

Multi-bond Graphs

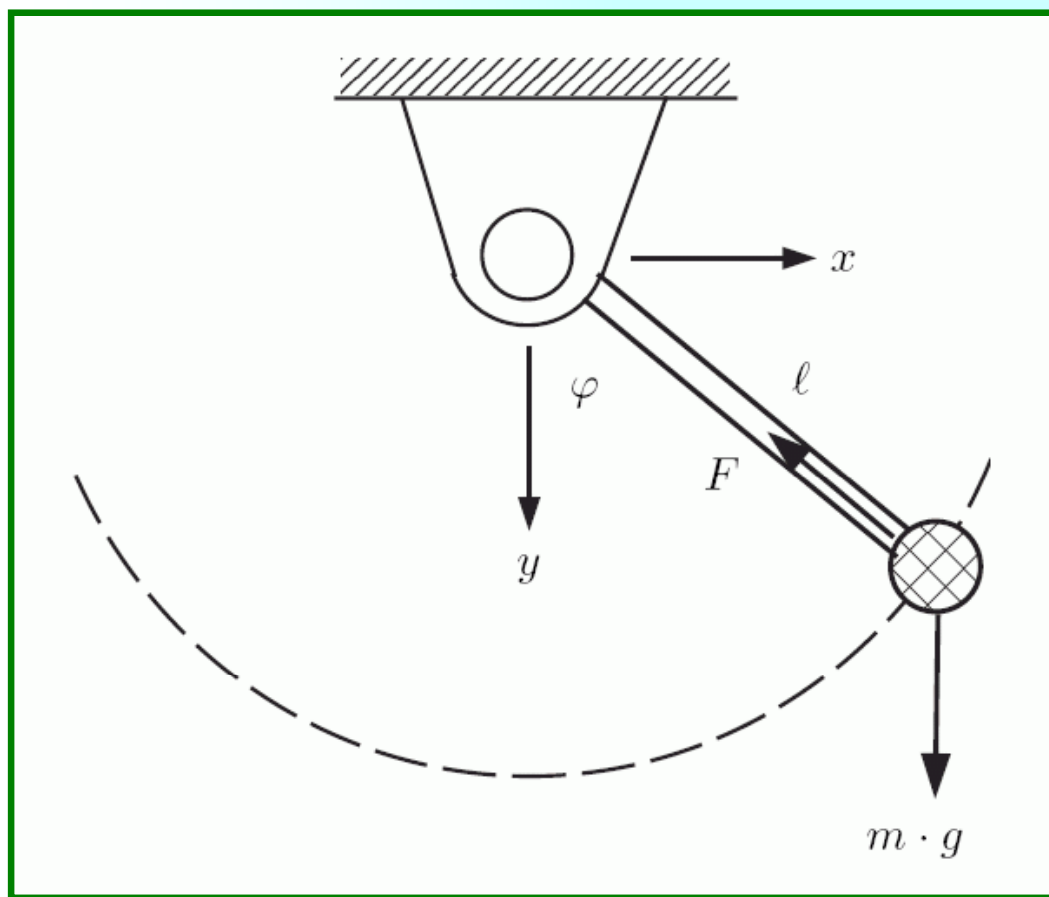
- We shall today look at vectors of bonds, called multi-bonds.
- Especially when dealing with 2D and 3D mechanics, the d'Alembert principle must be applied to each degree of freedom separately.
- Each equation looks structurally the same.
- This leads naturally to a demand for multi-bond graphs.

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A Planar Pendulum

- Let us model the following planar pendulum:



$$\begin{aligned} m \cdot \frac{dv_x}{dt} &= -\frac{F \cdot x}{\ell} \\ m \cdot \frac{dv_y}{dt} &= m \cdot g - \frac{F \cdot y}{\ell} \\ \frac{dx}{dt} &= v_x \\ \frac{dy}{dt} &= v_y \end{aligned}$$

$$x^2 + y^2 = \ell^2$$

**Holonomic
Constraint**

$$\begin{aligned} m \cdot \frac{dv_x}{dt} &= -\frac{F \cdot x}{\ell} \\ m \cdot \frac{dv_y}{dt} &= m \cdot g - \frac{F \cdot y}{\ell} \\ \frac{dx}{dt} &= v_x \\ \frac{dy}{dt} &= v_y \\ x &= \ell \cdot \sin(\varphi) \\ y &= \ell \cdot \cos(\varphi) \end{aligned}$$

