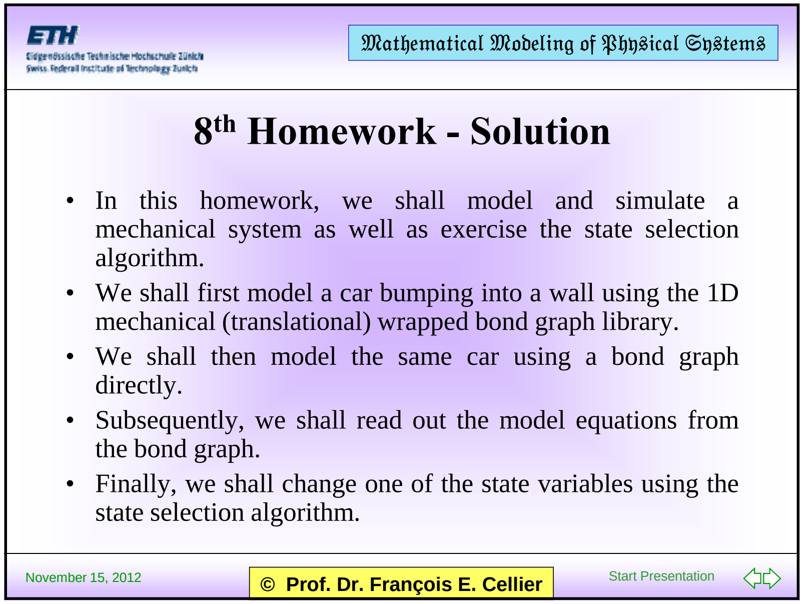
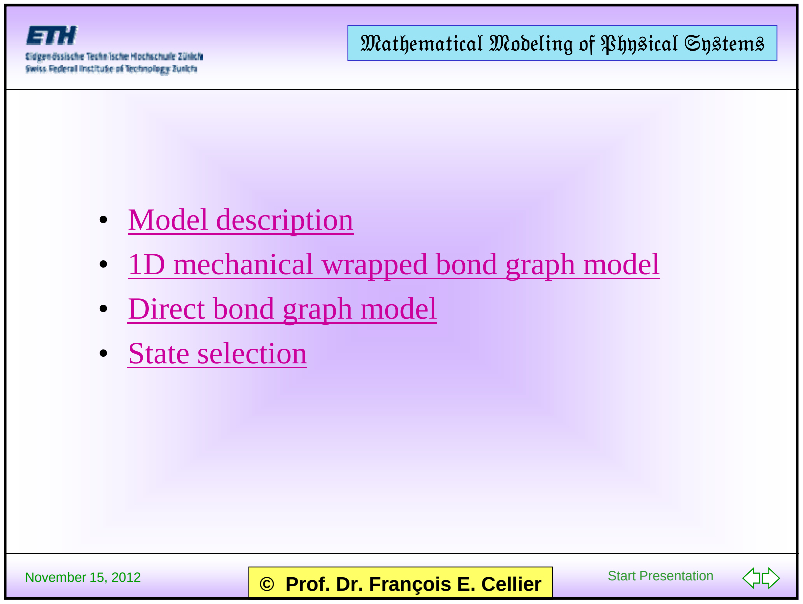
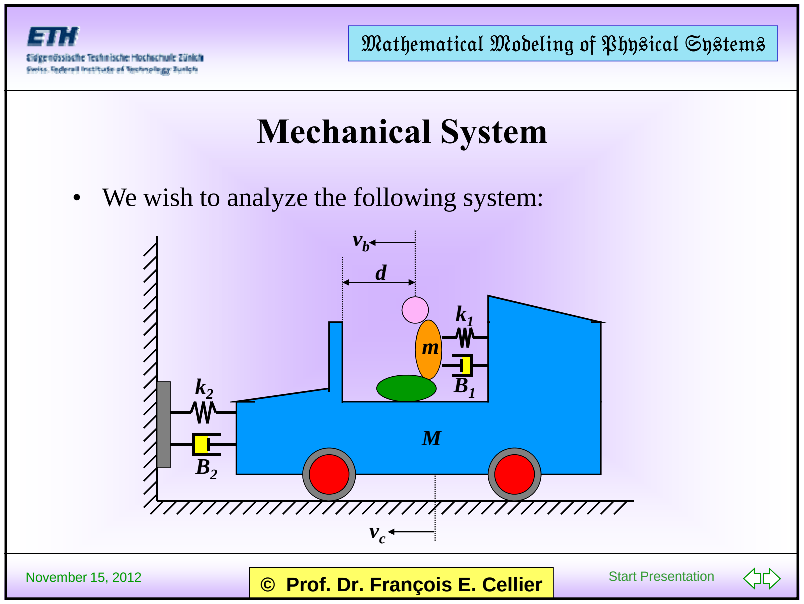


