Treatment of Discontinuities II

- We shall today once more look at the ***modeling of discontinuous systems***.
- First, an additional method to their mathematical description shall be discussed. This method makes use of a ***parameterized description of curves***.
- Subsequently, we shall deal with the problem of variable causality.
- Finally, a method shall be discussed that permits to solve causality problems elegantly.

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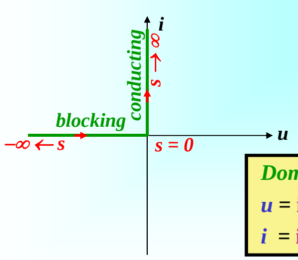
Parameterized Curve Descriptions

- It is always possible to describe discontinuous functions by means of parameterized curves. This technique shall be illustrated by means of the diode characteristic.



Domain:	Condition:	Equations:
blocking:	$s < 0$	$u = s;\ i = 0$
conducting:	$s > 0$	$u = 0;\ i = s$

Domain = **if** *s* < *0* **then** *blocking* **else** *conducting*;
u = **if** *Domain* == *blocking* **then** *s* **else** *0* ;
i = **if** *Domain* == *blocking* **then** *0* **else** *s* ;

The Causality of the Switch Equation I

- Let us consider once more the switch equation in its algebraic form:

$$0 = s \cdot i + (1 - s) \cdot u$$

Switch open: $s = 1$
Switch closed: $s = 0$

- We can solve this equation either for ***u*** or for ***i*** :

	$u = \frac{s}{s-1} \cdot i$	$i = \frac{s-1}{s} \cdot u$
Switch open: *Switch closed:*	*Division by 0!* $u = 0$	$i = 0$ *Division by 0!*

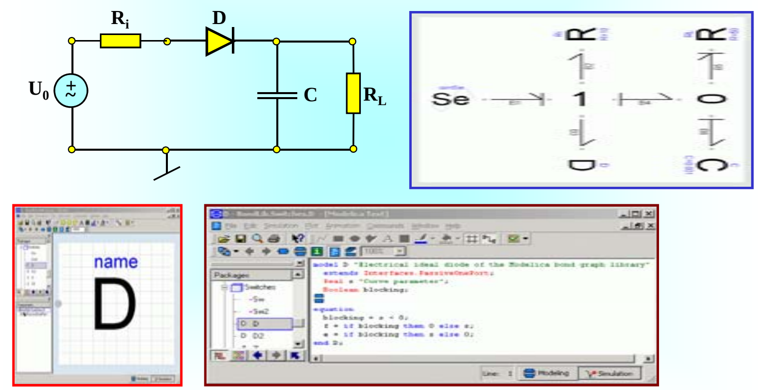
The Causality of the Switch Equation II

- Neither of the two causal equations can be used in both switch positions. Either one or the other switch position leads to a ***division by 0***.
- This is exactly what happens in the simulation, when the causality of the switch equation is fixed.

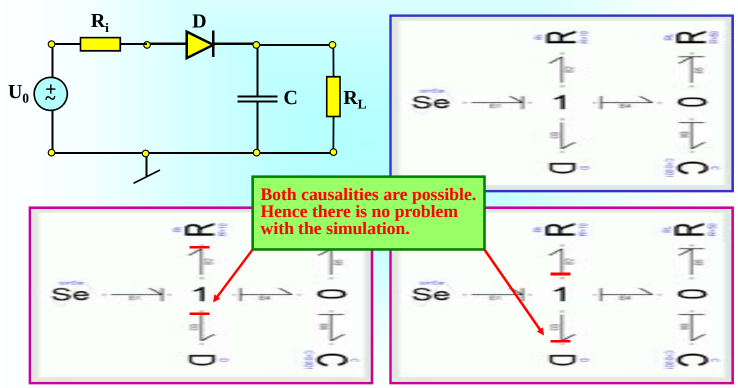
⇒ ***The causality of the switch equation must always be free.***