$$m \frac{dv_x}{dt} = -F \sin(\varphi)$$

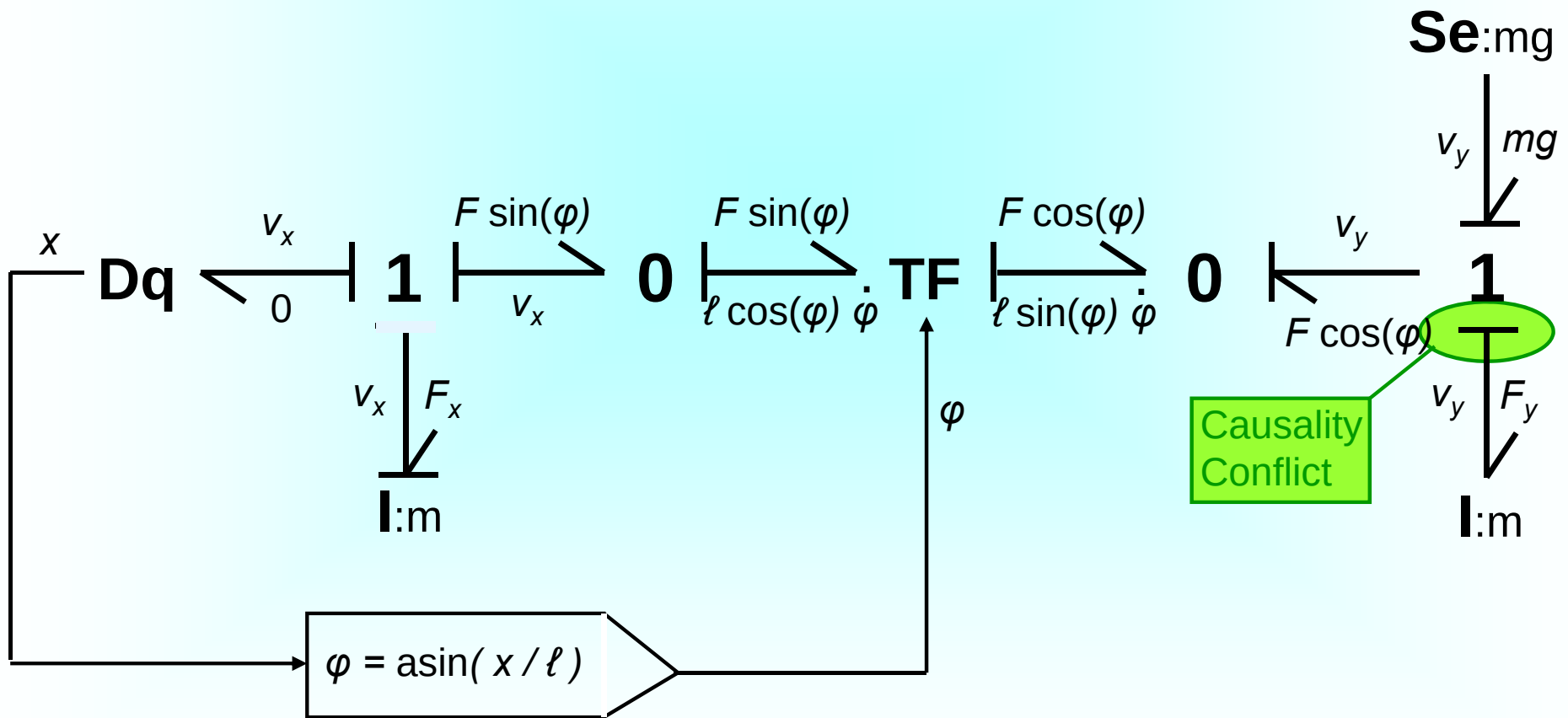
$$x = \ell \sin(\varphi)$$

$$v_x = \ell \cos(\varphi) \dot{\varphi}$$

$$y = \ell \cos(\varphi)$$

$$v_y = -\ell \sin(\varphi) \dot{\varphi}$$

$$m \frac{dv_y}{dt} = -F \cos(\varphi) + mg$$



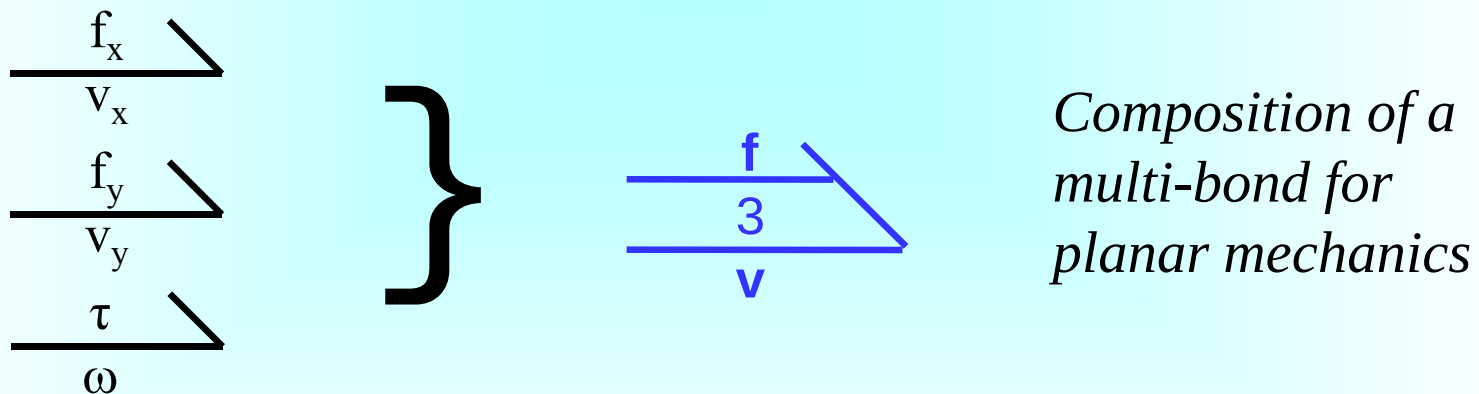
Analysis

It has been possible to describe the motion of the planar pendulum by a bond graph enhanced by activated bonds for the description of the *holonomic constraint*. Unfortunately, the bond graph doesn't tell us much that we didn't know already.

- ❖ We shouldn't have to derive the equations first in order to be able to derive the bond graph from them.
- ❖ The resulting bond graph didn't preserve the topological properties of the system in any recognizable form.

Multi-bonds

- Multi-bond graphs are a vector extension of the regular bond graphs.
- A multi-bond contains a freely selectable number of regular bonds of identical or similar domains.

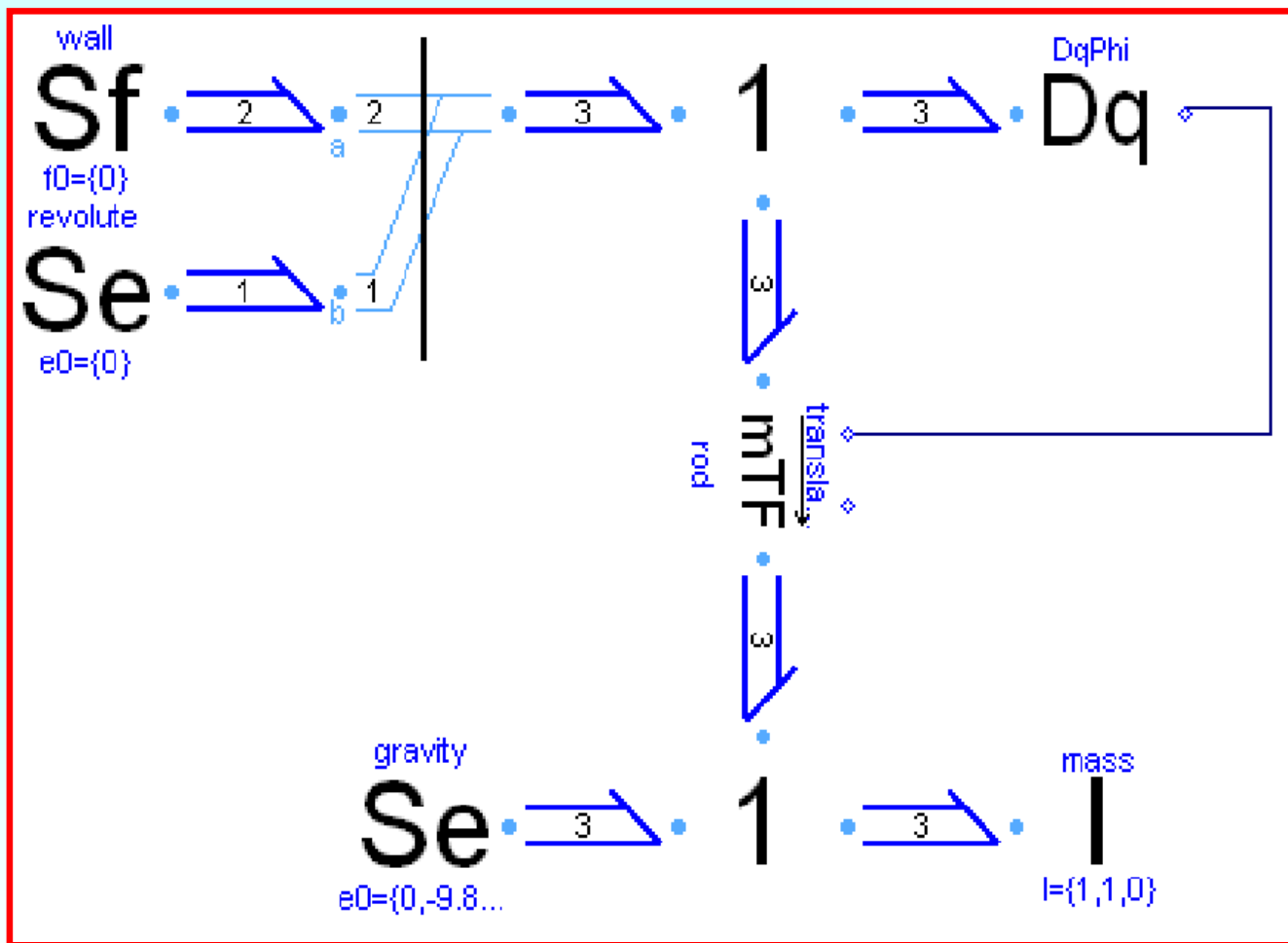
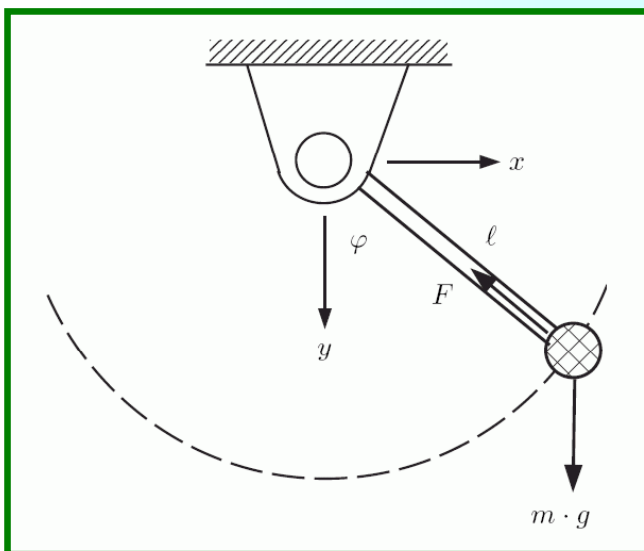


- All bond graph component models are adjusted in a suitable fashion.

