Thermodynamics

- Until now, we have ignored the thermal domain. However, it is fundamental for the understanding of physics.
- We mentioned that energy can neither be generated nor destroyed ... yet, we immediately turned around and introduced elements such as ***sources*** and ***resistors***, which shouldn't exist at all in accordance with the above statement.
- In today's lecture, we shall analyze these phenomena in more depth.

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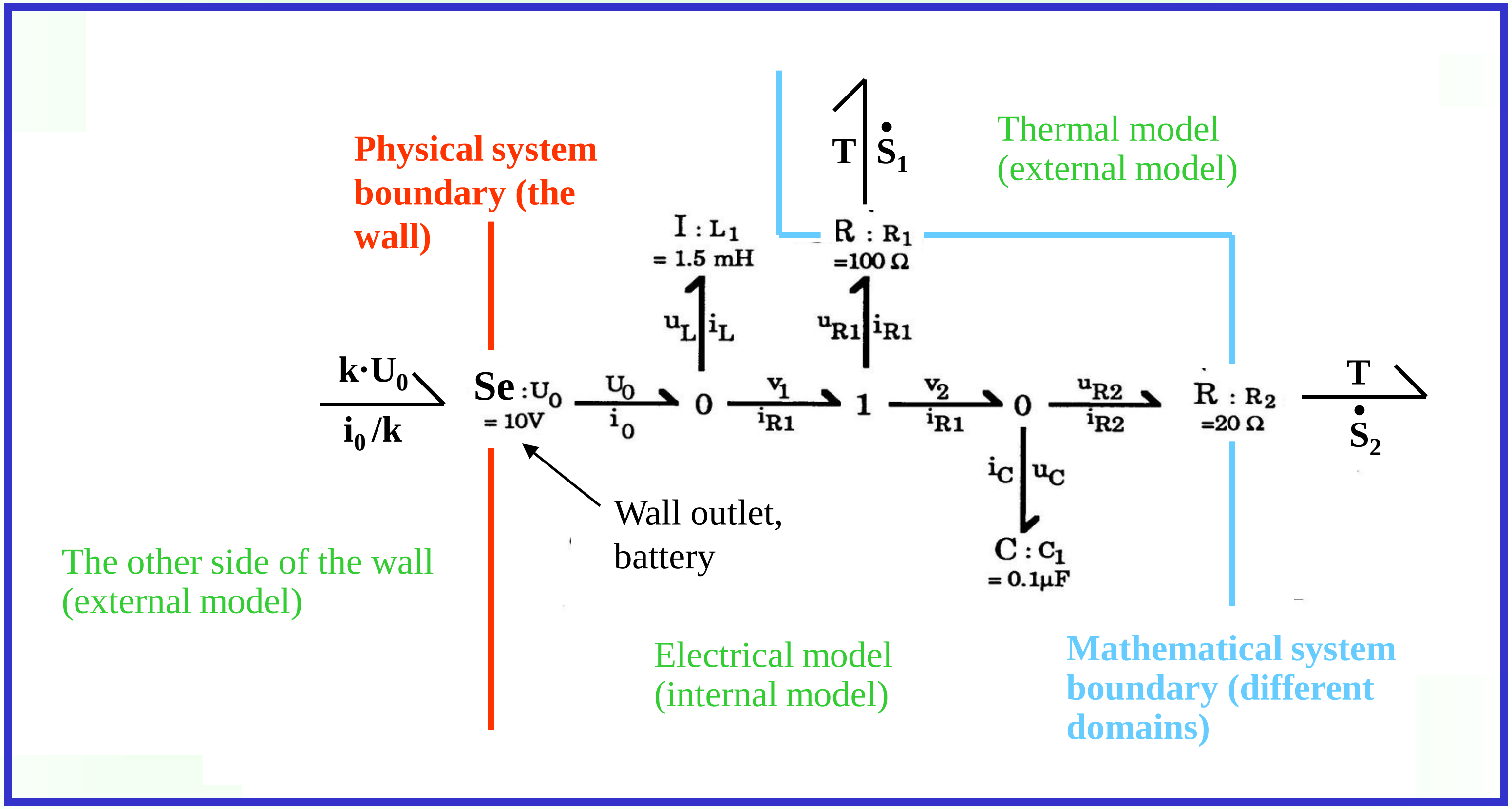
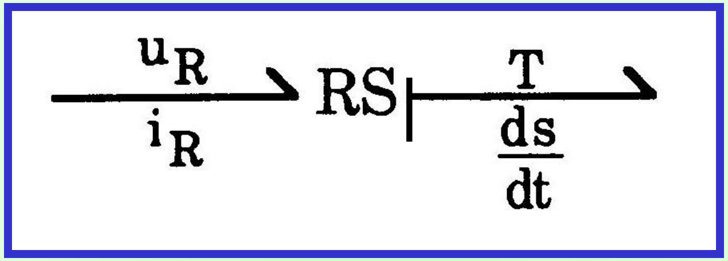
Energy Sources and Sinks



