The Theoretical Underpinnings of the Bond Graph Methodology

- In this lecture, we shall look more closely at the theoretical underpinnings of the bond graph methodology: the four base variables, the properties of capacitive and inductive storage elements, and the duality principle.
- We shall also introduce the two types of energy transducers: the transformers and the gyrators, and we shall look at hydraulic bond graphs.

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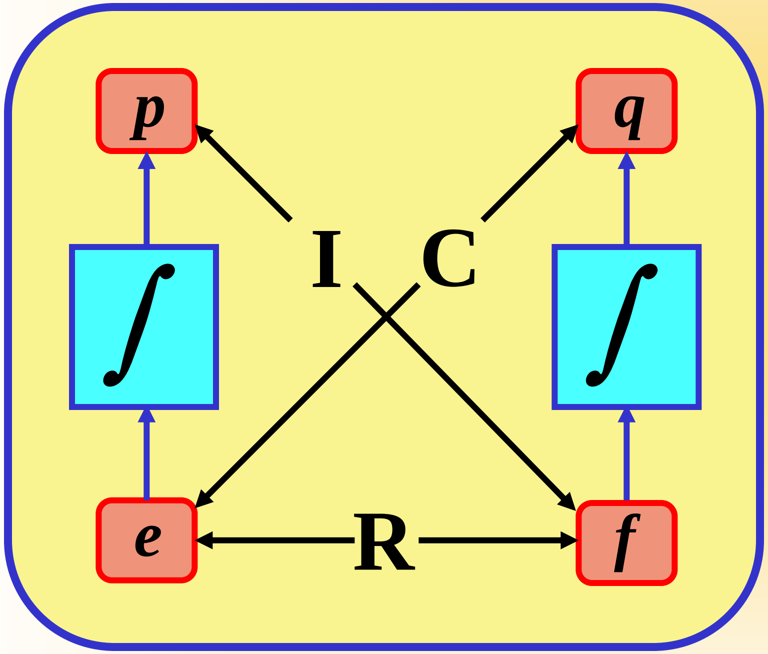
The Four Base Variables of the Bond Graph Methodology

- Beside from the two adjugate variables ***e*** and ***f***, there are two additional physical quantities that play an important role in the bond graph methodology:

Generalized Momentum: $p = \int e \cdot dt$

Generalized Position: $q = \int f \cdot dt$

Relations Between the Base Variables



Resistor: $e = R(f)$

Capacity: $q = C(e)$

Inductivity: $p = I(f)$

⇑

Arbitrarily non-linear functions in 1st and 3rd quadrants

⇒ *There cannot exist other storage elements besides* C *and* I.

Linear Storage Elements

General capacitive equation: $q = C(e)$

Linear capacitive equation: $q = C \cdot e$

Linear capacitive equation differentiated: $f = C \cdot \frac{de}{dt}$

↑ *"Normal" capacitive equation, as hitherto commonly encountered.*

	Effort	**Flow**	**Generalized Momentum**	**Generalized Position**
	e	***f***	***p***	***q***
Electrical Circuits	Voltage u (V)	Current i (A)	Magnetic Flux Φ (V·sec)	Charge q (A·sec)
Translational Systems	Force F (N)	Velocity v (m / sec)	Momentum M (N·sec)	Position x (m)
Rotational Systems	Torque T (N·m)	Angular Velocity ω (rad / sec)	Torsion T (N·m·sec)	Angle φ (rad)
Hydraulic Systems	Pressure p (N / m^2)	Volume Flow q (m^3 / sec)	Pressure Momentum Γ (N·sec / m^2)	Volume V (m^3)
Chemical Systems	Chem. Potential μ (J / mol)	Molar Flow ν (mol/sec)	-	Number of Moles n (mol)
Thermodynamic Systems	Temperature T (K)	Entropy Flow S' (W / K)	-	Entropy S (J / K)

Hydraulic Bond Graphs I

- In hydrology, the two adjugate variables are the ***pressure*** p and the ***volume flow*** q. Here, the pressure is considered the potential variable, whereas the volume flow plays the role of the flow variable.