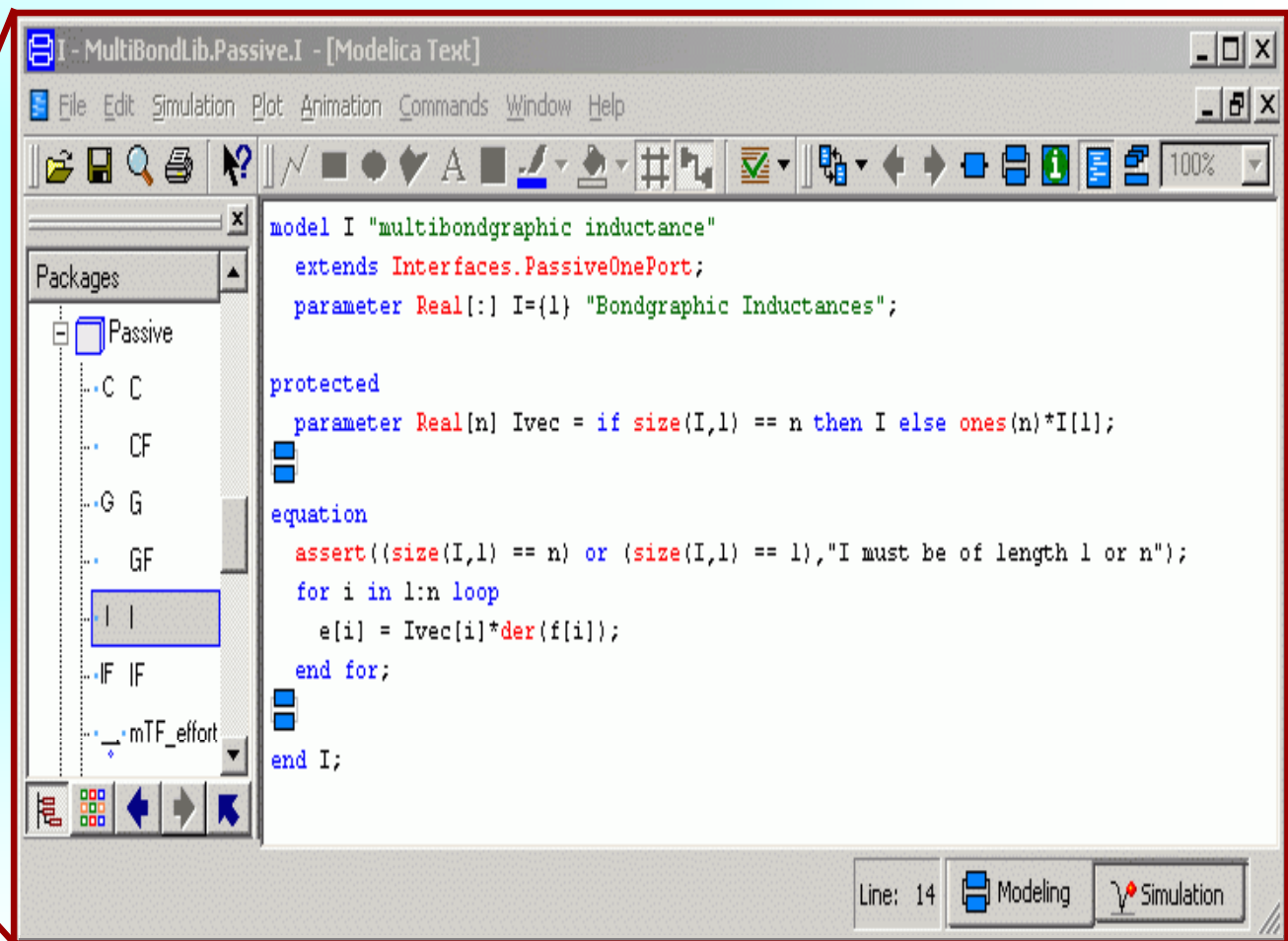
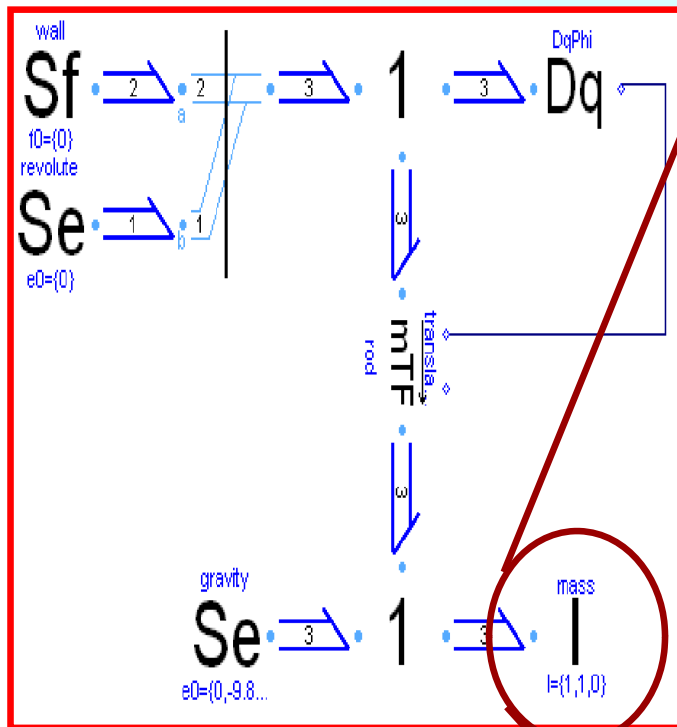
Multi-bond Graph Library

- A *Dymola* library for modeling systems by means of multi-bond graphs has been developed.
- The library has been designed with an interface that looks as much as possible like that of the original *BondLib* library.
- Just like the original library, also the new multi-bond graph library contains sub-libraries supporting modelers in modeling systems from particular application domains, especially from mechanics.

Planar Pendulum III



Planar Pendulum IV



Planar Pendulum V

The image displays the Modelica IDE interface with three windows open:

- Top Window: I - MultiBondLib.Passive.I - [Modelica Text]**

```

model I multiBondgraphic inductance
  extends Interfaces.PassiveOnePort;
  parameter Real[:] I={1} "Bondgraphic Inductances";

protected
  parameter Real[n] Ivec = if size(I,1) == n then I else ones(n)*I[1];
end I

```
- Bottom Window: PassiveOnePort - MultiBondLib.Interfaces.PassiveOnePort - [Modelica Text]**

```

partial model PassiveOnePort
  "partial model invoking one bondgraphic connector"

  final parameter Integer N = MBG_defaults.n;
  parameter Integer n = N "Cardinality of Bond connection";

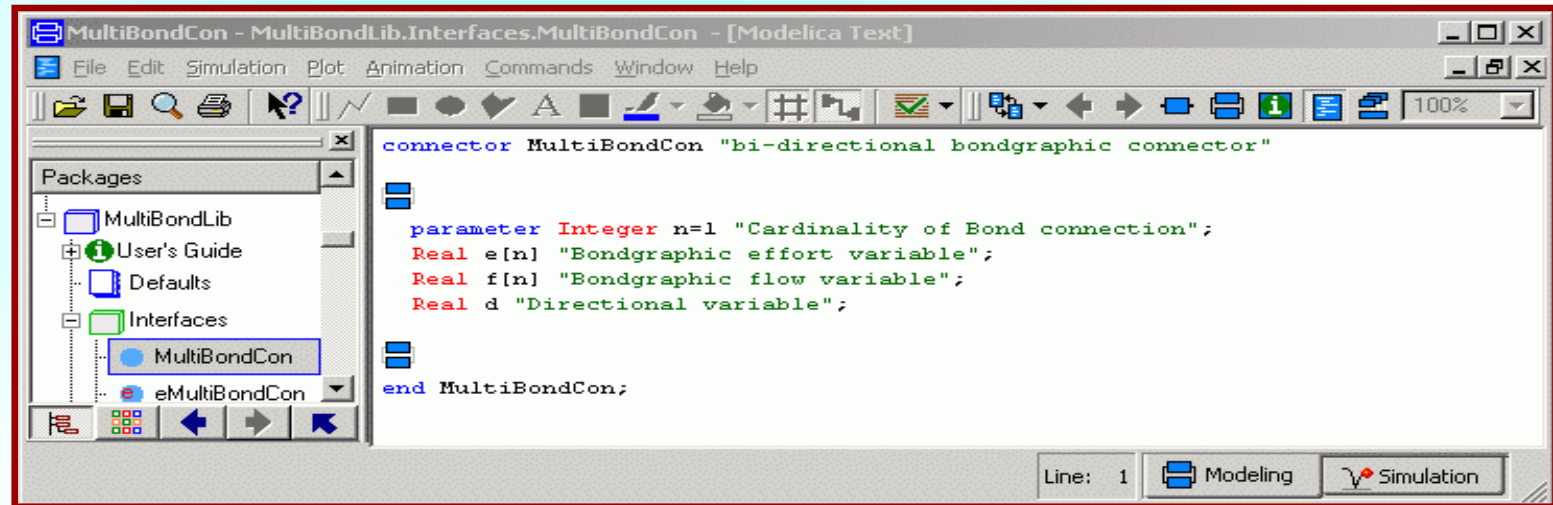
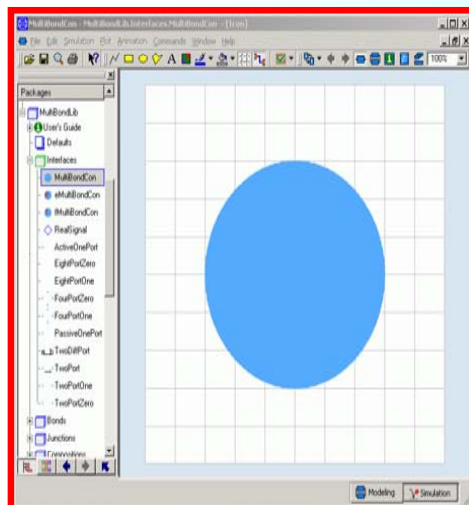
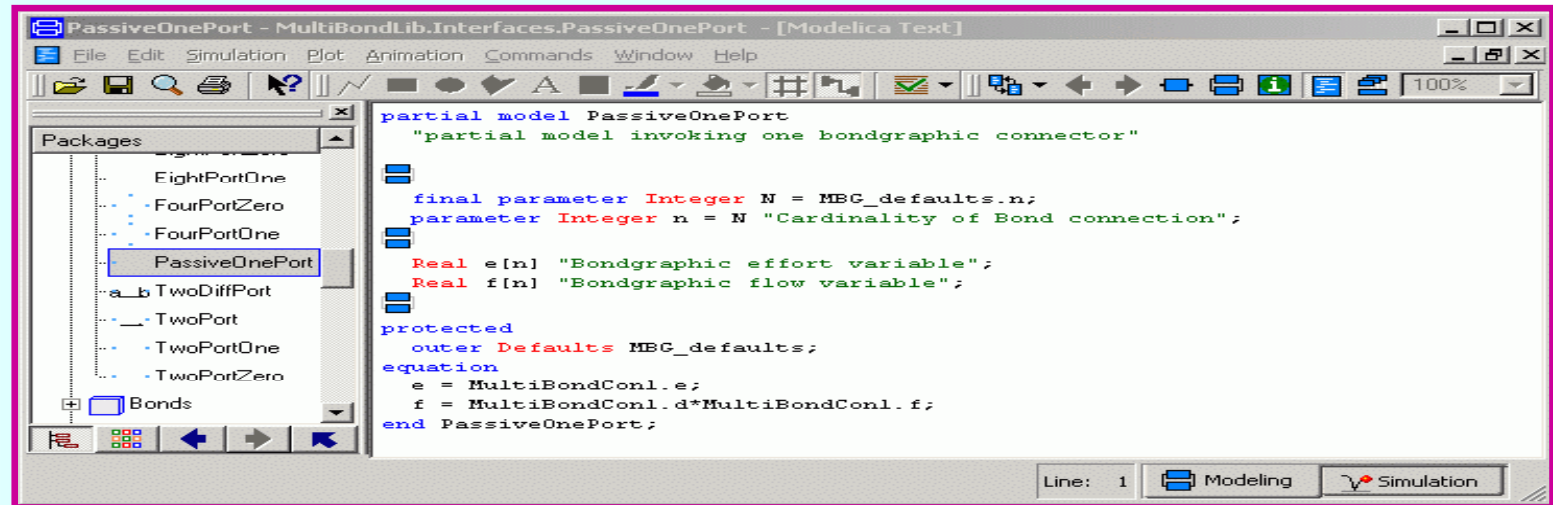
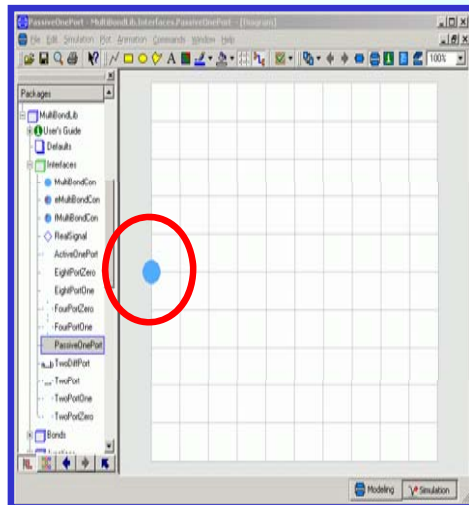
  Real e[n] "Bondgraphic effort variable";
  Real f[n] "Bondgraphic flow variable";

protected
  outer Defaults MBG_defaults;
equation
  e = MultiBondCon1.e;
  f = MultiBondCon1.d*MultiBondCon1.f;
end PassiveOnePort;

