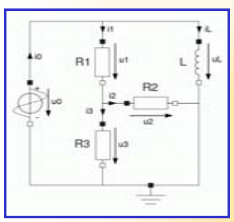
Bond Graphs II

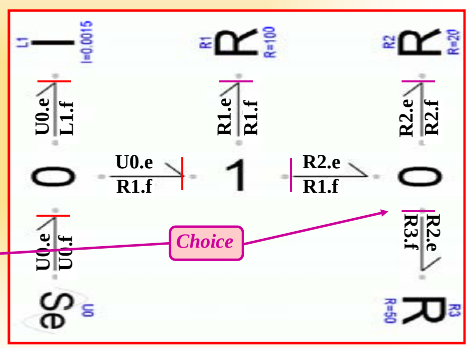
- In this class, we shall deal with the effects of algebraic loops and structural singularities on the bond graphs of physical systems.
- We shall also analyze the description of mechanical systems by means of bond graphs.

Table of Contents

Algebraic Loops

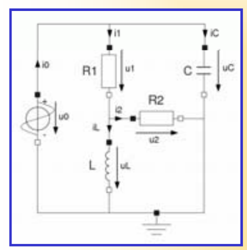


$U0.e = f(t)$
$U0.f = L1.f + R1.f$
$dL1.f/dt = U0.e / L1$
$R3.f = R1.f - R2.f$
$R2.e = R3 \cdot R3.f$
$R2.f = R2.e / R2$
$R1.f = R1.e / R1$
$R1.e = U0.e - R2.e$

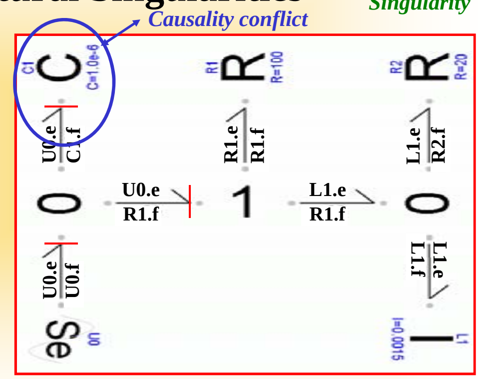


Structural Singularities

⇒ *Structural Singularity*



$U0.e = f(t)$
$U0.f = C1.f + R1.f$



Bond Graphs of Mechanical Systems I

- The two adjugate variables of the mechanical translational system are the ***force f*** as well as the ***velocity v***.
- You certainly remember the classical question posed to students in grammar school: *If one eagle flies at an altitude of 100 m above ground, how high do two eagles fly?* Evidently, ***position*** and ***velocity*** are ***intensive variables*** and therefore should be treated as ***potentials***.
- However, if one eagle can carry one sheep, two eagles can carry two sheep. Consequently, the ***force*** is an ***extensive variable*** and therefore should be treated as a ***flow variable***.

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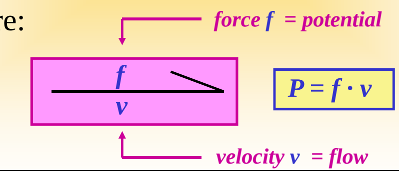
Bond Graphs of Mechanical Systems II

- Sadly, the bond graph community chose the reverse definition. ***"Velocity"*** gives the impression of a ***movement*** and therefore of a ***flow***.
- We shall show that it is always possible mathematically to make either of the two assumptions ***(duality principle)***.
- Therefore:



$P = f \cdot v$

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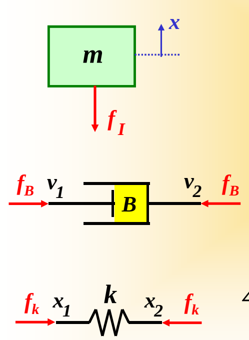
Passive Mechanical Elements in Bond Graph Notation

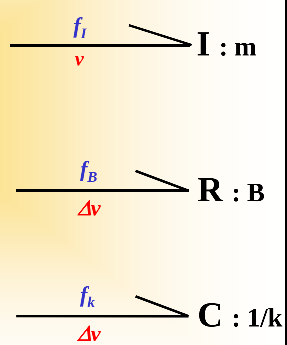


$f_I = m \cdot dv/dt$

$f_B = B \cdot \Delta v$

$\Delta x = f_k / k$

$\Rightarrow \Delta v = (1/k) \cdot df_k/dt$



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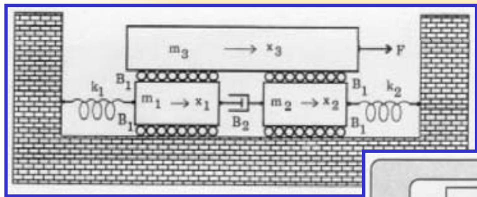
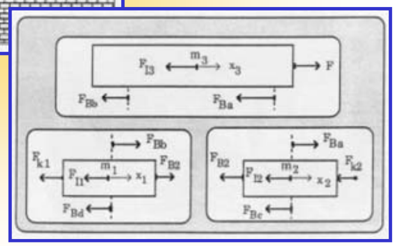
Selection of State Variables

- The ***"classical" representation of mechanical systems*** makes use of the ***absolute motions of the masses*** (position and velocity) as its state variables.
- The ***multi-body system representation in Dymola*** makes use of the ***relative motions of the joints*** (position and velocity) as its state variables.
- The ***bond graph representation*** selects the ***absolute velocities of masses*** as one type of state variable, and the ***spring forces*** as the other.

An Example I

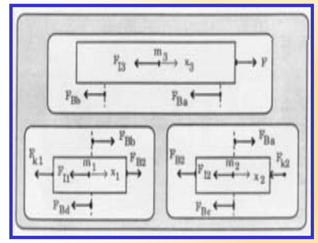


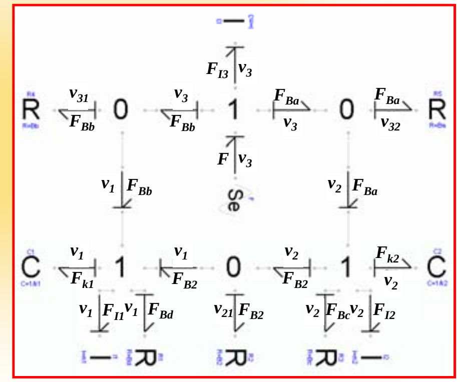
The cutting forces are represented by springs and friction elements that are placed between bodies at a 0-junction.



The D'Alembert principle is formulated in the bond graph representation as a grouping of all forces that attack a body around a junction of type 1.

An Example II





The sign rule follows here automatically, and the modeler rarely makes any mistake relating to it.

References

- Borutzky, W. and F.E. Cellier (1996), "Tearing Algebraic Loops in Bond Graphs," *Trans. of SCS*, **13**(2), pp. 102-115.
- Borutzky, W. and F.E. Cellier (1996), "Tearing in Bond Graphs With Dependent Storage Elements," *Proc. Symposium on Modelling, Analysis, and Simulation*, CESA'96, IMACS MultiConference on Computational Engineering in Systems Applications, Lille, France, vol. 2, pp. 1113-1119.