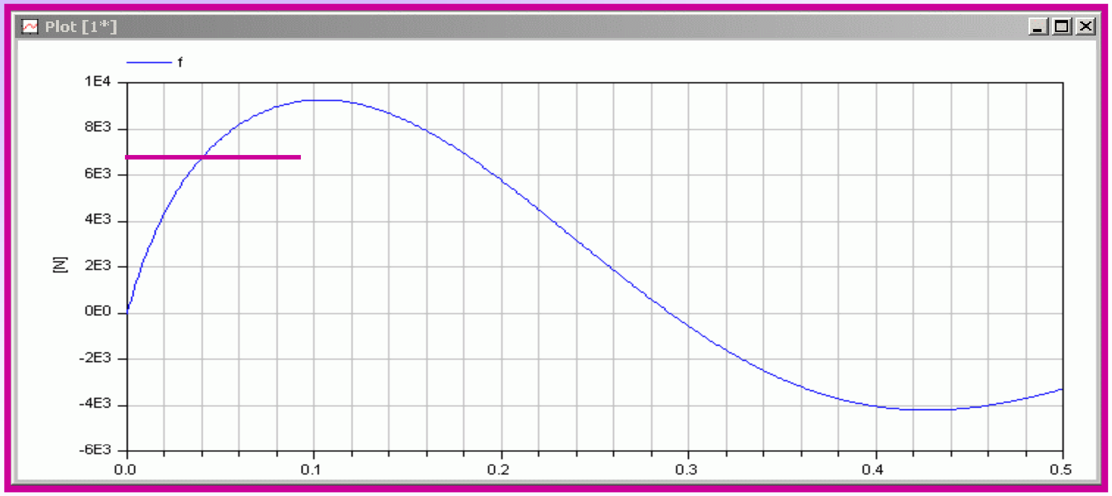



# Mechanical System VI

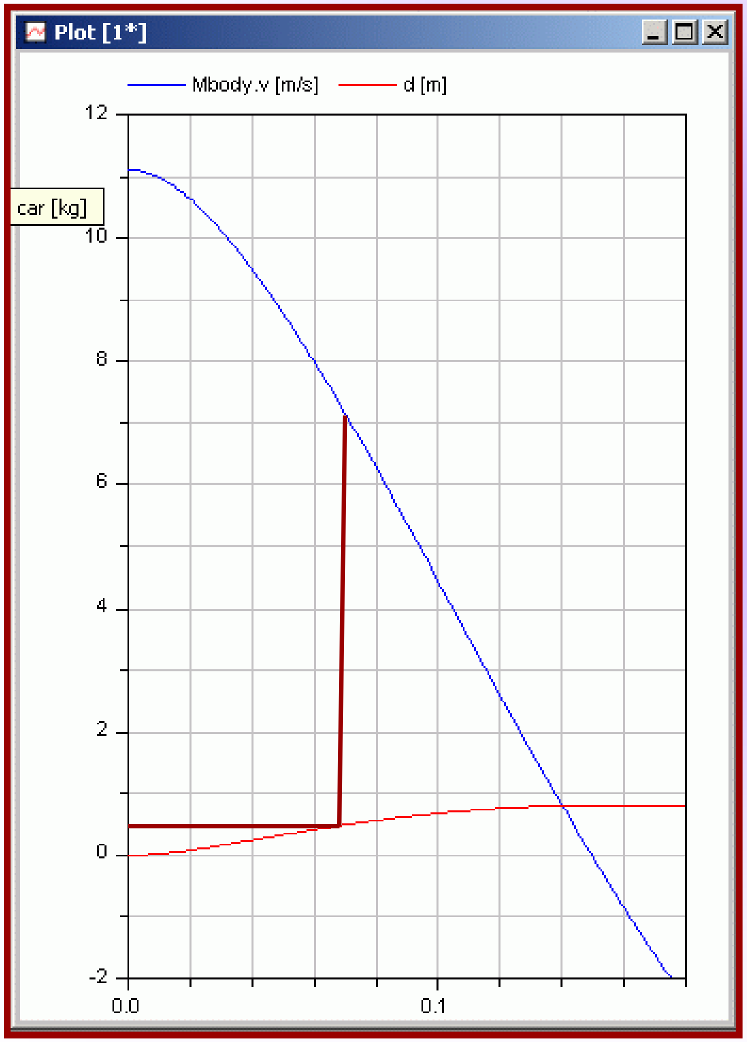



The driver will break his ribs.