```
- Right Window: A graphical representation of a pendulum.** It shows a blue dot representing the mass, connected by a line to a pivot point, with a grid background.

The Package browser on the left shows the hierarchy: **Passive** > **I**. The Package browser in the bottom window shows: **Bonds** > **PassiveOnePort**.

Planar Pendulum VI



Planar Pendulum VII

PassiveOnePort - MultiBondLib.Interfaces.PassiveOnePort - [Modelica Text]

```

partial model PassiveOnePort
  "partial model invoking one bondgraphic connector"
  final parameter Integer n = MBG_defaults.n;
  parameter Integer n = N "Cardinality of Bond connection"
  Real e[n];
  Real f[n];
  protected
    outer Def
  equation
    e = Multi
    f = Multi
  end Passive
  
```

Defaults - MultiBondLib.Defaults - [Documentation]

Default settings for all Bond Graphic elements

Information

This is the Default model of the MultiBondLib.

This model sets the default cardinality of all bondgraphic elements. Place this model in every multibondgraphic model and set the default cardinality via the parameter menu. The default cardinality is then shown in the model icon and is used in all basic elements downwards the model hierarchy if not stated otherwise.

Note: all basic bondgraphic elements expect an inner declaration of this Default model with the exact name MBG_defaults. This model must be part of all complete multibondgraphic models.

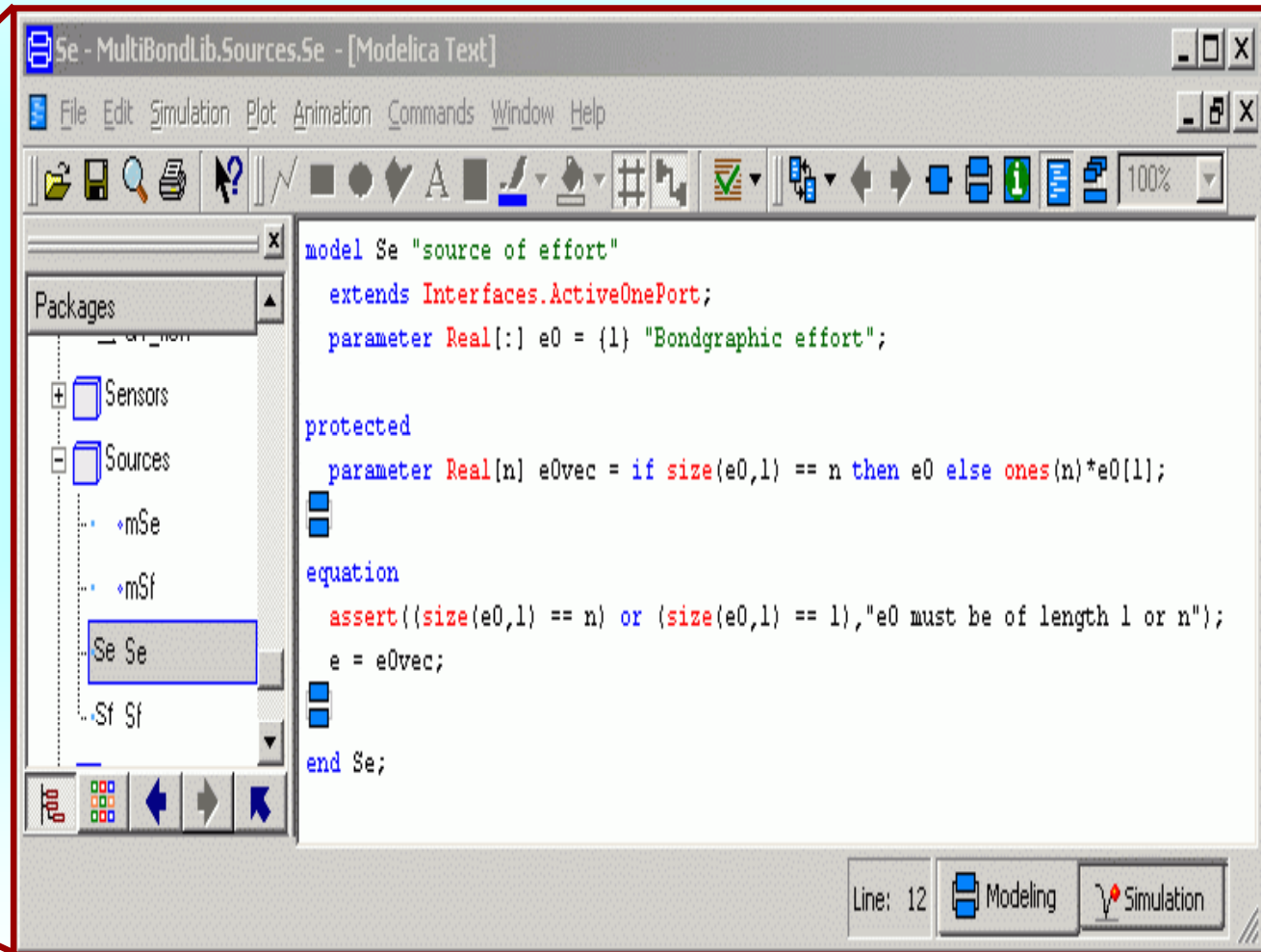
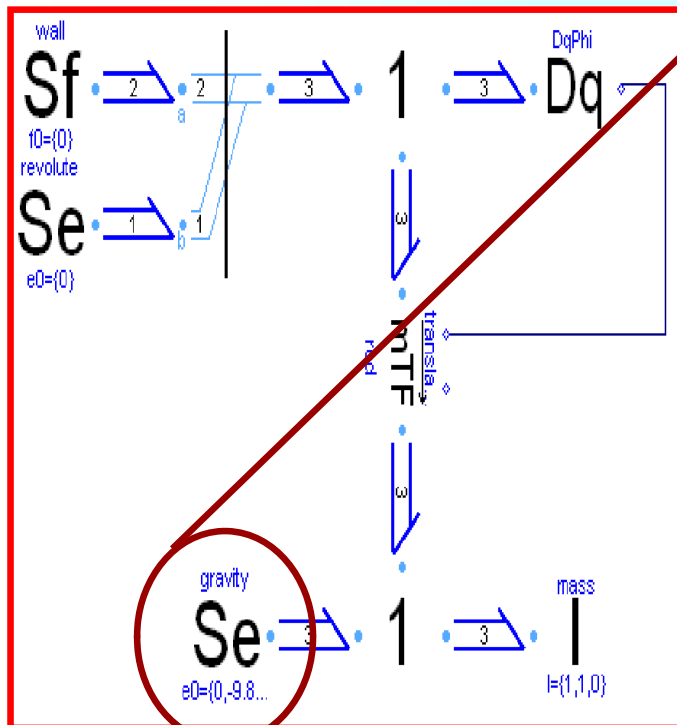
Parameters