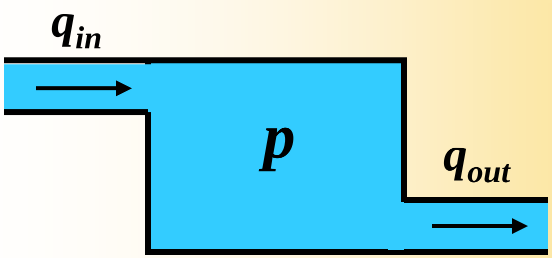
$$P_{hydr} = p \cdot q$$

$$\begin{aligned}[W] &= [N/m^2] \cdot [m^3/s] \\ &= kg \cdot m^{-1} \cdot s^{-2}] \cdot [m^3 \cdot s^{-1}] \\ &= [kg \cdot m^2 \cdot s^{-3}]\end{aligned}$$

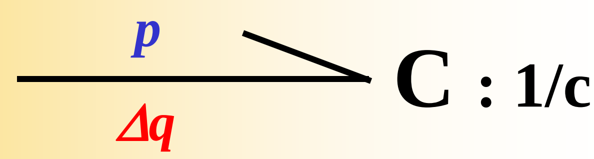
- The ***capacitive storage*** describes the compressibility of the fluid as a function of the pressure, whereas the ***inductive storage*** models the inertia of the fluid in motion.

Hydraulic Bond Graphs II

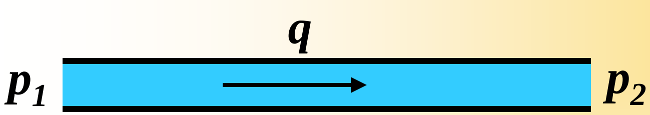
Compression:



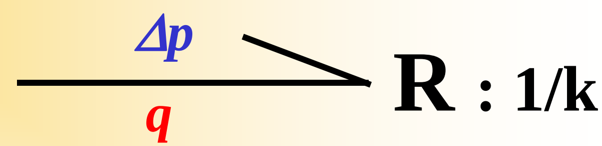
$$\frac{dp}{dt} = c \cdot (q_{in} - q_{out})$$



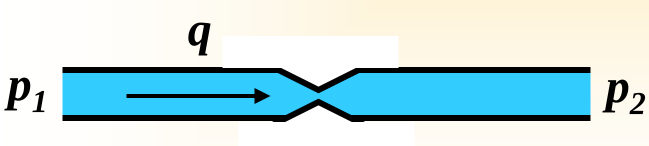
Laminar Flow:



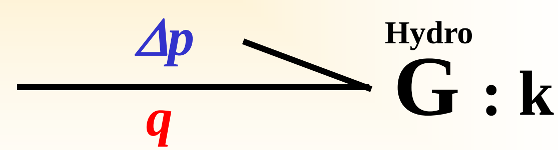
$$q = k \cdot \Delta p = k \cdot (p_1 - p_2)$$



Turbulent Flow:



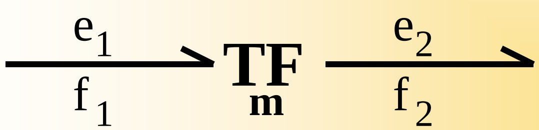
$$q = k \cdot \text{sign}(\Delta p) \cdot \sqrt{|\Delta p|}$$



Energy Conversion

- Beside from the elements that have been considered so far to describe the ***storage of energy*** (***C*** and ***I***) as well as its ***dissipation (conversion to heat)*** (***R***), two additional elements are needed, which describe the general ***energy conversion***, namely the ***Transformer*** and the ***Gyrator***.
- Whereas resistors describe the ***irreversible conversion of free energy into heat***, transformers and gyrators are used to model ***reversible energy conversion phenomena*** between identical or different forms of energy.