The driver's head will hit the windshield.



# Mechanical System VII




At the time of the impact with the windshield, the head of the driver has a velocity of ***7 m/s = 25.2 km/h***.

# Mechanical System VIII

**The results are so depressing that a simulation with an initial velocity of 80 km/h is hardly necessary!**

# Mechanical System IX

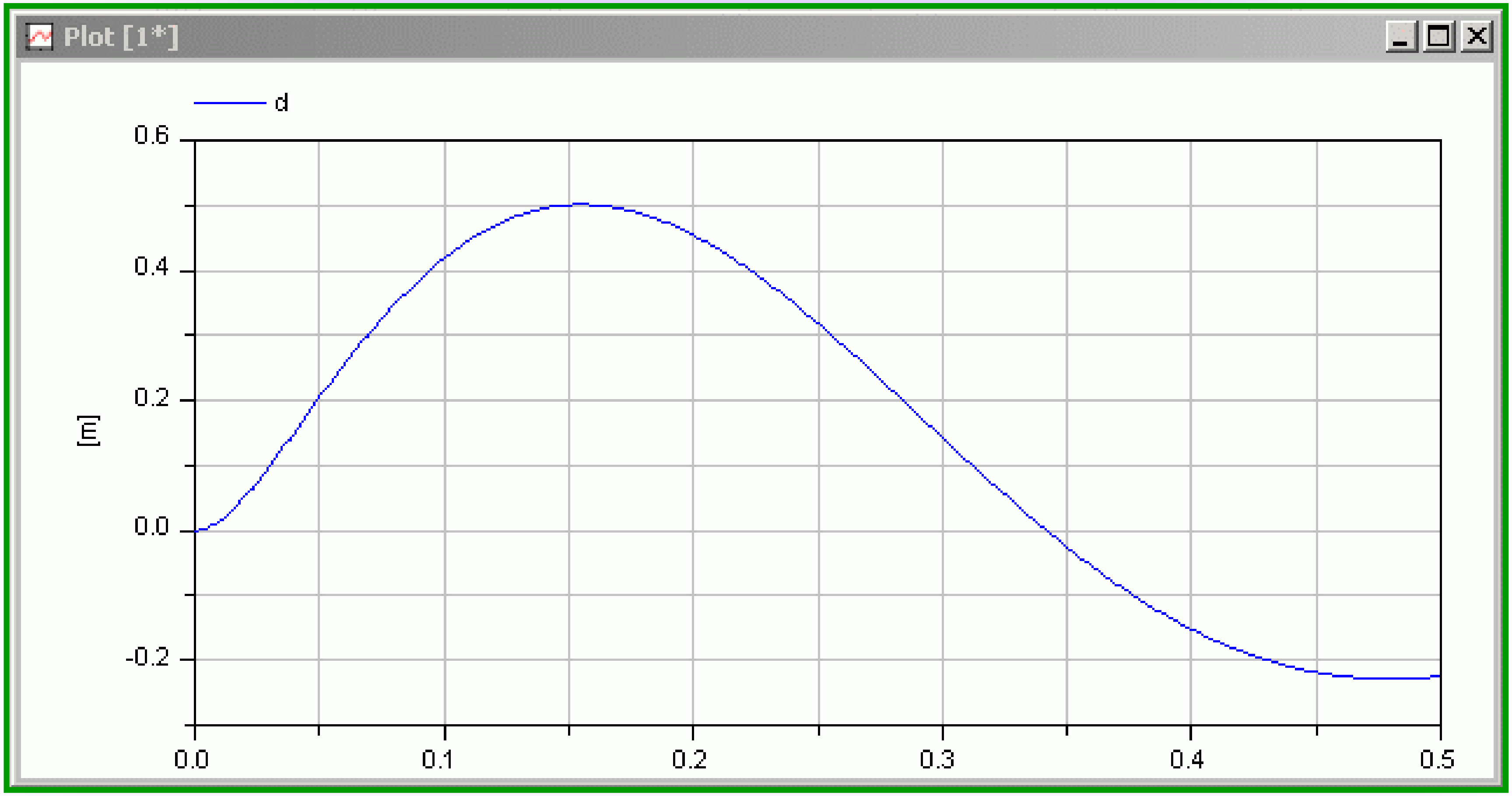
- The system is linear. Hence we can compute the maximal velocity below which the head won't hit the windshield:

$$v_0 = 40*0.5/0.81 = 24.7\ km/h$$

where 0.81 m is the largest distance in the simulation that we just carried out.

- Let us verify the results.

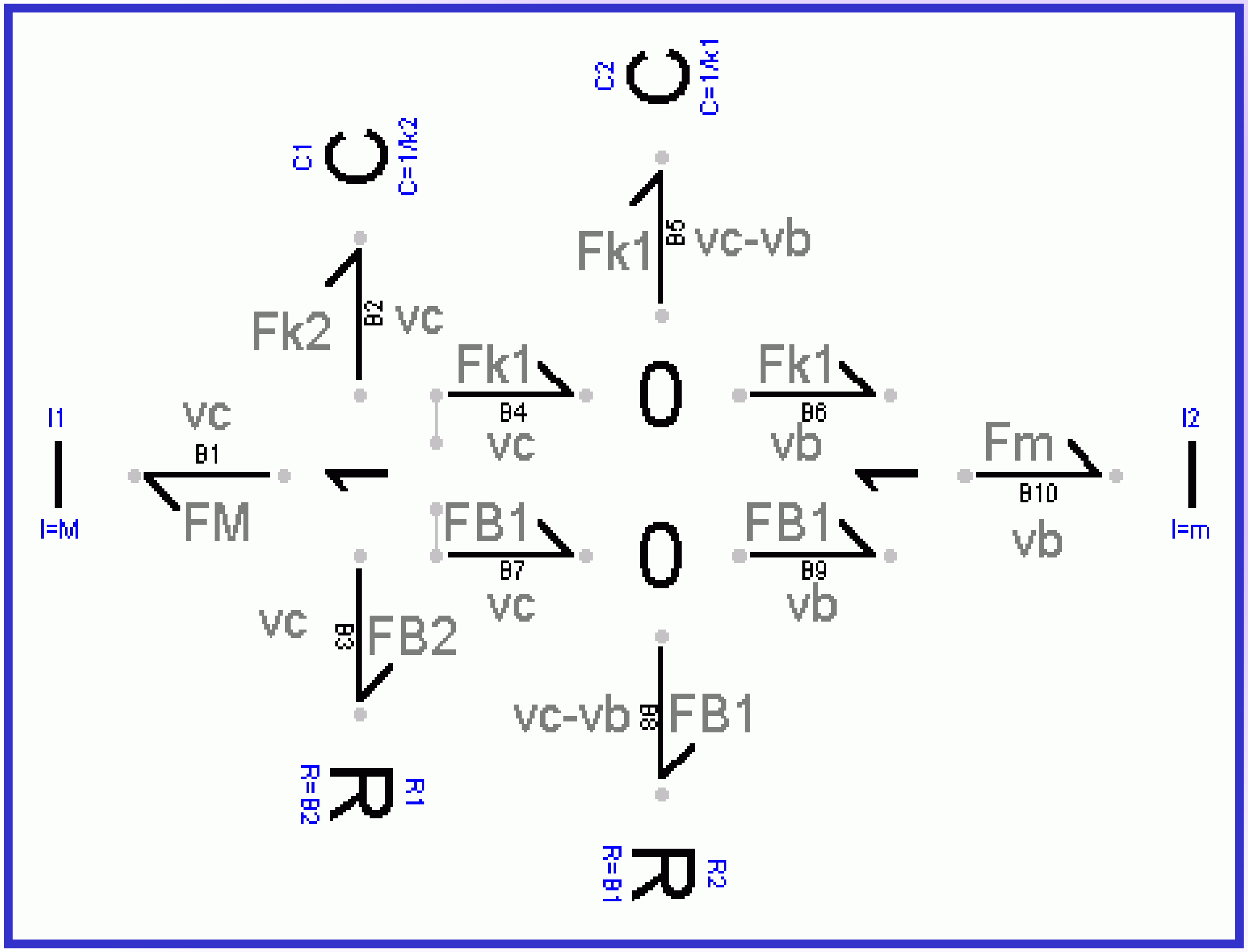
# Mechanical System X



# Mechanical System XI

- Draw a bond graph of this system.
- Simplify the bond graph using the diamond property.
- Add causality strokes.
- Simulate the simplified bond graph model using ***BondLib***, and compare the results with those obtained earlier.