Type	Name	Default	Description
Integer	n	1	default cardinality of Bondgraphic elements

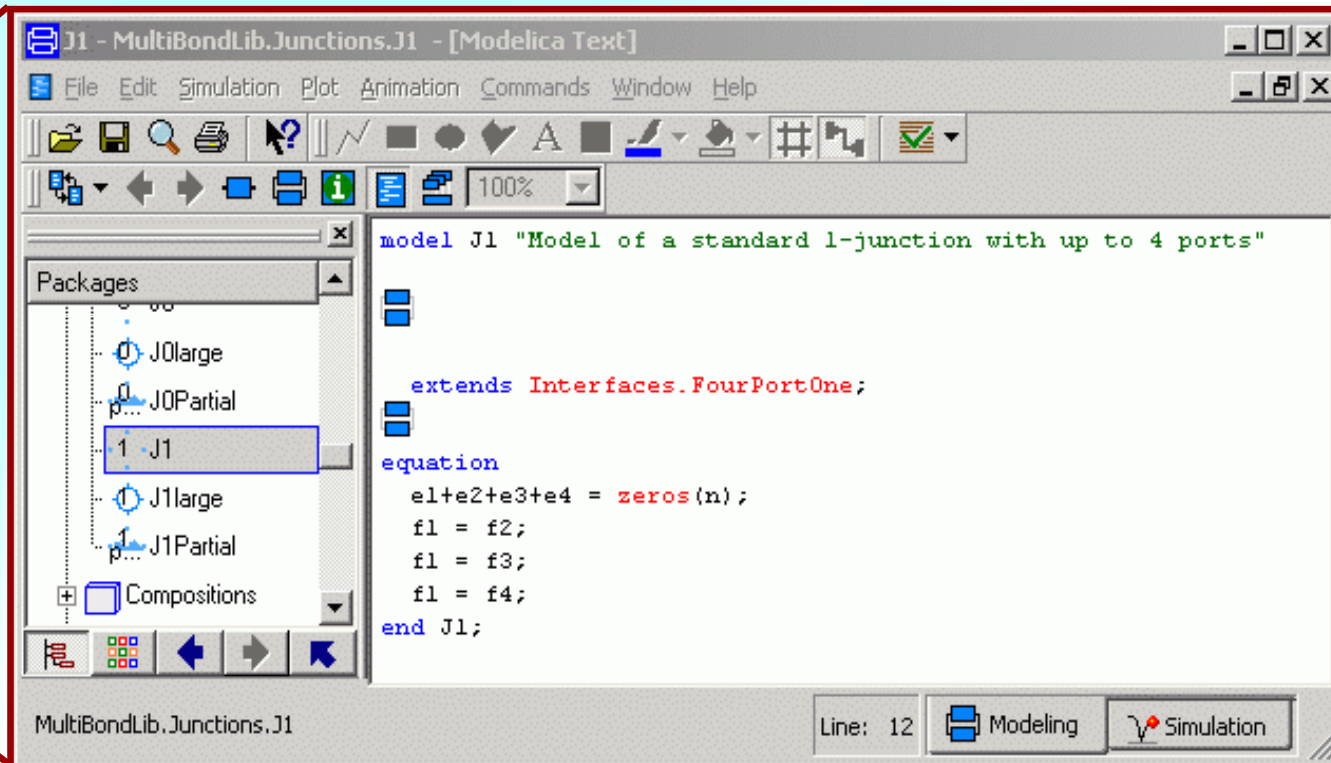
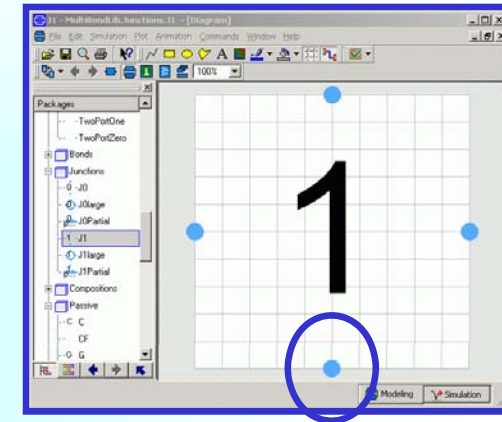
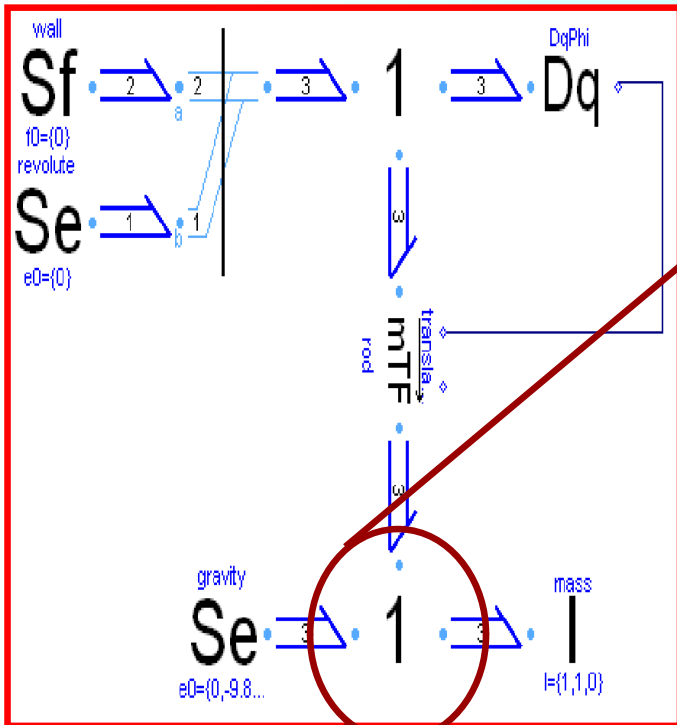
Name: Defaults
Path: MultiBondLib.Defaults
Filename: C:/Cellier/Students/Zimmer/MultiBondLib_v1_2/MultiBondLib.mo
Version: 1.2
Uses: BondLib (version="1.1"), Modelica (version="2.2")

Modelica Text Editor (Bottom Left): Shows a graphical representation of a bond with a large blue '1' and the word 'name' above it, indicating the cardinality and name of the bond.

Planar Pendulum VIII



Planar Pendulum IX



Planar Pendulum X

The image displays two windows from the Modelica IDE, illustrating the implementation of a Planar Pendulum X.

Left Window: J1 - MultiBondLib.Junctions.J1 - [Modelica Text]

The code defines a model J1 that extends the `Interfaces.FourPortOne` interface:

```
model J1 "Model of a standard 1-junction with"
  extends Interfaces.FourPortOne;
  equation
    e1+e2+e3+e4 = zeros(n);
    f1 = f2;
    f1 = f3;
    f1 = f4;
  end J1;
```

A green box highlights the `extends Interfaces.FourPortOne;` line, with a green arrow pointing to the `FourPortOne` package in the right window's package browser.