8th Homework - Solution

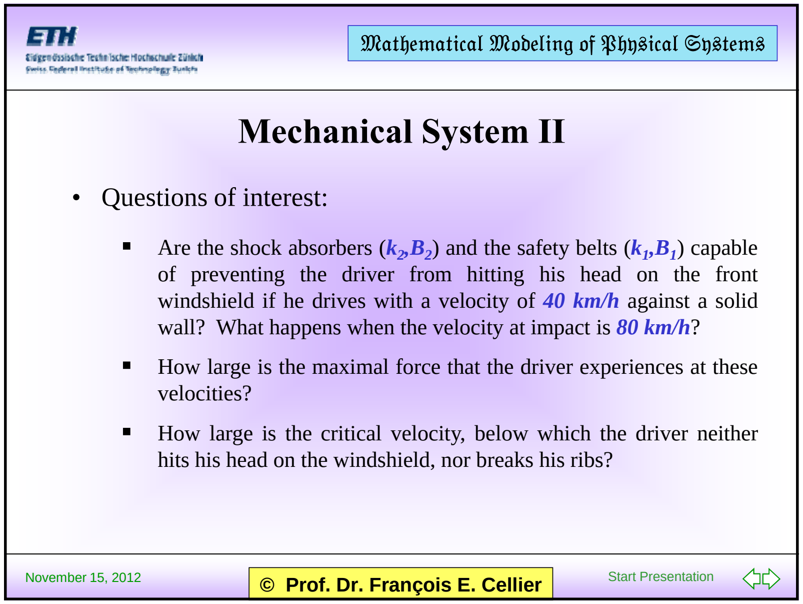
- In this homework, we shall model and simulate a mechanical system as well as exercise the state selection algorithm.
- We shall first model a car bumping into a wall using the 1D mechanical (translational) wrapped bond graph library.
- We shall then model the same car using a bond graph directly.
- Subsequently, we shall read out the model equations from the bond graph.
- Finally, we shall change one of the state variables using the state selection algorithm.





Mechanical System

- We wish to analyze the following system:



Mechanical System II

- Questions of interest:
  - Are the shock absorbers (k_2,B_2) and the safety belts (k_1,B_1) capable of preventing the driver from hitting his head on the front windshield if he drives with a velocity of *40 km/h* against a solid wall? What happens when the velocity at impact is *80 km/h*?
  - How large is the maximal force that the driver experiences at these velocities?
  - How large is the critical velocity, below which the driver neither hits his head on the windshield, nor breaks his ribs?

Mechanical System III

- Data:

  Mass of vehicle (M) = 1500 kg
  Mass of driver (m) = 100 kg
  Stiffness of safety belt (k_1) = 10’000 N/m
  Stiffness of shock absorber (k_2) = 300’000 N/m
  Damping of safety belt (B_1) = 500 Ns/m
  Damping of shock absorber (B_2) = 80’000 Ns/m

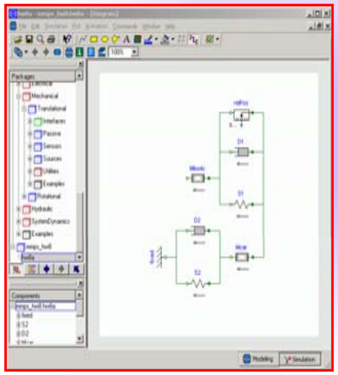
- Limit values:

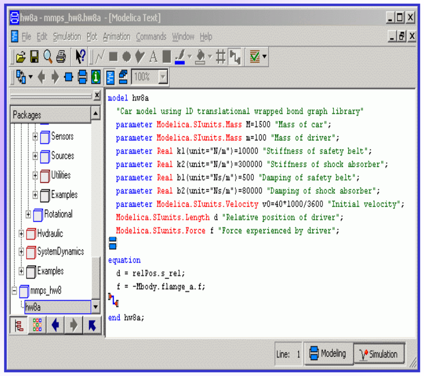
  Safety belt tested up to (F_1) < 13’340 N
  Ribs break beyond (F_2) > 6670 N
  Distance to windshield (d) = 0.5 m

Mechanical System IV

- Model the car and the driver using two sliding masses of the translational sub-library of the mechanical sub-library of ***BondLib***.
- Simulate the system across ***0.5 sec*** of simulated time, and answer the questions that were raised before.