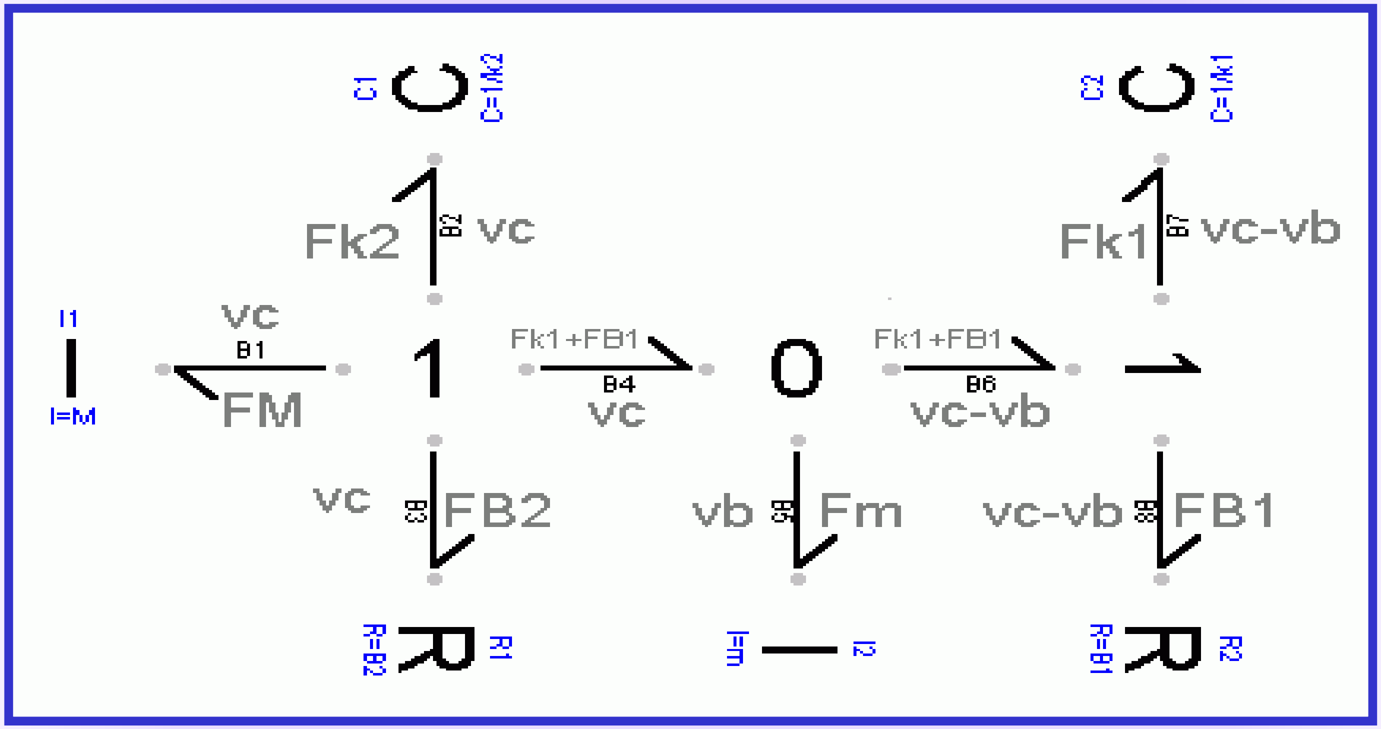
# Mechanical System XII




The velocity of the car is entered as initial condition to the two inertias.