Right Window: FourPortOne - MultiBondLib.Interfaces.FourPortOne - [Modelica Text]

The code defines the `FourPortOne` interface:

```
partial model FourPortOne
  "partial model invoking 4 optional bondgraphic connectors"
  final parameter Integer N = MBG_defaults.n;
  parameter Integer n = N "Cardinality of Bond connections";

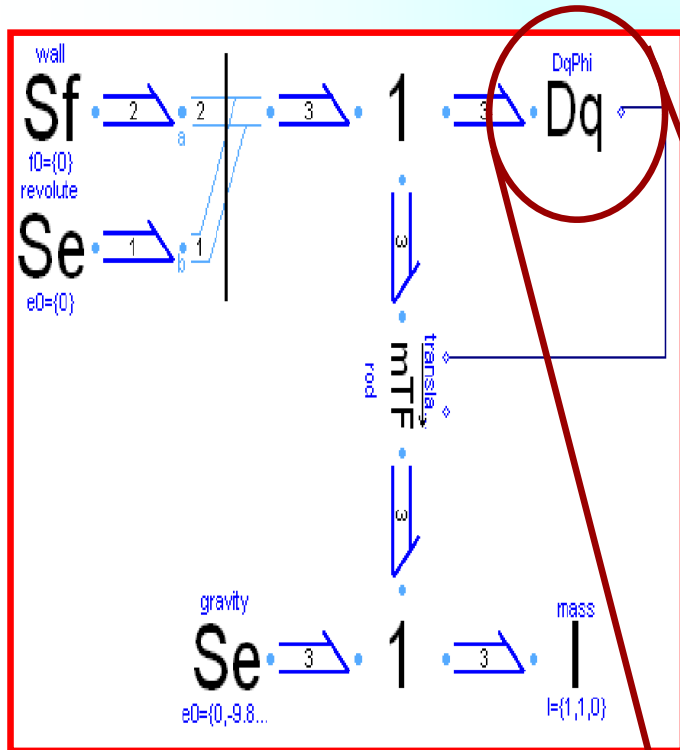
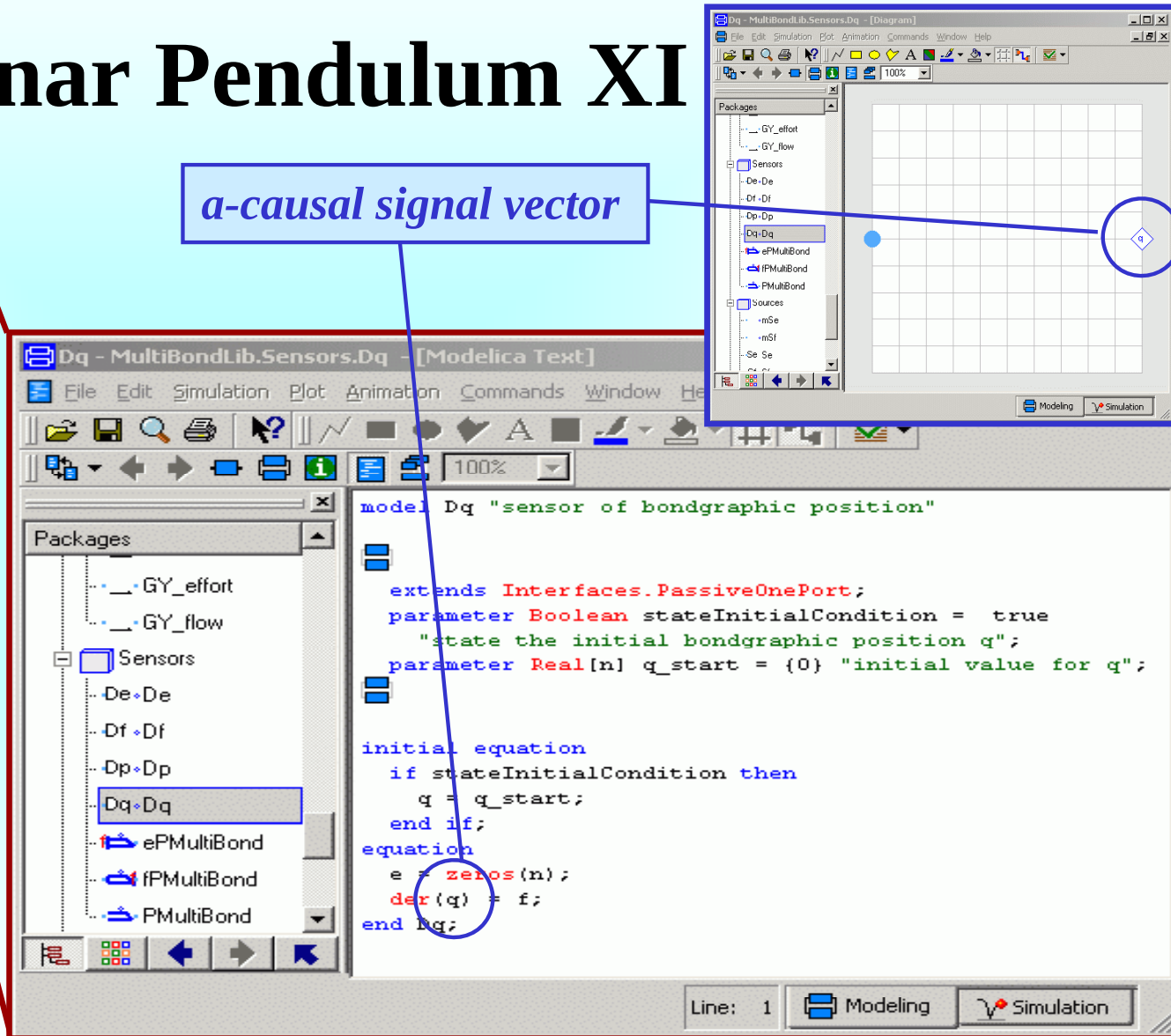
  //define bondgraphic variables
  Real e1[n];
  Real f1[n];
  Real e2[n];
  Real f2[n];
  Real e3[n];
  Real f3[n];
  Real e4[n];
  Real f4[n];

  protected
    outer Defaults MBG_defaults;
  equation

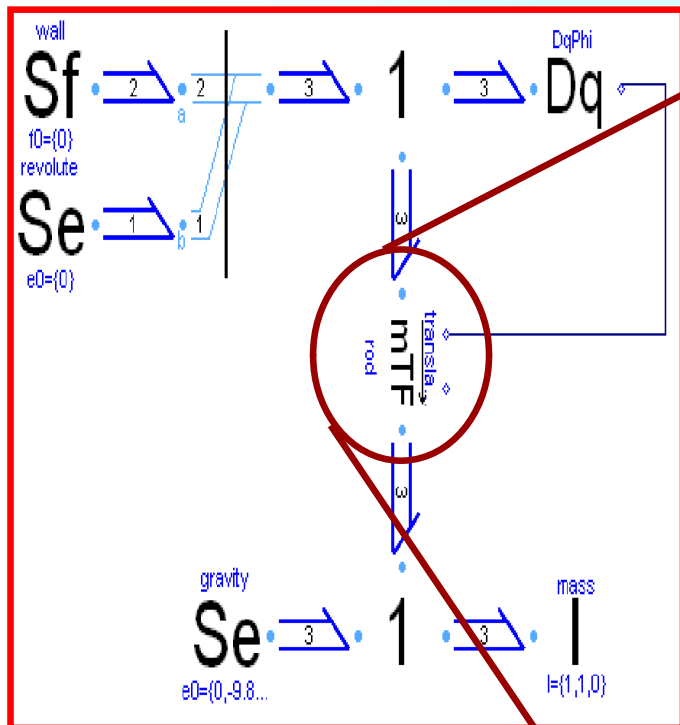
    //assign bondgraphic variables to optional connectors
    e1 = MultiBondCon1.d*MultiBondCon1.e;
    f1 = MultiBondCon1.f;
    e2 = MultiBondCon2.d*MultiBondCon2.e;
    f2 = MultiBondCon2.f;
    e3 = MultiBondCon3.d*MultiBondCon3.e;
    f3 = MultiBondCon3.f;
    e4 = MultiBondCon4.d*MultiBondCon4.e;
    f4 = MultiBondCon4.f;
    if cardinality(MultiBondCon1) == 0 then
      MultiBondCon1.d = 1;
      e1 = zeros(n);
    end if;
    if cardinality(MultiBondCon2) == 0 then
      MultiBondCon2.d = 1;
      e2 = zeros(n);
    end if;
    if cardinality(MultiBondCon3) == 0 then
      MultiBondCon3.d = 1;
```

The package browser on the left of the right window shows the hierarchy: `Interfaces` > `FourPortOne`.