The Resistive Source



- The resistor converts free energy irreversibly into entropy.
- This fact is represented in the bond graph by a ***resistive source***, the RS-element.
- The causality of the thermal side is always such that the resistor is seen there as a ***source of entropy***, never as a source of temperature.
- Sources of temperature are non-physical.

Heat Conduction I

- Heat conduction in a well insulated rod can be described by the one-dimensional heat equation:

$$\frac{\partial T}{\partial t} = \sigma \frac{\partial^2 T}{\partial x^2}$$

- Discretization in space leads to:

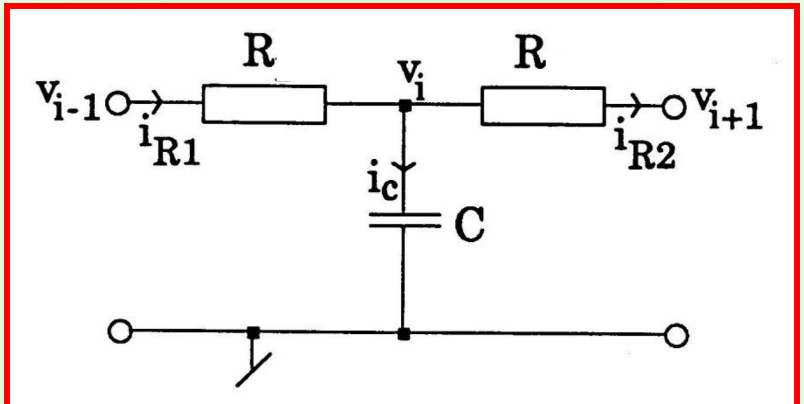
$$\frac{\partial^2 T(t, x_k)}{\partial x^2} \approx \frac{T(t, x_{k+1}) - 2T(t, x_k) + T(t, x_{k-1})}{\Delta x^2}$$

$$\frac{dT_k(t)}{dt} = \frac{\sigma}{\Delta x^2}[T_{k+1}(t) - 2T_k(t) + T_{k-1}(t)], \quad k \in \{1\}, \ldots, \{n\}$$

$$(\frac{\Delta x^2}{\sigma}) \cdot \frac{dT_i}{dt} = T_{i+1} - 2T_i + T_{i-1}$$

Heat Conduction II

- Consequently, the following electrical equivalence circuit may be considered:



$$dv_i/dt = i_C/C$$
$$i_C = i_{R1} - i_{R2}$$
$$v_{i-1} - v_i = R \cdot i_{R1}$$
$$v_i - v_{i+1} = R \cdot i_{R2}$$

$\Rightarrow$

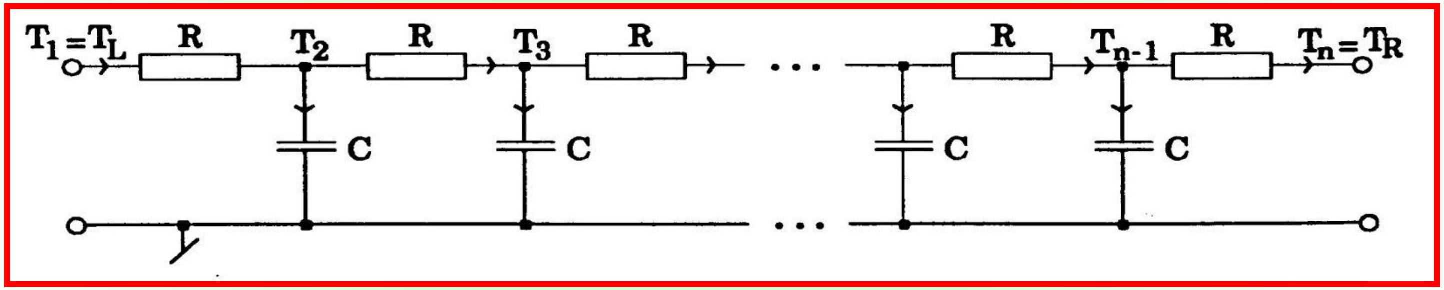
$$dv_i/dt = (i_{R1} - i_{R2})/C = (v_{i+1} - 2 \cdot v_i + v_{i-1})/(R \cdot C)$$