Transformers



Transformation: $e_1 = m \cdot e_2$ **(1)**

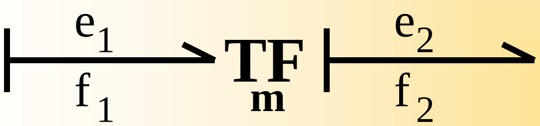
Energy Conservation: $e_1 \cdot f_1 = e_2 \cdot f_2$ **(2)**

$\Rightarrow (m \cdot e_2) \cdot f_1 = e_2 \cdot f_2$ **(3)**

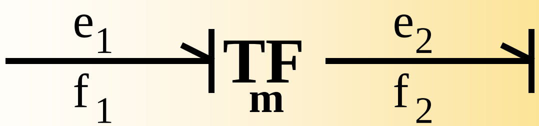
$\Rightarrow f_2 = m \cdot f_1$ **(4)**

$\Rightarrow$ ***The transformer may either be described by means of equations (1) and (2) or using equations (1) and (4).***

The Causality of the Transformer



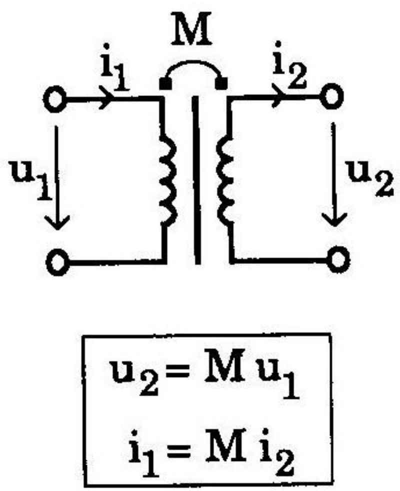
$$e_1 = m \cdot e_2$$
$$f_2 = m \cdot f_1$$



$$e_2 = e_1 / m$$
$$f_1 = f_2 / m$$

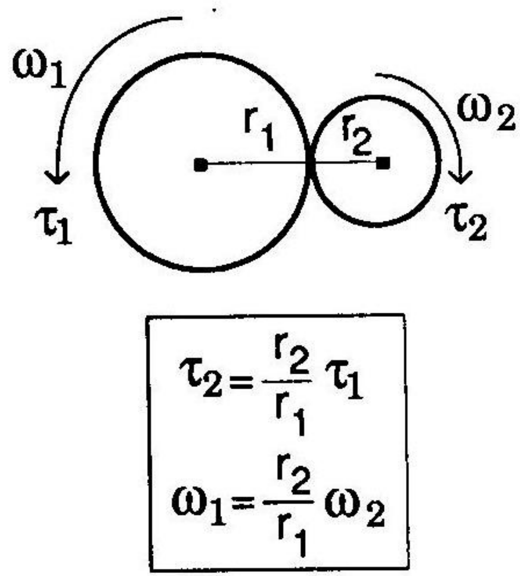
$\Rightarrow$ *As we have exactly one equation for the effort and another for the flow, it is mandatory that the transformer compute one effort variable and one flow variable. Hence there is one causality stroke at the TF element.*

Examples of Transformers



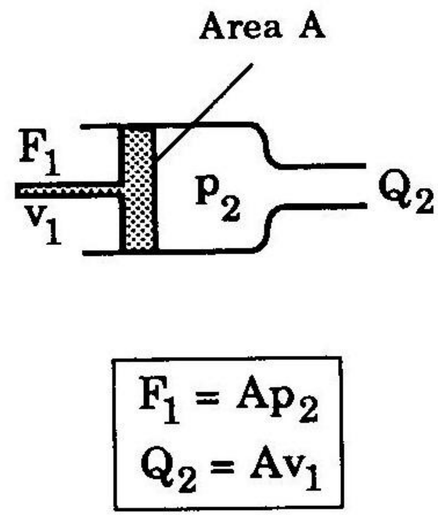
m = 1/M

Electrical Transformer
(in AC mode)



$m = r_1/r_2$

Mechanical Gear



m = A

Hydraulic Shock Absorber

Gyrators

Transformation: $e_1 = r \cdot f_2$ (1)

Energy Conservation: $e_1 \cdot f_1 = e_2 \cdot f_2$ (2)

$\Rightarrow (r \cdot f_2) \cdot f_1 = e_2 \cdot f_2$ (3)

$\Rightarrow e_2 = r \cdot f_1$ (4)

$\Rightarrow$ *The gyrator may either be described by means of equations (1) and (2) or using equations (1) and (4).*