⇒ ***The switch equation must always be placed in an algebraic loop.***

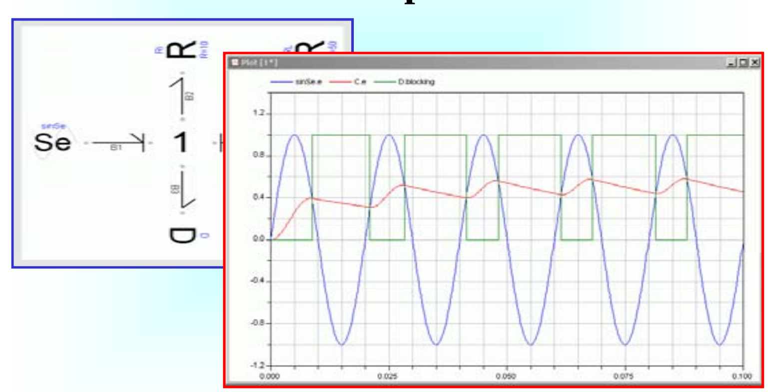
An Example I



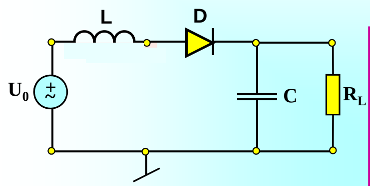
An Example II

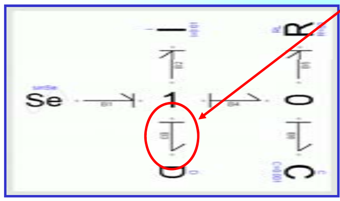


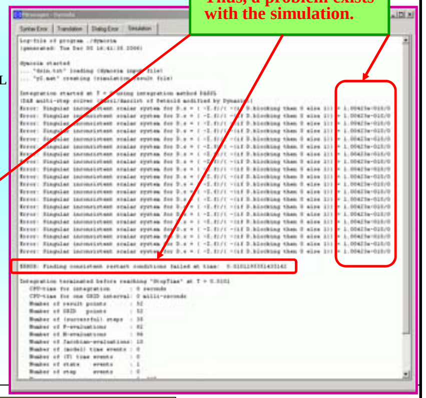
An Example III



A Second Example







The causality is fixed. Thus, a problem exists with the simulation.

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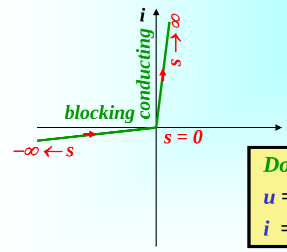
Not So Ideal Diode I

- One possibility for circumventing the causality problem consists in defining a ***leakage resistance*** R_{on} for the closed switch, as well as a ***leakage conductance*** G_{off} for the open switch.



Domain:	Condition:	Equations:
blocking:	$s < 0$	$u = s;\ i = G_{off} \cdot s$
conducting:	$s > 0$	$u = R_{on} \cdot s;\ i = s$

Domain = **if** $s < 0$ **then** *blocking* **else** *conducting*;
u = s*(**if** *Domain* == *blocking* **then** *1* **else** R_{on});
i = s*(**if** *Domain* == *blocking* **then** G_{off} **else** *1*);

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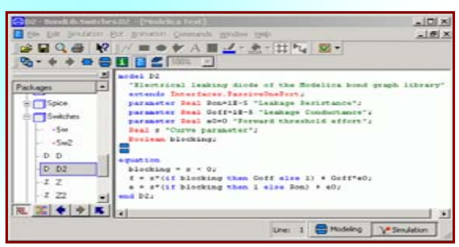
Not So Ideal Diode II

- This is the solution that was chosen in the standard library of ***Modelica***.
- The same solution is also offered in ***BondLib*** in the form of a "leaky" diode model.



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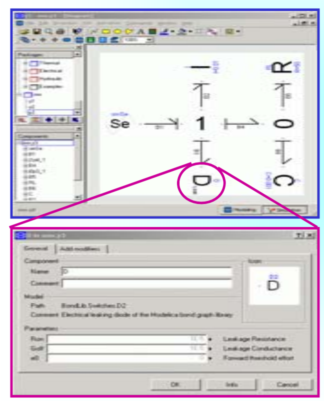
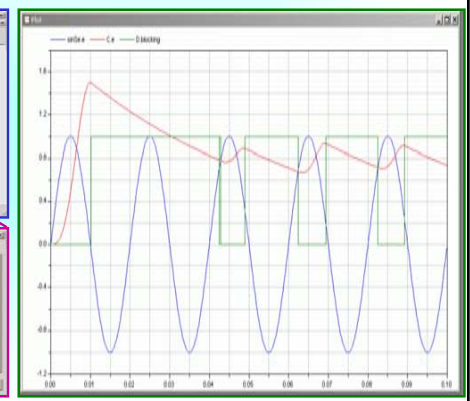
Not So Ideal Diode III