Planar Pendulum XI

*a-causal signal vector*

Planar Pendulum XII



translationalTF - MultiBondLib.PlanarMechanics.AdditionalMBG.translationalTF

File Edit Simulation Plot Animation Commands Window Help

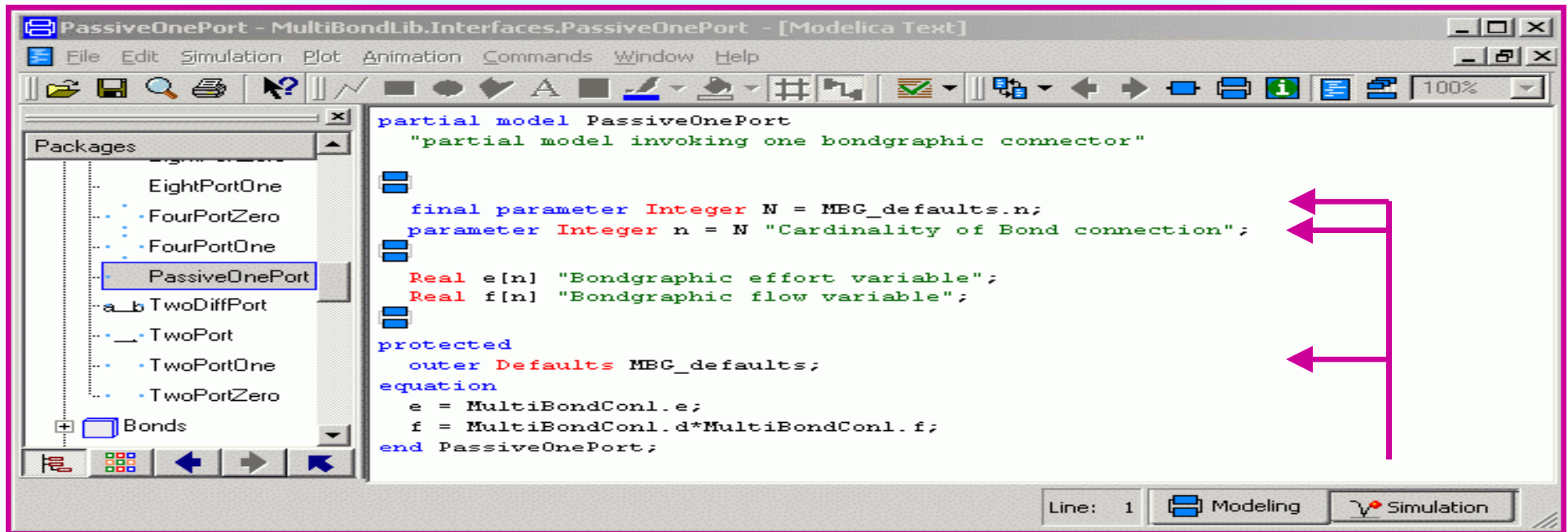
model translationalTF
 "optionally modulated transformer for a translation"
 extends MultiBondLib.Interfaces.TwoPort(final n=3);
 parameter Real d[2] = {1,0} a;
 equation
 if cardinality(amp1) == 0 then
 amp1 = 1;
 end if;
 f2[1] = f1[1] + (-sin(phi)*d[1] + cos(phi)*d[2])*amp1*f1[3];
 f2[2] = f1[2] + (-cos(phi)*d[1] - sin(phi)*d[2])*amp1*f1[3];
 f2[3] = f1[3];
 e1[1] = e2[1];
 e1[2] = e2[2];
 e1[3] = e2[3] + (-sin(phi)*d[1] + cos(phi)*d[2])*amp1*e2[1] + (-cos(phi)*d[1] - sin(phi)*d[2])*amp1*e2[2];
 end translationalTF;

Line: 1 Modeling Simulation

Multi-bond Graph Basics

- The basic multi-bond graph models contain little that is surprising. They represent essentially natural extensions of the regular bond graph models.
- A few points are worth mentioning though. First, there is the *defaults model* that must be included in each multi-bond graph model. It contains only a single parameter, the dimensional parameter, *n*, that specifies, how many bonds each multi-bond contains by default.
- The defaults model must be referenced in each multi-bond graph model as an *outer model*.

Multi-bond Graph Basics II

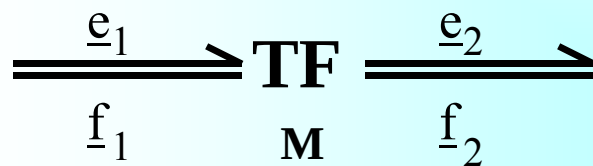


- If the multi-bond graph model inherits one of the partial models, this has already been taken care of.

Multi-bond Graph Basics III

- A second difference concerns the use of junctions. Whereas the general bond graph library provides separate junction models for 2..6 bond connections, the multi-bond graph library offers only junctions with either 4 or 8 connectors. Yet, individual connectors may be left unconnected as needed.
- A third difference is in the use of transformers and gyrators. The multi-bond graph library offers a much larger variety of different transformer and gyrator models when compared to the regular bond graph library.

Multi-port Transformers



Transformation: $\underline{e}_1 = M \cdot \underline{e}_2$ (1)

Energy Conservation: $\underline{e}_1^T \cdot \underline{f}_1 = \underline{e}_2^T \cdot \underline{f}_2$ (2)

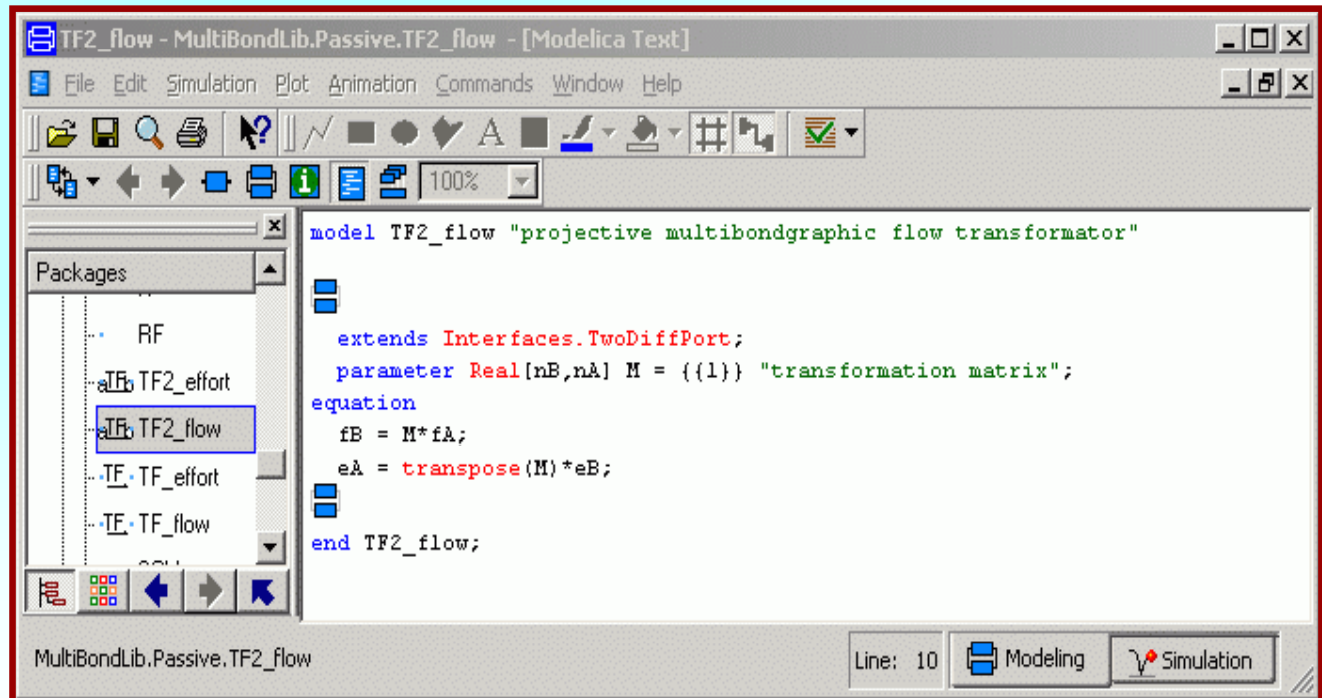
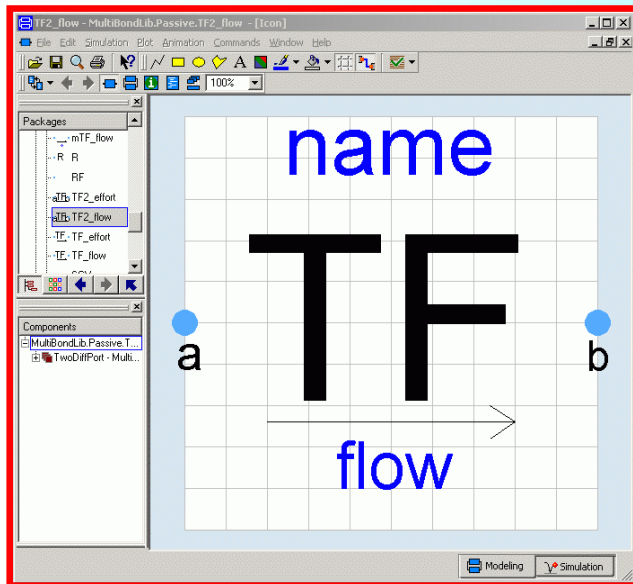
$\Rightarrow (M \cdot \underline{e}_2)^T \cdot \underline{f}_1 = \underline{e}_2^T \cdot \underline{f}_2$ (3)

$\Rightarrow \underline{f}_2 = M^T \cdot \underline{f}_1$ (4)

$\Rightarrow$ The transformer may either be described by means of equations (1) and (2) or using equations (1) and (4).