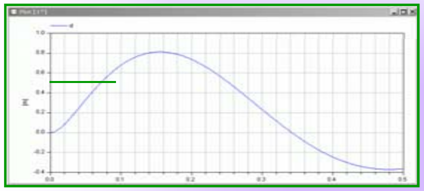
Mechanical System V



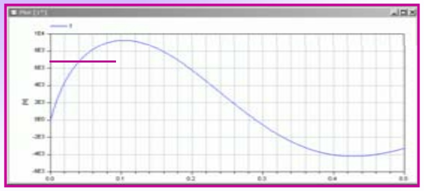


```
model hw8a
  "Car model using 1D translational wrapped bond graph library"
  parameter Modelica.SIunits.Mass M=1500 "Mass of car";
  parameter Modelica.SIunits.Mass m=100 "Mass of driver";
  parameter Real k1(unit="N/m")=10000 "Stiffness of safety belt";
  parameter Real k2(unit="N/m")=300000 "Stiffness of shock absorber";
  parameter Real b1(unit="Ns/m")=500 "Damping of safety belt";
  parameter Real b2(unit="Ns/m")=80000 "Damping of shock absorber";
  parameter Modelica.SIunits.Velocity v0=40*1000/3600 "Initial velocity";
  Modelica.SIunits.Length d "Relative position of driver";
  Modelica.SIunits.Force f "Force experienced by driver";

equation
  d = relPos.s_rel;
  f = -Mbody.flange_a.f;

end hw8a;
```

Mechanical System VI

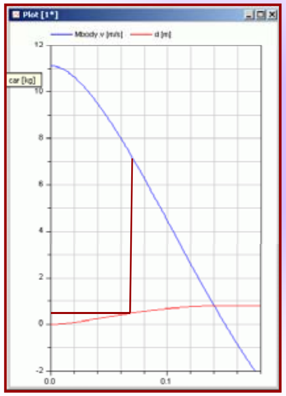


The driver will break his ribs.



The driver’s head will hit the windshield.

Mechanical System VII



At the time of the impact with the windshield, the head of the driver has a velocity of *7 m/s = 25.2 km/h*.

Mechanical System VIII

The results are so depressing that a simulation with an initial velocity of 80 km/h is hardly necessary!

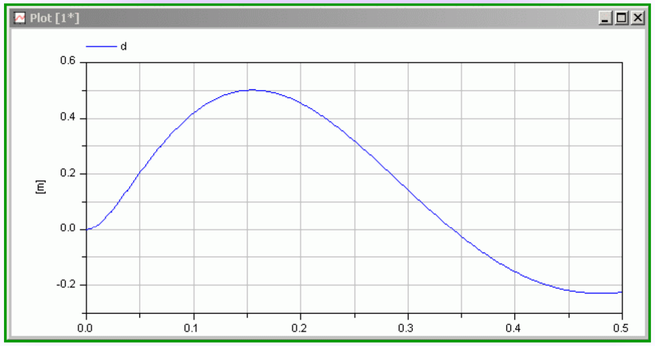
Mechanical System IX

- The system is linear. Hence we can compute the maximal velocity below which the head won't hit the windshield:

  $$v_0 = 40*0.5/0.81 = 24.7 \text{ km/h}$$

  where 0.81 m is the largest distance in the simulation that we just carried out.
- Let us verify the results.

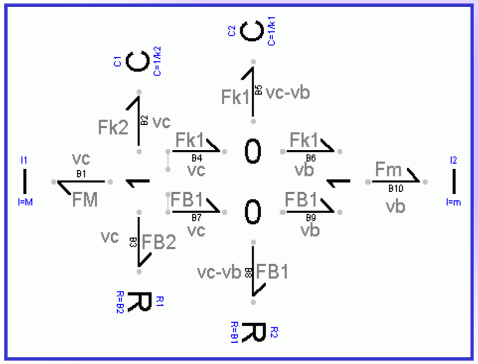
Mechanical System X



Mechanical System XI

- Draw a bond graph of this system.
- Simplify the bond graph using the diamond property.
- Add causality strokes.
- Simulate the simplified bond graph model using ***BondLib***, and compare the results with those obtained earlier.