$\Rightarrow$

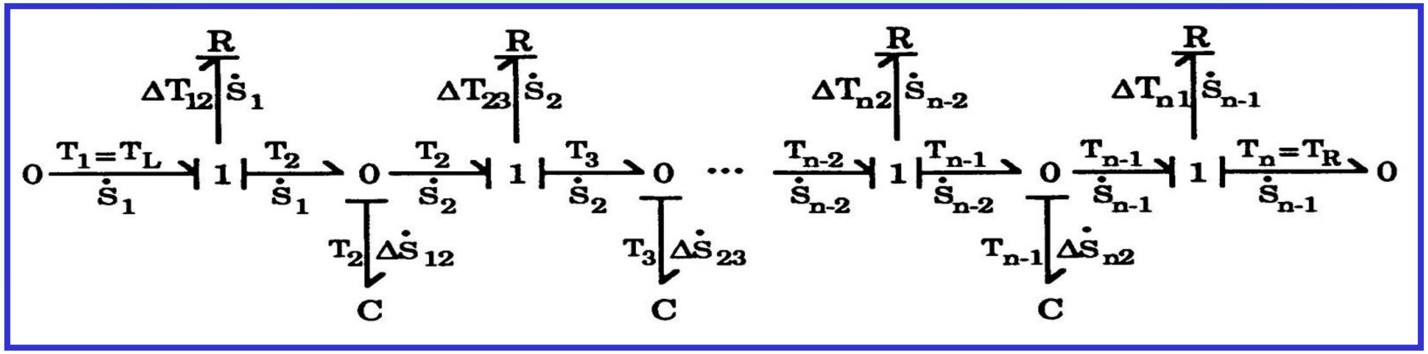
$$(R \cdot C) \cdot \frac{dv_i}{dt} = v_{i+1} - 2 \cdot v_i + v_{i-1}$$

Heat Conduction III

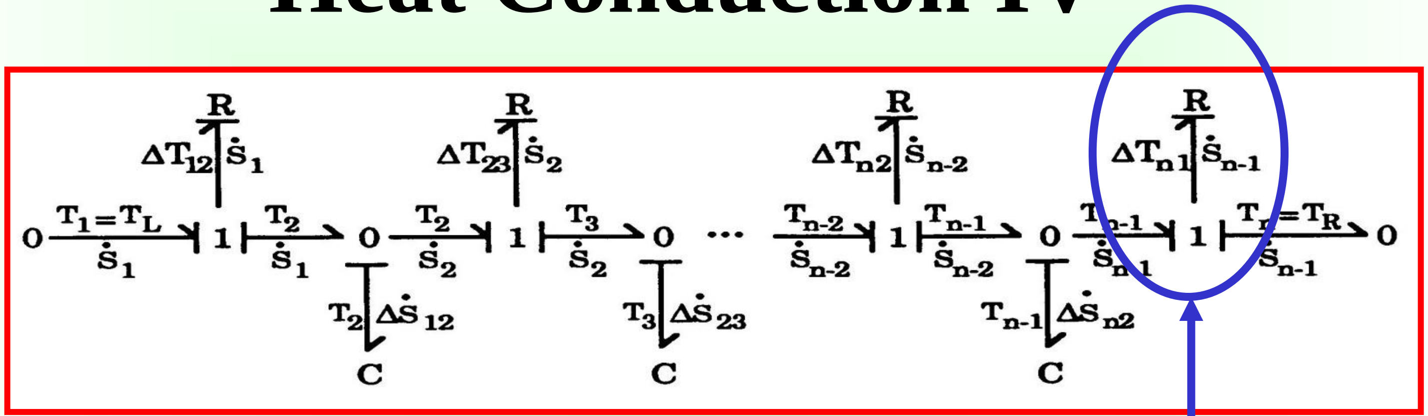
- As a consequence, heat conduction can be described by a series of such T-circuits:



In bond graph representation:



Heat Conduction IV



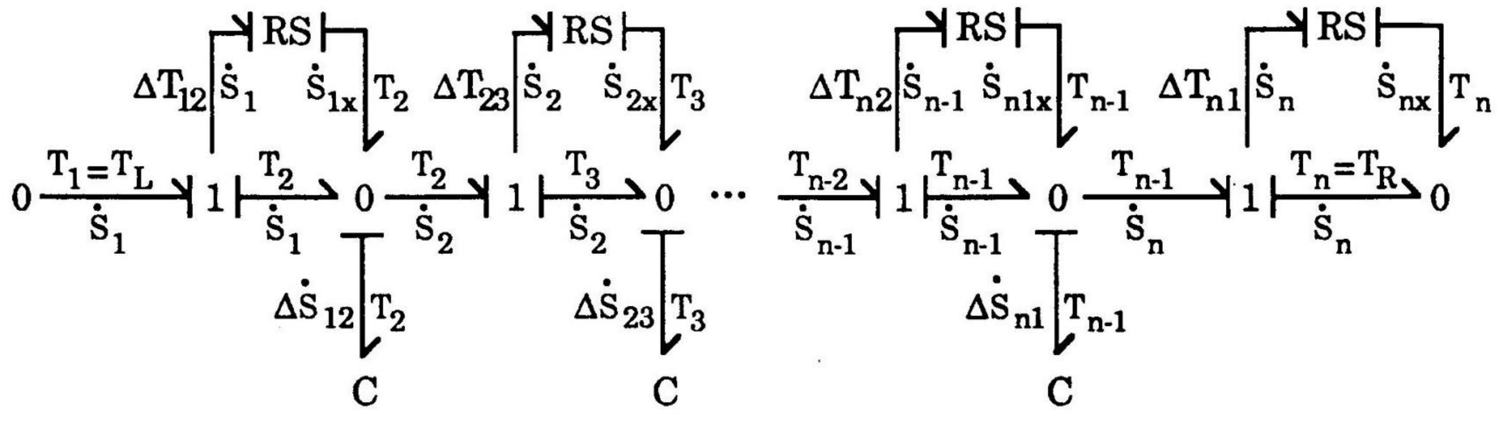
- This bond graph is exceedingly beautiful ...
  It only has one drawback ...
  It is most certainly incorrect!

- There are no energy sinks!

  A resistor may make sense in an electrical circuit, if the heating of the circuit is not of interest, but it is most certainly not meaningful, when the system to be described is itself in the thermal domain.

Heat Conduction V

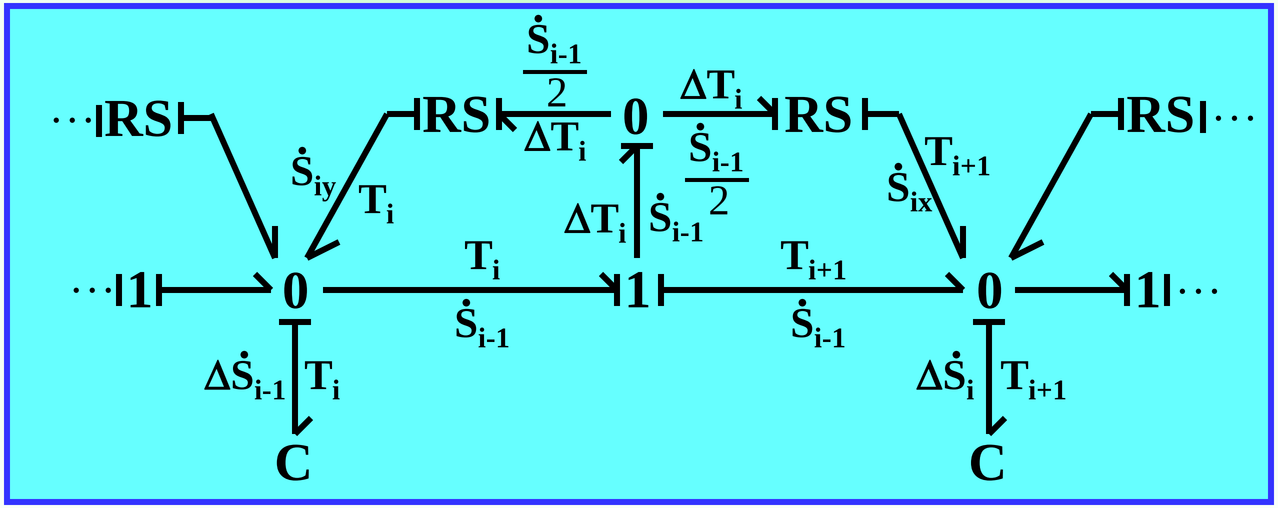
- The problem can be corrected easily by replacing each resistor by a resistive source.



