Multi-bond Graphs

- We shall today look at vectors of bonds, called multi-bonds.
- Especially when dealing with 2D and 3D mechanics, the d'Alembert principle must be applied to each degree of freedom separately.
- Each equation looks structurally the same.
- This leads naturally to a demand for multi-bond graphs.

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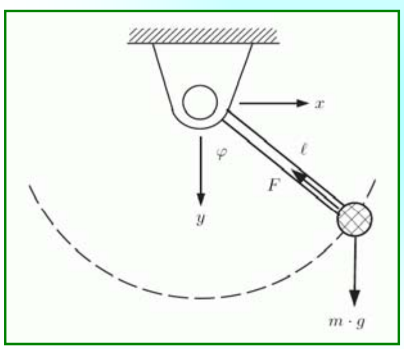
A Planar Pendulum

- Let us model the following planar pendulum:



$$m \cdot \frac{dv_x}{dt} = -\frac{F \cdot x}{\ell}$$
$$m \cdot \frac{dv_y}{dt} = m \cdot g - \frac{F \cdot y}{\ell}$$
$$\frac{dx}{dt} = v_x$$
$$\frac{dy}{dt} = v_y$$
$$x^2 + y^2 = \ell^2$$

Holonomic Constraint

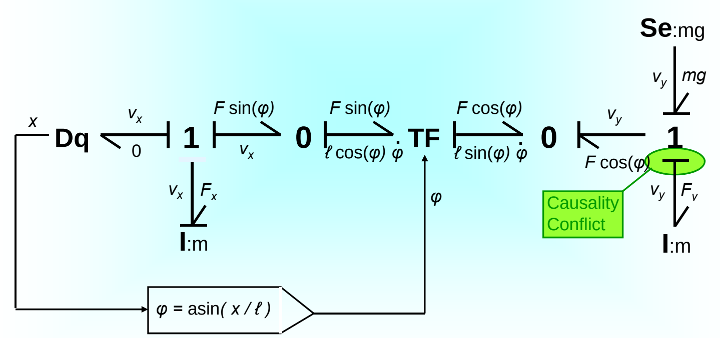
$$m \cdot \frac{dv_x}{dt} = -\frac{F \cdot x}{\ell}$$
$$m \cdot \frac{dv_y}{dt} = m \cdot g - \frac{F \cdot y}{\ell}$$
$$\frac{dx}{dt} = v_x$$
$$\frac{dy}{dt} = v_y$$
$$x = \ell \cdot \sin(\varphi)$$
$$y = \ell \cdot \cos(\varphi)$$

$m \frac{dv_x}{dt} = -F \sin(\varphi)$

$x = \ell \sin(\varphi)$, $v_x = \ell \cos(\varphi)\,\dot{\varphi}$

$y = \ell \cos(\varphi)$, $v_y = -\ell \sin(\varphi)\,\dot{\varphi}$

$m \frac{dv_y}{dt} = -F \cos(\varphi) + mg$



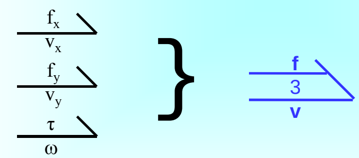
Analysis

It has been possible to describe the motion of the planar pendulum by a bond graph enhanced by activated bonds for the description of the ***holonomic constraint***. Unfortunately, the bond graph doesn't tell us much that we didn't know already.

- ❖ We shouldn't have to derive the equations first in order to be able to derive the bond graph from them.
- ❖ The resulting bond graph didn't preserve the topological properties of the system in any recognizable form.

Multi-bonds

- Multi-bond graphs are a vector extension of the regular bond graphs.
- A multi-bond contains a freely selectable number of regular bonds of identical or similar domains.



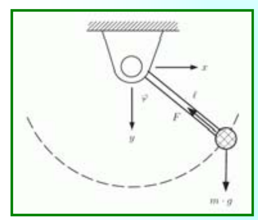
Composition of a multi-bond for planar mechanics

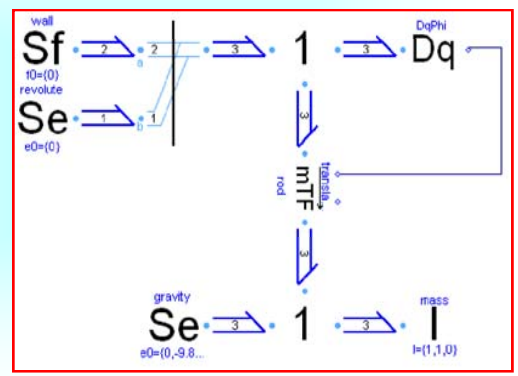
- All bond graph component models are adjusted in a suitable fashion.