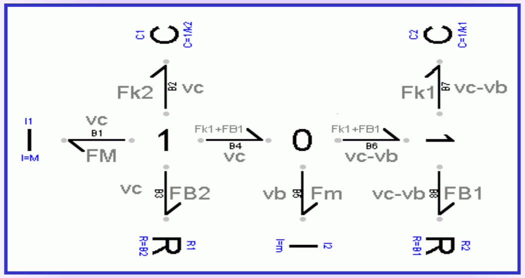
Mechanical System XII



The velocity of the car is entered as initial condition to the two inertias.

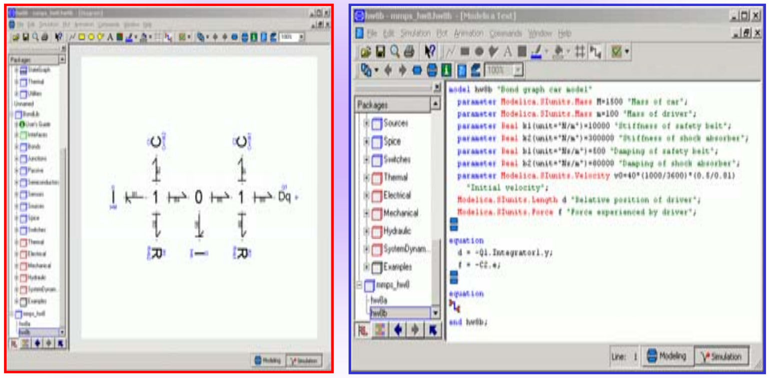
The bond graph can be simplified somewhat using the diamond property.

Mechanical System XIII



We can now introduce the causality strokes.

Mechanical System XIV

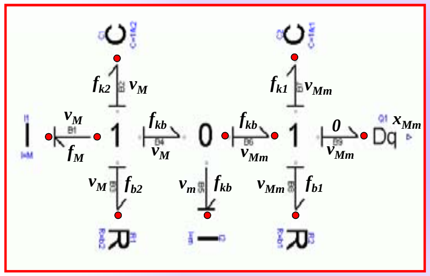


```
model hw0b "Bond graph car model"
  parameter Modelica.SIunits.Mass M=1500 "Mass of car";
  parameter Modelica.SIunits.Mass m=100 "Mass of driver";
  parameter Real k1(unit="N/m")=10000 "Stiffness of safety belt";
  parameter Real k2(unit="N/m")=300000 "Stiffness of shock absorber";
  parameter Real b1(unit="Ns/m")=600 "Damping of safety belt";
  parameter Real b2(unit="Ns/m")=60000 "Damping of shock absorber";
  parameter Modelica.SIunits.Velocity v0=40*(1000/3600)*(0.5/0.01)
    "Initial velocity";
  Modelica.SIunits.Length d "Relative position of driver";
  Modelica.SIunits.Force f "Force experienced by driver";

equation
  d = -Q1.Integrator1.y;
  f = -C2.e;

equation

end hw0b;
```

Mechanical System XV

- Read the model equations out of the simplified bond graph.
- What is the model order?
- Which are the natural state variables?
- We now wish to include the relative position and the relative velocity of the spring representing the seat belt among the set of desired state variables.
- Use the state selection algorithm to derive a modified set of equations that make use of the desired state variables.

Mechanical System XVI



$dv_M/dt = f_M/M$
$f_M = -(f_{k2} + f_{b2} + f_{kb})$
$df_{k2}/dt = v_M/k_2$
$f_{b2} = b_2 \cdot v_M$
$dv_m/dt = f_{kb}/m$
$v_{Mm} = v_M - v_m$
$f_{kb} = f_{k1} + f_{b1}$
$df_{k1}/dt = v_{Mm}/k_1$
$f_{b1} = b_1 \cdot v_{Mm}$
$dx_{Mm}/dt = v_{Mm}$

Mechanical System XVI

$dv_M/dt = f_M/M$
$f_M = -(f_{k2} + f_{b2} + f_{kb})$
$df_{k2}/dt = v_M/k_2$
$f_{b2} = b_2 \cdot v_M$
$dv_m\cancel{/dt} = f_{kb}/m$
$v_{Mm} = v_M - v_m$ ← Differentiate this equation.
$f_{kb} = f_{k1} + f_{b1}$
$df_{k1}/dt = v_{Mm}/k_1$
$f_{b1} = b_1 \cdot v_{Mm}$
$dx_{Mm}/dt = v_{Mm}$ √

$dv_{Mm}/dt = dv_M/dt - dv_m\cancel{/dt}$