- The temperature gradient leads to additional entropy, which is re-introduced at the nearest 0-junction.

Heat Conduction VI

- This provides a good approximation of the physical reality. Unfortunately, the resulting bond graph is asymmetrical, although the heat equation itself is symmetrical.
- A further correction removes the asymmetry.



Heat Flow

- The thermal power is the heat flow ***dQ/dt***. It is commonly computed as the product of two adjugate thermal variables, i.e.:

$$P = \dot{Q} = T \cdot \dot{S}$$

- It is also possible to treat heat flow as the primary physical phenomenon, and derive consequently from it an equation for computing the entropy:

$$\dot{S} = \dot{Q} / T$$

The Computation of *R* and *C* I

- The capacity of a long well insulated rod to conduct heat is proportional to the temperature gradient.

$$\Delta T = \theta \cdot \dot{Q} = \theta \cdot (T \cdot \dot{S}) = (\theta \cdot T) \cdot \dot{S} = R \cdot \dot{S}$$

$\Rightarrow$ $R = \theta \cdot T$ $\qquad$ θ = ***thermal resistance***

- where: $\theta = \dfrac{1}{\lambda} \cdot \dfrac{l}{A}$

λ = ***specific thermal conductance***

l = ***length of the rod***

A = ***cross-section of the rod***

$\Rightarrow$ $R = \theta \cdot T = \dfrac{\Delta x \cdot T}{\lambda \cdot A}$ $\qquad$ Δx = ***length of a segment***

The Computation of *R* and *C* II

- The capacity of a long well insulated rod to store heat satisfies the capacitive law:

explained at a later time

$$\Delta\dot{Q} = \gamma \cdot \frac{dT}{dt} = \Delta(T \cdot \dot{S}) = T \cdot \Delta\dot{S} \quad \Rightarrow \quad \Delta\dot{S} = \frac{\gamma}{T} \cdot \frac{dT}{dt} = C \cdot \frac{dT}{dt}$$

$$\Rightarrow \quad C = \gamma / T$$

γ = heat capacity

- where:

$$\gamma = c \cdot m$$

$$m = \rho \cdot V$$

c = specific heat capacity
m = mass of the rod

ρ = material density
V = volume of a segment

The Computation of *R* and *C* III

$$\Rightarrow \boxed{C = \gamma / T = c \cdot \rho \cdot V / T = c \cdot \rho \cdot A \cdot \Delta x / T}$$

$$\Rightarrow \boxed{R \cdot C = \theta \cdot \gamma = \frac{c \cdot \rho}{\lambda} \cdot \Delta x^2 = \frac{1}{\sigma} \cdot \Delta x^2}$$

- The diffusion time constant ***R·C*** is independent of temperature.
- The thermal resistance is proportional to the temperature.
- The thermal capacity is inverse proportional to the temperature.
- The thermal ***R*** and ***C*** elements are, contrary to their electrical and mechanical counterparts, ***not constant***.

Is the Thermal Capacity truly capacitive?