Multi-bond Graph Library

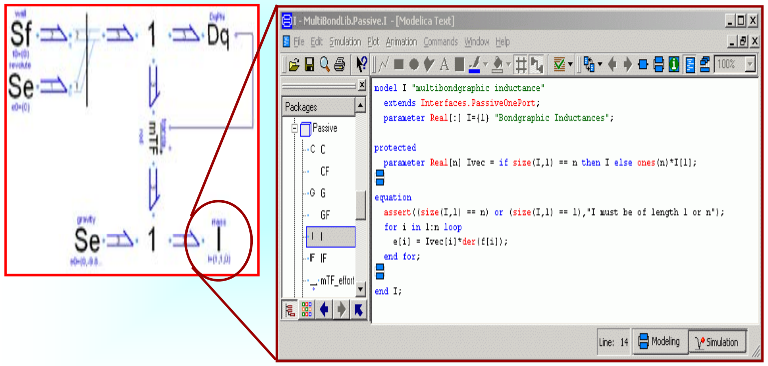
- A ***Dymola*** library for modeling systems by means of multi-bond graphs has been developed.
- The library has been designed with an interface that looks as much as possible like that of the original ***BondLib*** library.
- Just like the original library, also the new multi-bond graph library contains sub-libraries supporting modelers in modeling systems from particular application domains, especially from mechanics.

Planar Pendulum III





Planar Pendulum IV



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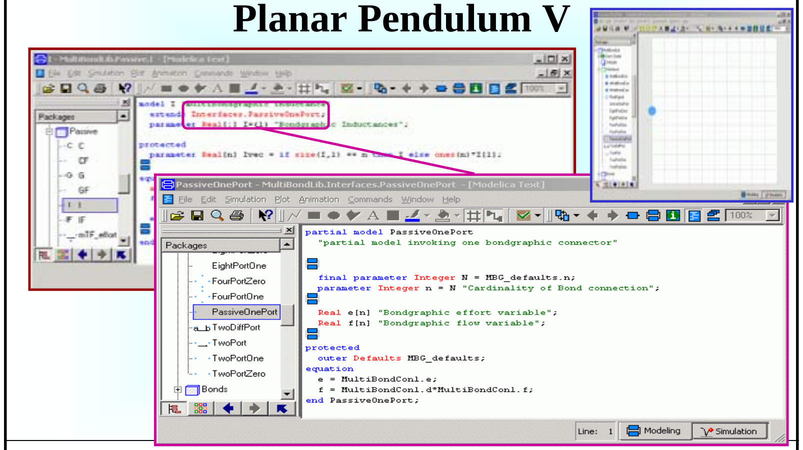
Planar Pendulum V



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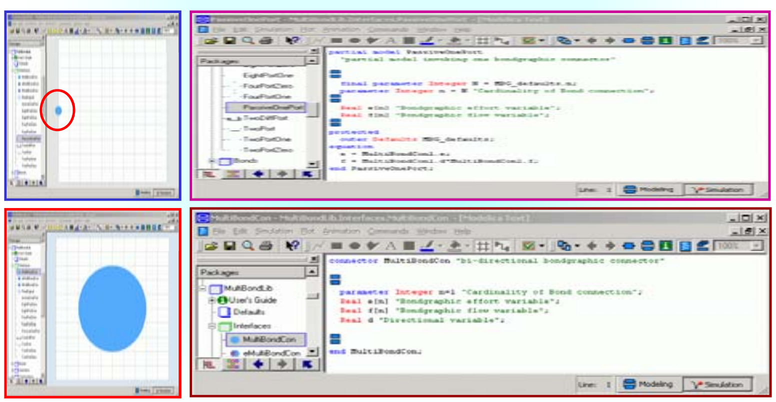
Planar Pendulum VI