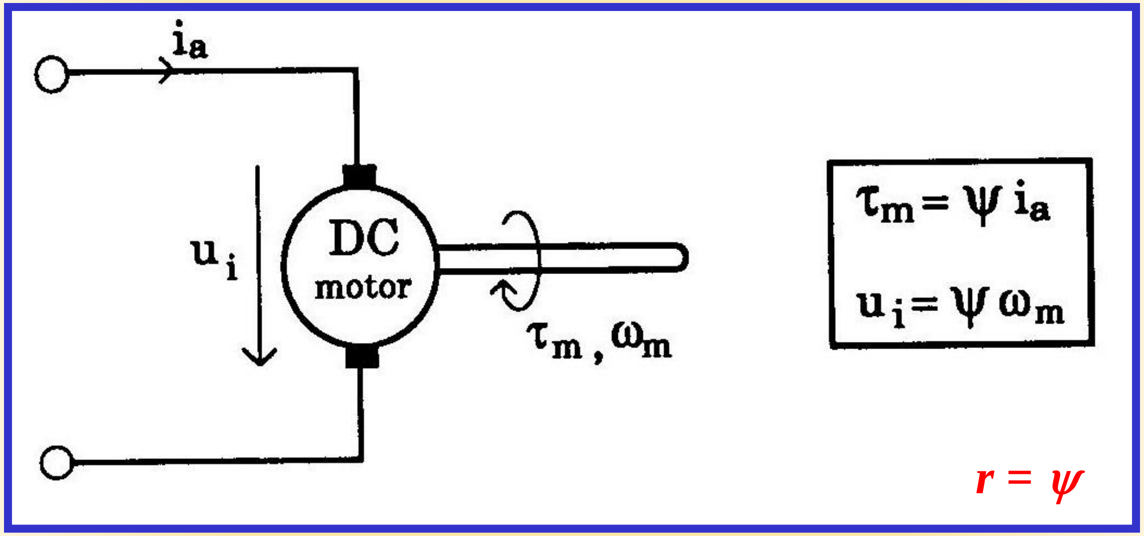
The Causality of the Gyrator

$$e_1 = r \cdot f_2$$
$$e_2 = r \cdot f_1$$

$$f_2 = e_1 / r$$
$$f_1 = e_2 / r$$

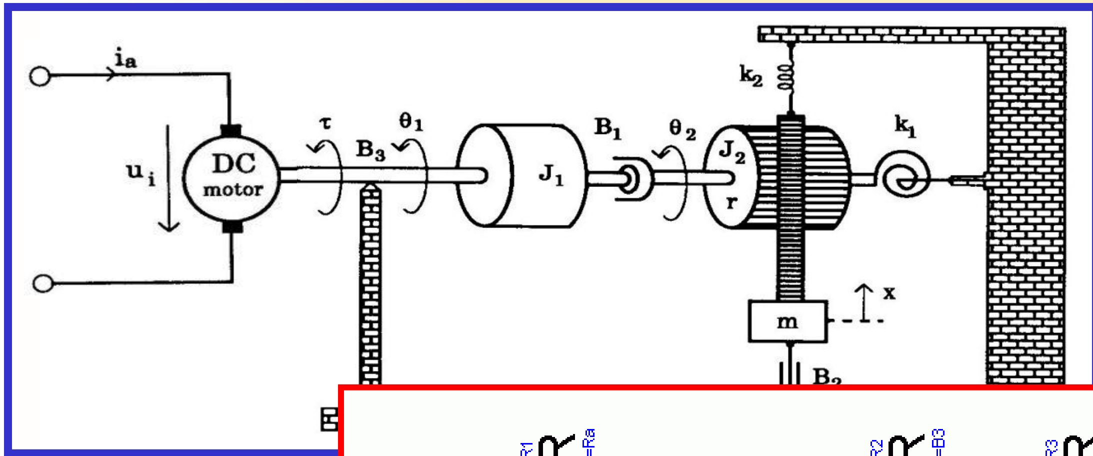
$\Rightarrow$ *As we must compute one equation to the left, the other to the right of the gyrator, the equations may either be solved for the two effort variables or for the two flow variables.*