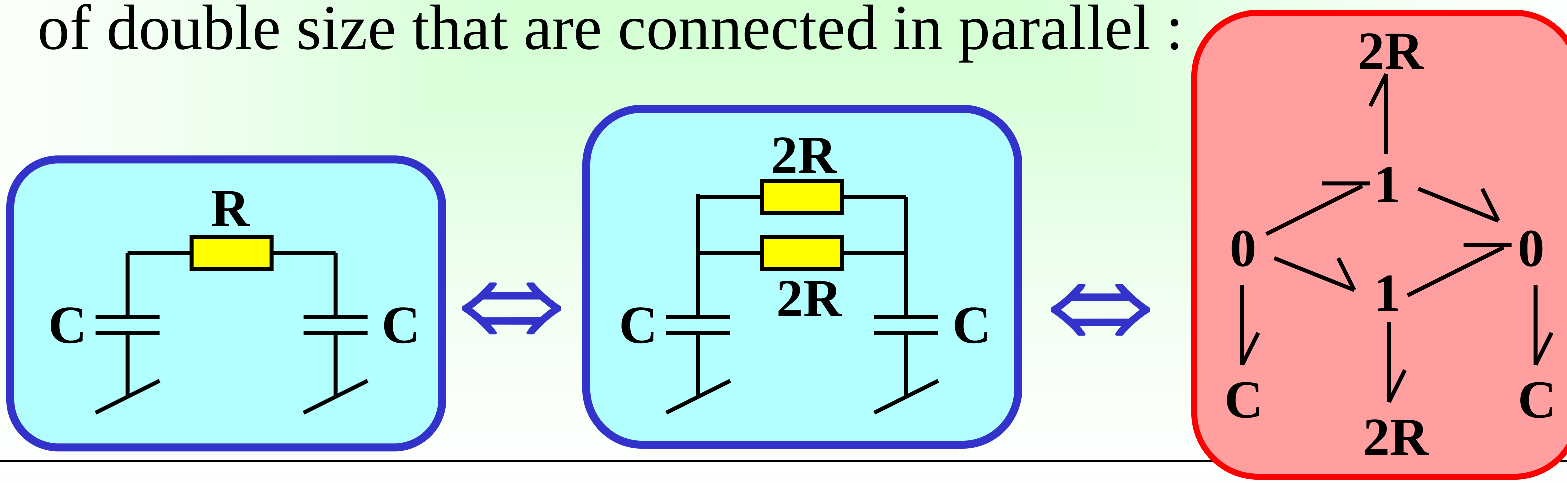
- We have to verify that the derived capacitive law is not in violation of the general rule of capacitive laws.

$$\Delta \dot{S} = \frac{\gamma}{T} \cdot \frac{dT}{dt} \Rightarrow f = \frac{\gamma}{e} \cdot \frac{de}{dt} \Rightarrow q = \gamma \cdot \ln(e/e_0)$$

***q* is indeed a (non-linear) function of *e*. Therefore, the derived law satisfies the general rule for capacitive laws.**

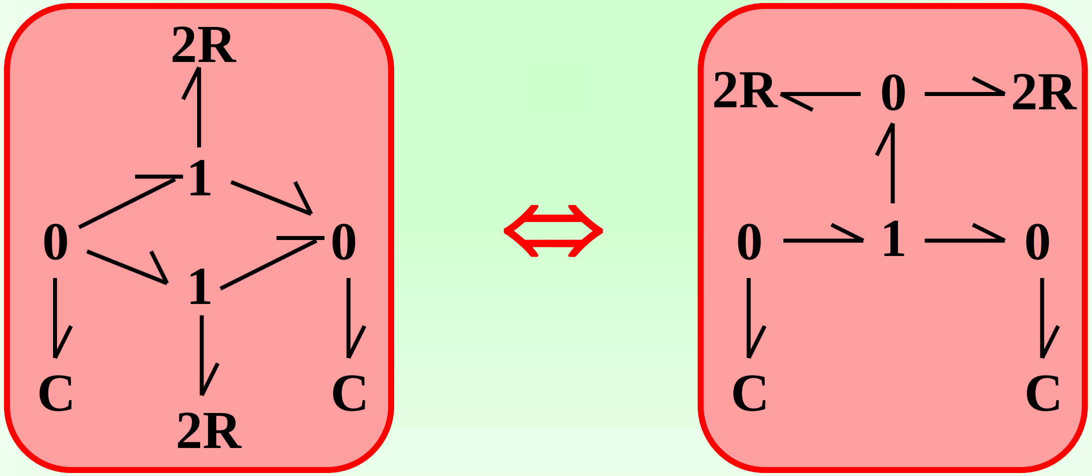
Computation of *R* for the Modified Bond Graph

- The resistor value has been computed for the original circuit configuration. We need to analyze, what the effects of the symmetrization of the bond graph have on the computation of the resistor value.
- We evidently can replace the original resistor by two resistors of double size that are connected in parallel :



Modification of the Bond Graph

- The bond graph can be modified by means of the ***diamond rule***:



This is exactly the structure in use.