Problems I

- For ***electrical applications***, the solution with the leaking diode is frequently acceptable.
- One problem has to do with the numerics. When a circuit using the ideal diode is plagued by division problems, the circuit with the leaking diode leads invariably to a ***stiff system***.
- Stiff systems can be integrated in ***Modelica*** by means of the (standard) ***DASSL integration algorithm***.
- However, this is time consuming and may not be suitable, at least for real-time applications.

Problems II

- In the case of ***mechanical applications***, the method is less suitable, since for example friction characteristics must frequently be computed rather accurately, and since in mechanical applications, the causalities are almost invariably fixed.
- The masses (and inertias) determine all velocities, and the friction as well as spring forces (and torques) must therefore be determined by the ***R***- and ***C***-elements in a pre-set causality.
- Consequently, another solution approach should be sought for these applications.

"Inline" Integration Algorithm

- When using ***Inline Integration***, the integration algorithm is directly substituted into the model equations (or inversely: the model equations are being substituted into the integration algorithm).
- Let us consider an inductor integrated by means of the ***implicit Euler algorithm***.

$$u_L = L \cdot di_L/dt$$
$$i_L(t) = i_L(t-h) + h \cdot di_L(t)/dt$$

$$\Rightarrow \quad i_L(t) = i_L(t-h) + (h/L) \cdot u_L(t)$$

The Causality of Inline Integration

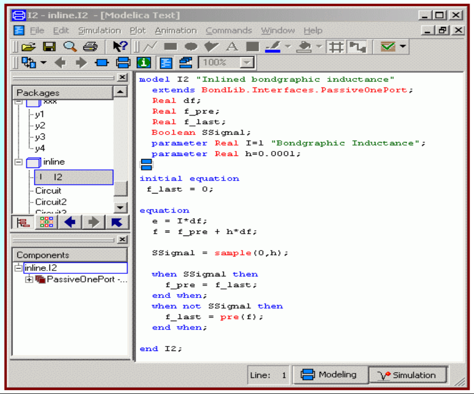
$$i_L(t) = i_L(t-h) + (h/L) \cdot u_L(t)$$

Known, since computed in the past.

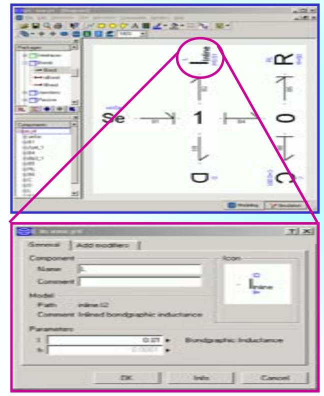
This constitutes an algebraic relation between *i* and *u*. This now looks like a resistor. Hence the causality is now free.

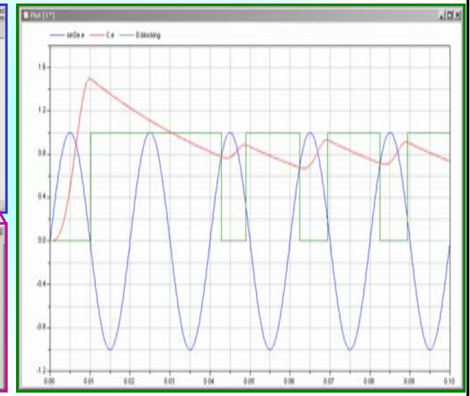
When using the inline integration algorithm, the causalities of the so integrated storage elements are being freed up. Consequently, the division by zero problem disappears.

Ideal Diode With Inline Integration I



Ideal Diode With Inline Integration II





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- Cellier, F.E. and M. Krebs (2007), "Analysis and simulation of variable structure systems using bond graphs and inline integration," *Proc. ICBGM'07, 8th Intl. Conf. Bond Graph Modeling and Simulation*, San Diego, CA, pp. 29-34.