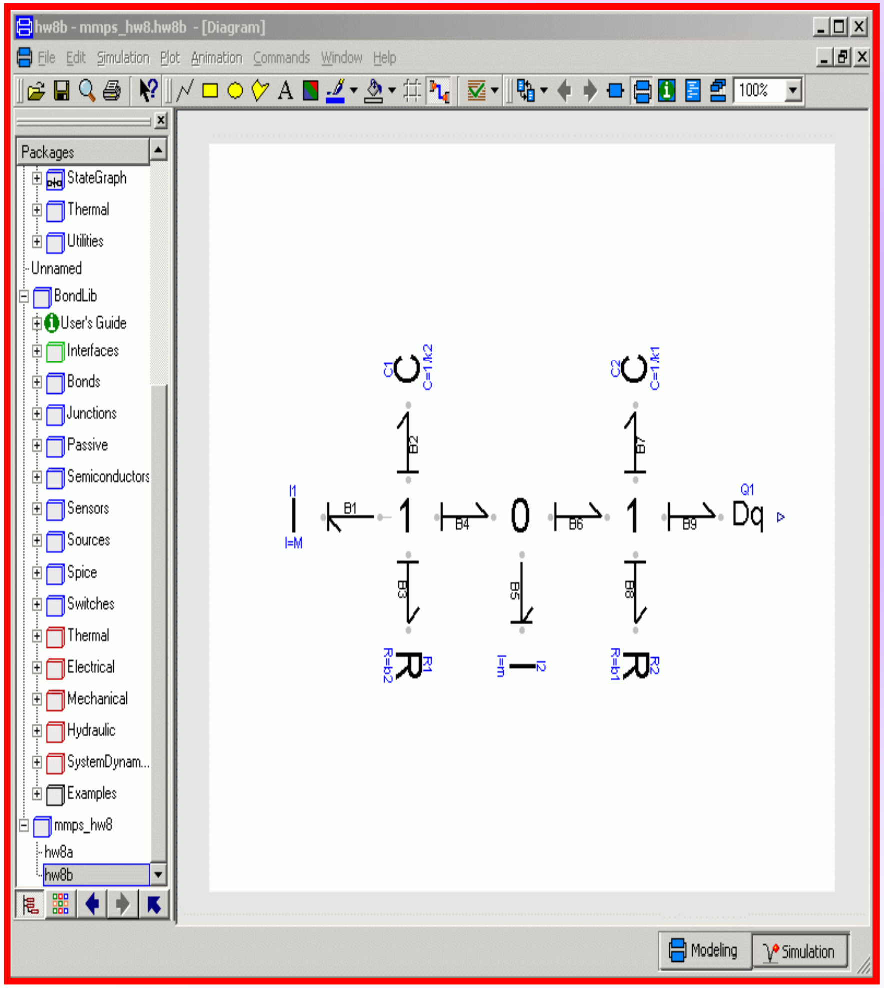
The bond graph can be simplified somewhat using the diamond property.