Examples of Gyrators

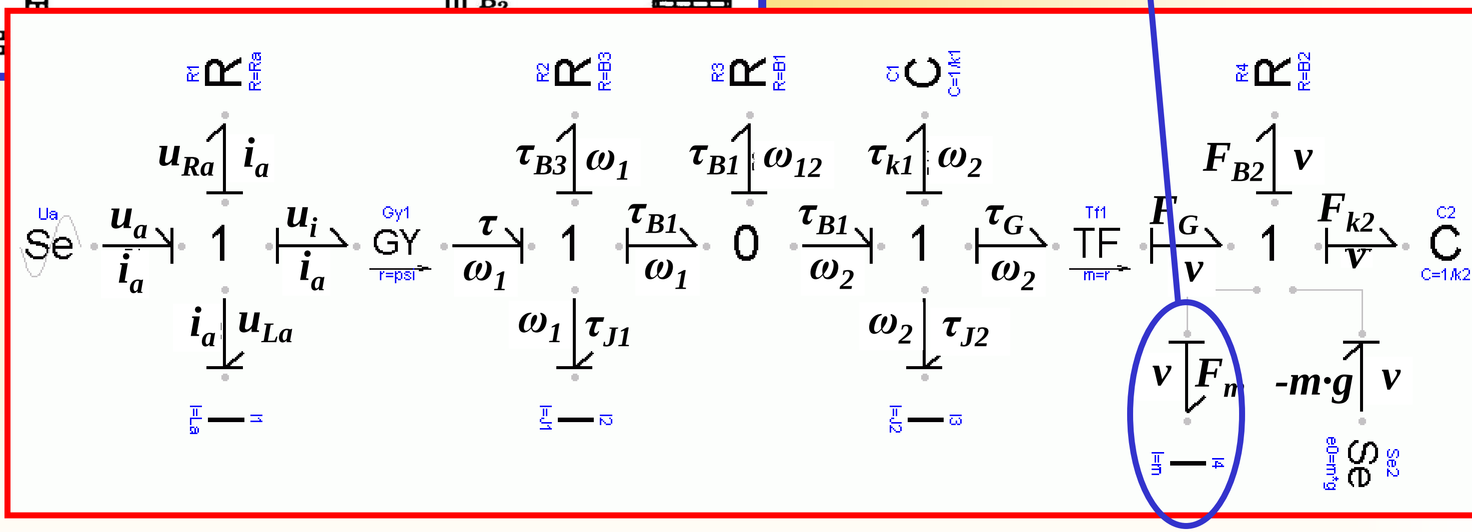


The DC motor generates a torque τ_m proportional to the armature current i_a, whereas the resulting induced Voltage u_i is proportional to the angular velocity ω_m.

Example of an Electromechanical System



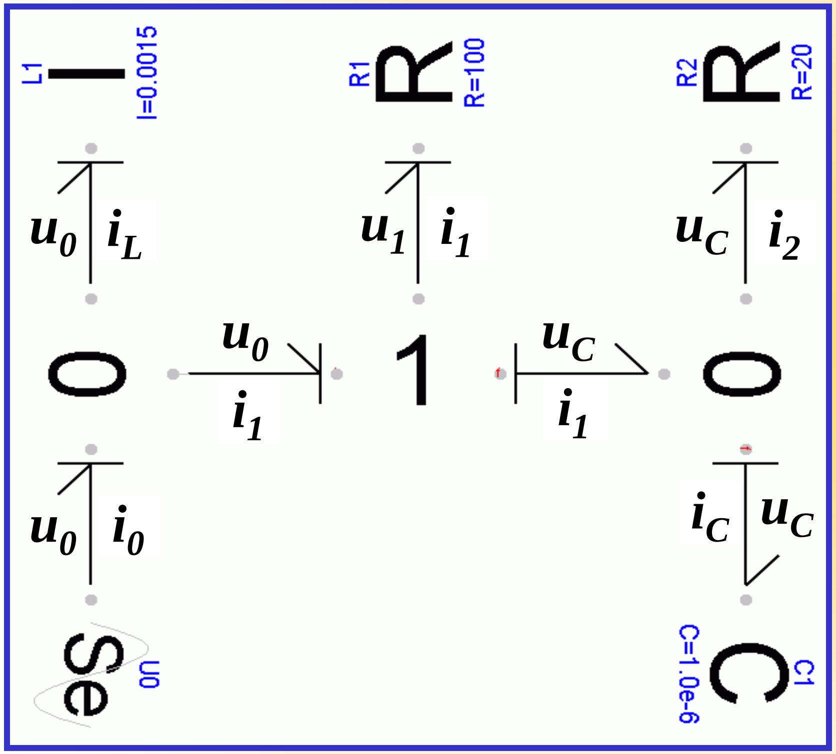
Causality conflict (caused by the mechanical gear)