Radiation I

- A second fundamental phenomenon of thermodynamics concerns the radiation. It is described by the ***law of Stephan-Boltzmann***.

$$\Re = \sigma \cdot T^4$$

- The emitted heat is proportional to the radiation and to the emitting surface.

$$\dot{Q} = \sigma \cdot A \cdot T^4$$

- Consequently, the emitted entropy is proportional to the third power of the absolute temperature.

$$\dot{S} = \sigma \cdot A \cdot T^3$$

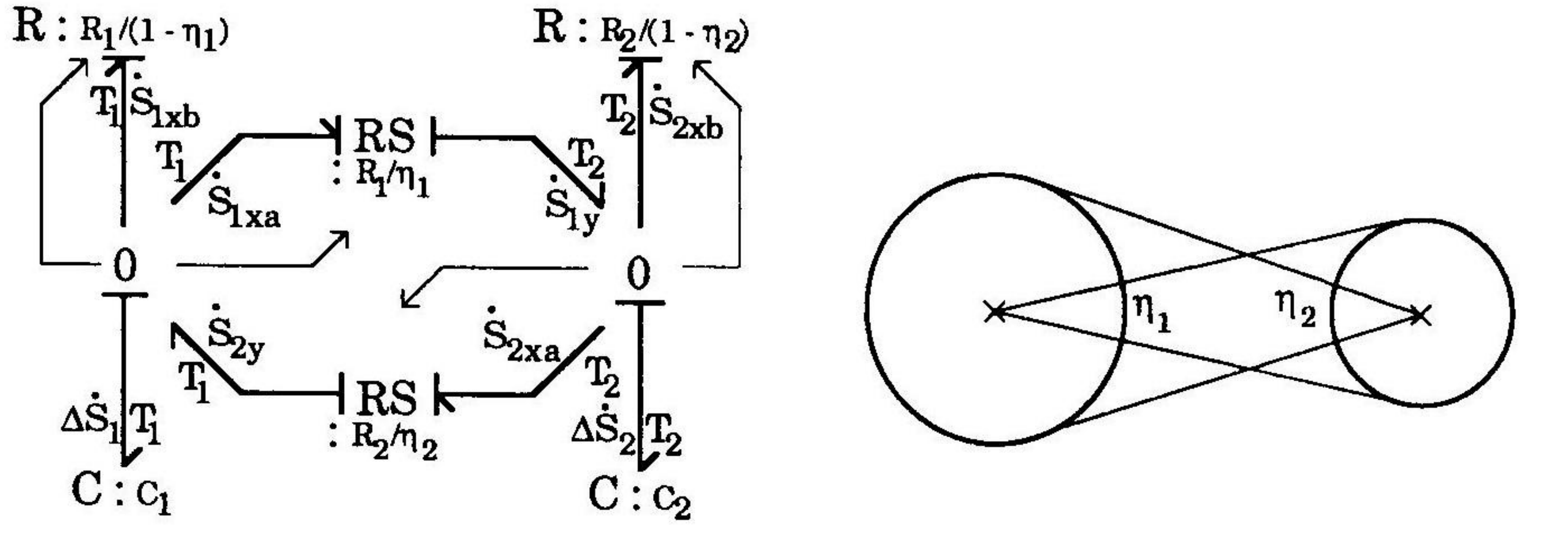
Radiation II

- Radiation describes a dissipative phenomenon (we know this because of its static relationship between T and $\dot{S}$).
- Consequently, the resistor can be computed as follows:

$$R = T / \dot{S} = 1 / (\sigma \cdot A \cdot T^2)$$

- The radiation resistance is thus inverse proportional to the square of the (absolute) temperature.

Radiation III



References

- Cellier, F.E. (1991), *Continuous System Modeling*, Springer-Verlag, New York, Chapter 8.