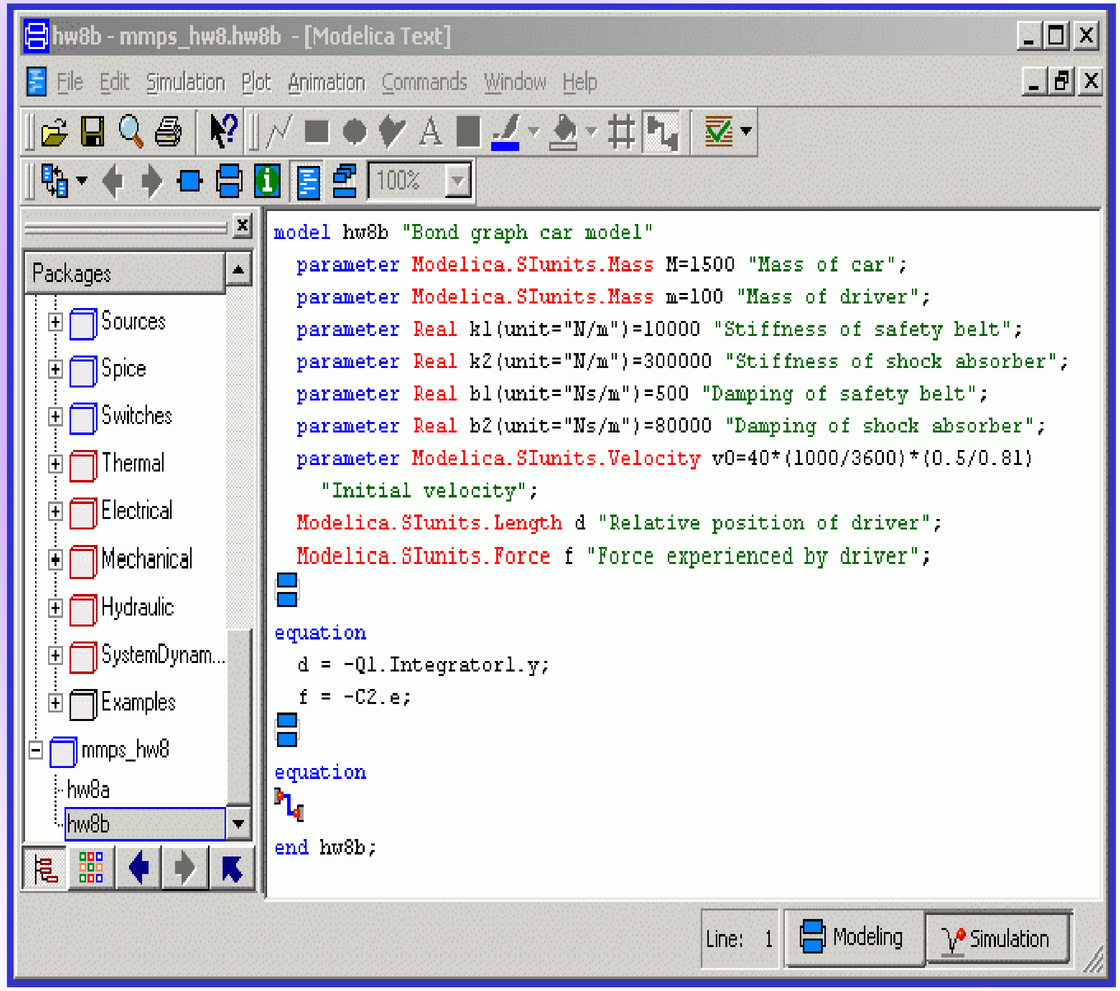
# Mechanical System XIII




We can now introduce the causality strokes.