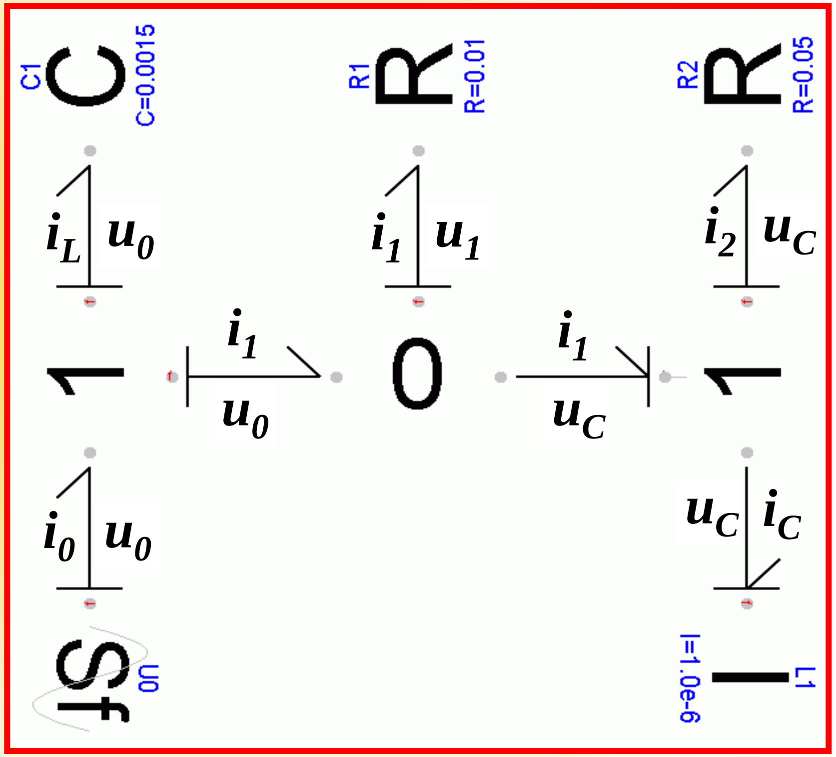


The Duality Principle

- It is always possible to ***"dualize"*** a bond graph by switching the definitions of the effort and flow variables.
- In the process of dualization, effort sources become flow sources, capacities turn into inductors, resistors are converted to conductors, and vice-versa.
- Transformers and gyrators remain the same, but their transformation values are inverted in the process.
- The two junctions exchange their type.
- The causality strokes move to the other end of each bond.