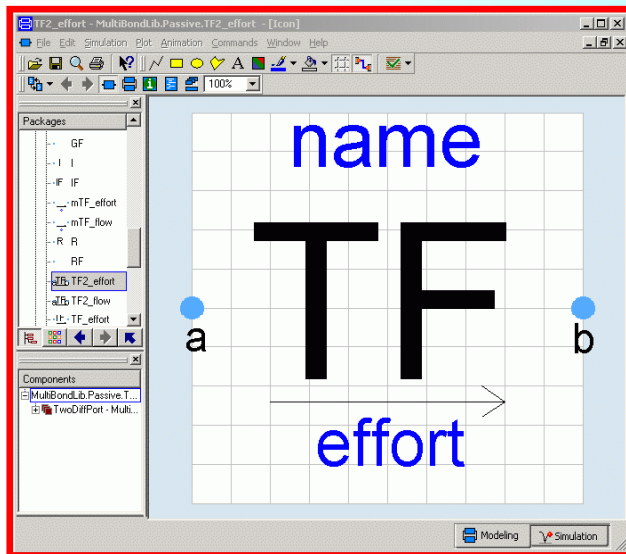
Multi-port Transformers II

- The transformer that looks most similar to the TF element of the regular bond graph library is the flow multi-port transformer. The cardinality of the bonds on the two sides doesn't have to be identical.



Multi-port Transformers III

- Yet, since M doesn't usually have an inverse, an effort transformer model must also be provided.



The image shows the Modelica Text editor for the 'TF2_effort' model. The title bar reads 'TF2_effort - MultiBondLib.Passive.TF2_effort - [Modelica Text]'. The code defines a projective multibondgraphic effort transformer. The 'Packages' list on the left shows 'MultiBondLib.Passive.TF2_effort' selected. The code is as follows:

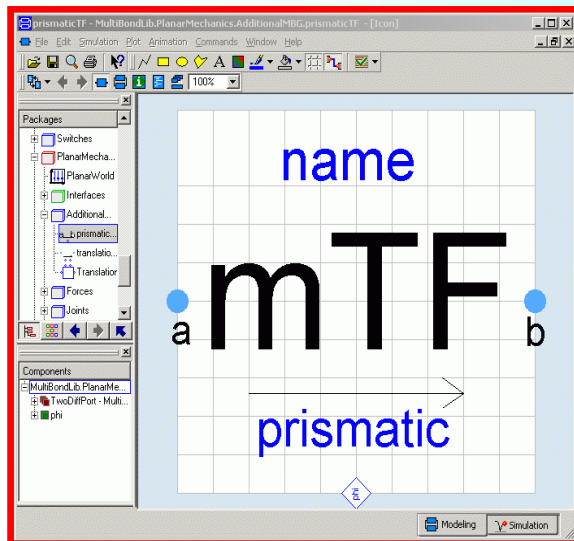
```
model TF2_effort "projective multibondgraphic effort transformer"
[
  extends Interfaces.TwoDiffPort;
  parameter Real[nB,nA] M = {{1}} "transformation matrix";
equation
  eB = M*eA;
  fA = transpose(M)*fB;
[
end TF2_effort;
```

At the bottom, the status bar shows 'Line: 9' and 'Modeling' and 'Simulation' buttons.

Multi-port Transformers IV

- Also offered are modulated versions of multi-port transformers and gyrators.
- Yet, this is still insufficient. Special transformers for particular purposes ought to be provided as well, since they are being used frequently in mechanics.
- We already met the *translational transformer*.
- Also provided is a *prismatic transformer*.
- The special transformers are contained in the 2D mechanics sub-library, since they are only useful in that context.

Multi-port Transformers V



```
model prismaticTF "modulated transformer for a prismatic joint"
  extends MultiBondLib.Interfaces.TwoDiffPort(final nA=1,final nB=2);
  parameter Real d[2] = {1,0} a;

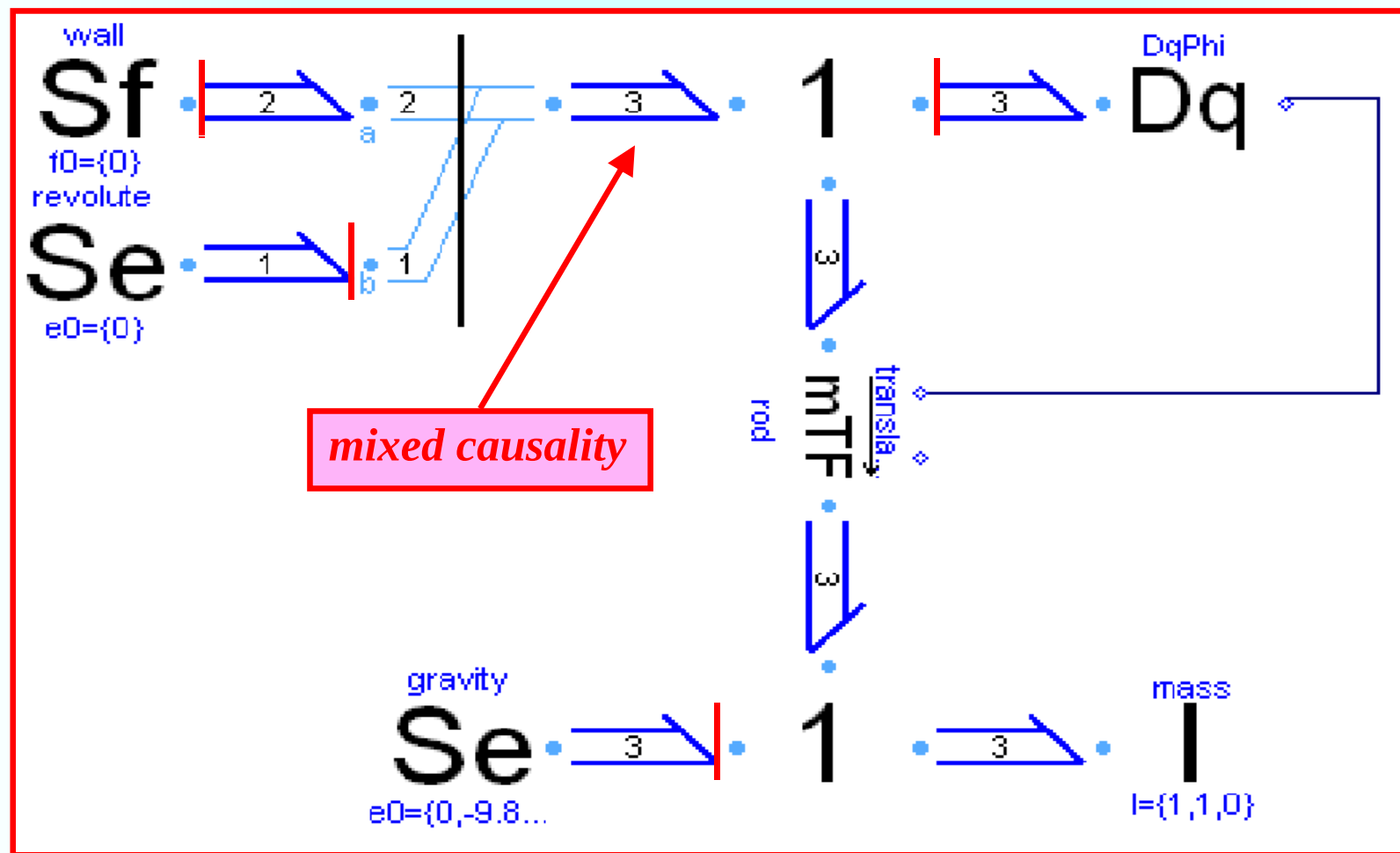
equation
  fB[1] = (cos(phi)*d[1] + sin(phi)*d[2])*fA[1];
  fB[2] = (-sin(phi)*d[1] + cos(phi)*d[2])*fA[1];
  eA[1] = (cos(phi)*d[1] + sin(phi)*d[2])*eB[1] + (-sin(phi)*d[1] + cos(phi)*d[2])*eB[2];

end prismaticTF;
```

Multi-bond Graph Basics IV

- Finally, although the library offers causal multi-bonds, these are much less useful than the causal regular bonds, because many multi-bonds have mixed computational causality. Hence causal multi-bonds are rarely used in practice.

Planar Pendulum XIII



References I

- Zimmer, D. (2006), *A Modelica Library for MultiBond Graphs and its Application in 3D-Mechanics*, MS Thesis, Dept. of Computer Science, ETH Zurich.
- Zimmer, D. and F.E. Cellier (2006), “The Modelica Multi-bond Graph Library,” *Proc. 5th Intl. Modelica Conference*, Vienna, Austria, Vol.2, pp. 559-568.

References II

- Cellier, F.E. and D. Zimmer (2006), “Wrapping Multi-bond Graphs: A Structured Approach to Modeling Complex Multi-body Dynamics,” *Proc. 20th European Conference on Modeling and Simulation*, Bonn, Germany, pp. 7-13.