# Mechanical System XIV

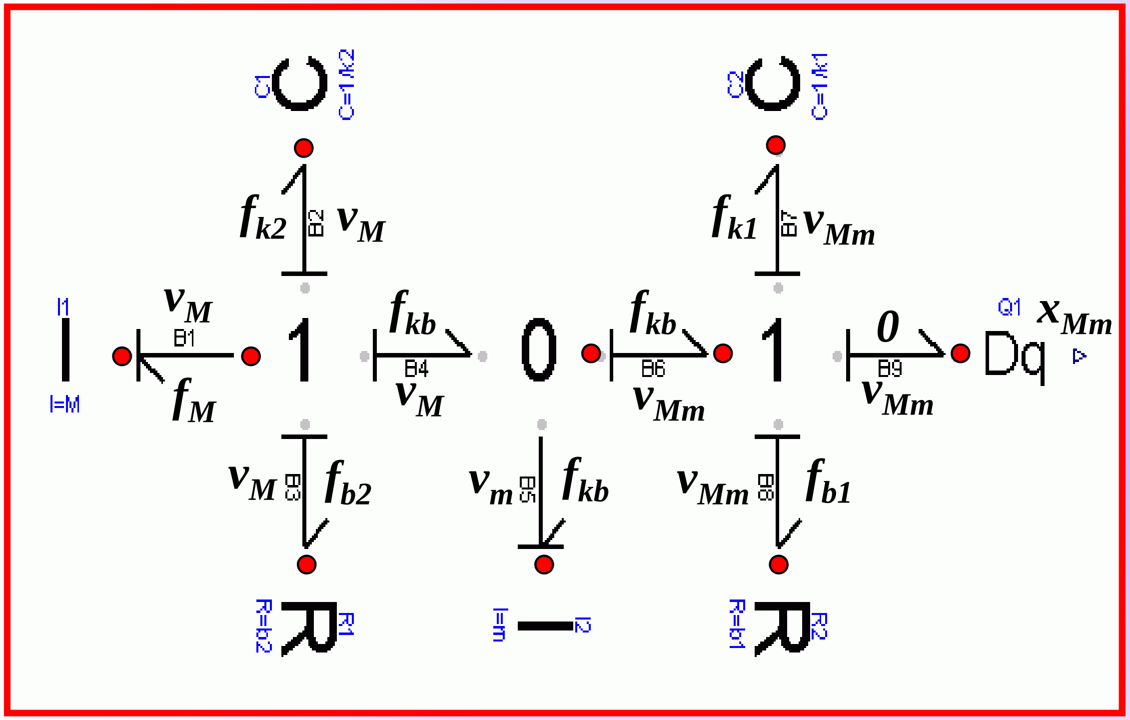


# Mechanical System XV

- Read the model equations out of the simplified bond graph.
- What is the model order?
- Which are the natural state variables?
- We now wish to include the relative position and the relative velocity of the spring representing the seat belt among the set of desired state variables.
- Use the state selection algorithm to derive a modified set of equations that make use of the desired state variables.

# Mechanical System XVI



$$dv_M/dt = f_M/M$$
$$f_M = -(f_{k2} + f_{b2} + f_{kb})$$
$$df_{k2}/dt = v_M/k_2$$
$$f_{b2} = b_2 \cdot v_M$$
$$dv_m/dt = f_{kb}/m$$
$$v_{Mm} = v_M - v_m$$
$$f_{kb} = f_{k1} + f_{b1}$$
$$df_{k1}/dt = v_{Mm}/k_1$$
$$f_{b1} = b_1 \cdot v_{Mm}$$
$$dx_{Mm}/dt = v_{Mm}$$

# Mechanical System XVI

$$dv_M/dt = f_M/M$$
$$f_M = -(f_{k2} + f_{b2} + f_{kb})$$
$$df_{k2}/dt = v_M/k_2$$
$$f_{b2} = b_2 \cdot v_M$$
$$dv_m\cancel{/dt} = f_{kb}/m$$
$$v_{Mm} = v_M - v_m$$
$$f_{kb} = f_{k1} + f_{b1}$$
$$df_{k1}/dt = v_{Mm}/k_1$$
$$f_{b1} = b_1 \cdot v_{Mm}$$
$$dx_{Mm}/dt = v_{Mm} \quad \surd$$

Differentiate this equation.

$$dv_{Mm}/dt = dv_M/dt - dv_m\cancel{/dt}$$