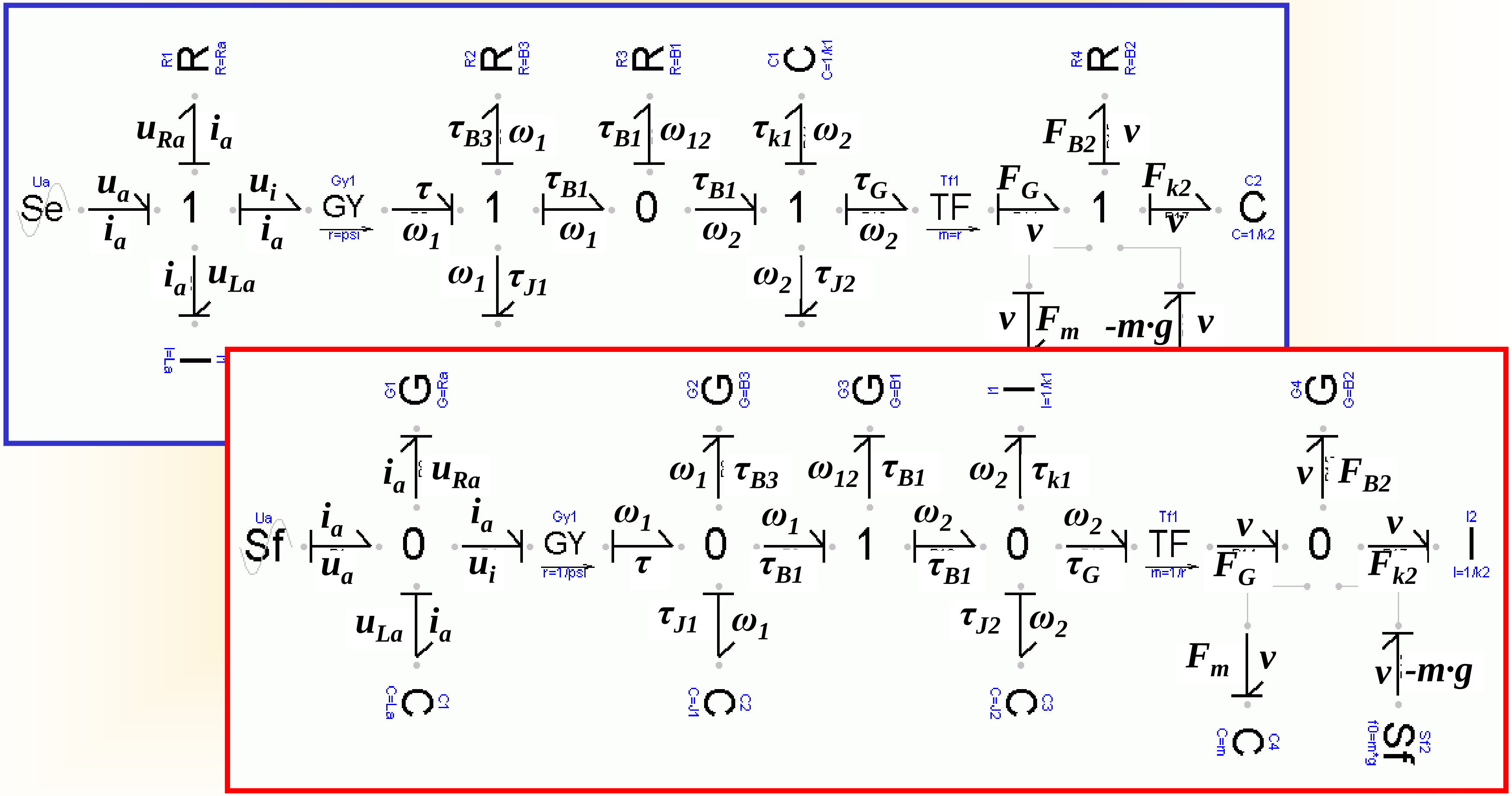
1st Example





The two bond graphs produce identical simulation results.

2nd Example



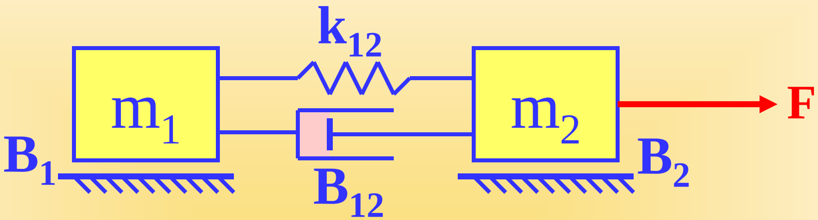
Partial Dualization

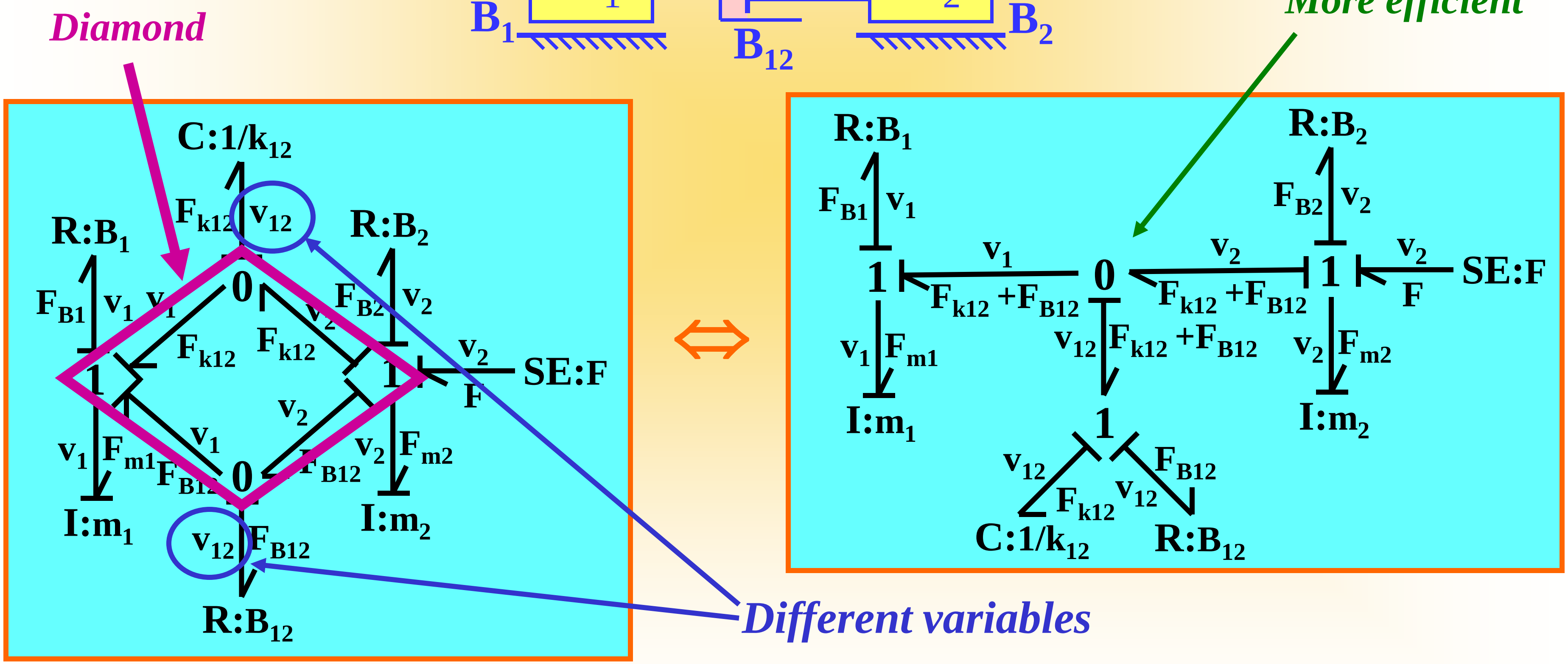
- It is always possible to dualize bond graphs only in parts.
  - It is particularly easy to partially dualize a bond graph at the transformers and gyrators. The two conversion elements thereby simply exchange their types.
  - For example, it may make sense to only dualize the mechanical side of an electromechanical bond graph, whereas the electrical side is left unchanged.
  - However, it is also possible to dualize the bond graph at any bond. Thereby, the "twisted" bond is turned into a gyrator with a gyration of ***r=1***.
  - Such a gyrator is often referred to as ***symplectic gyrator*** in the bond graph literature.

Manipulation of Bond Graphs

- Any physical system with concentrated parameters can be described by a bond graph.
- However, the bond graph representation is not unique, i.e., several different-looking bond graphs may represent identical equation systems.
- One type of ambiguity has already been introduced: the ***dualization***.
- However, there exist other classes of ambiguities that cannot be explained by dualization.

The Diamond Rule





References

- Cellier, F.E. (1991), *Continuous System Modeling*, Springer-Verlag, New York, Chapter 7.