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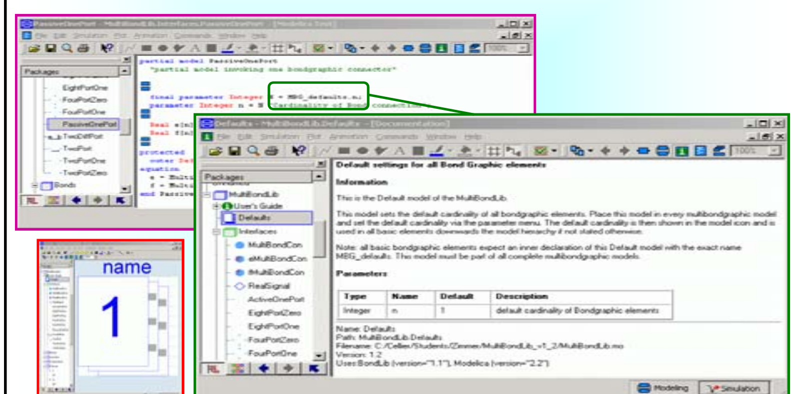
Planar Pendulum VII



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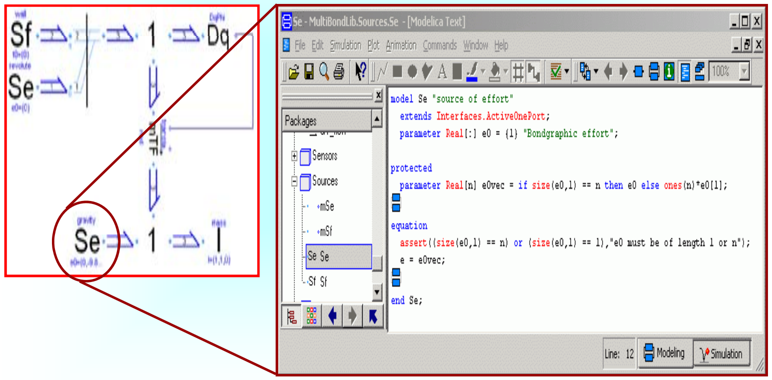
Planar Pendulum VIII



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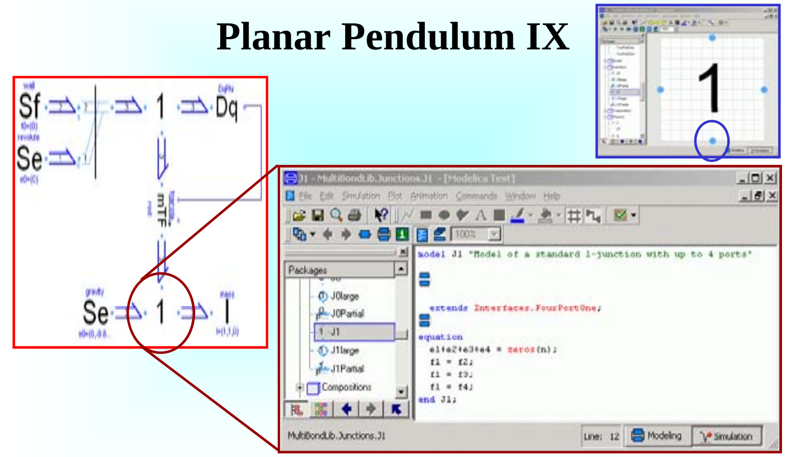
Planar Pendulum IX



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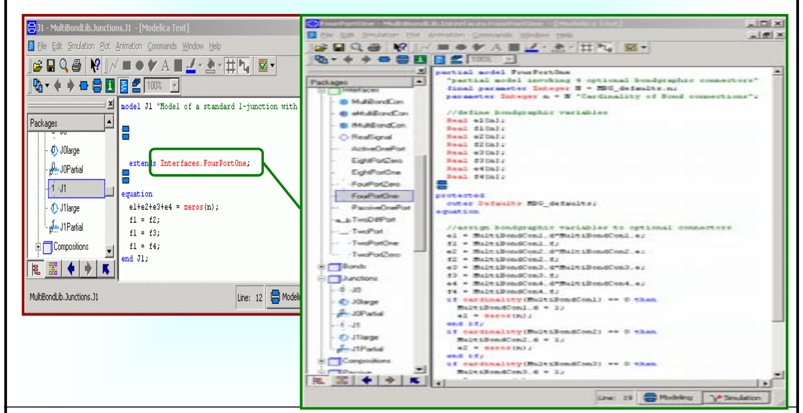
Planar Pendulum X



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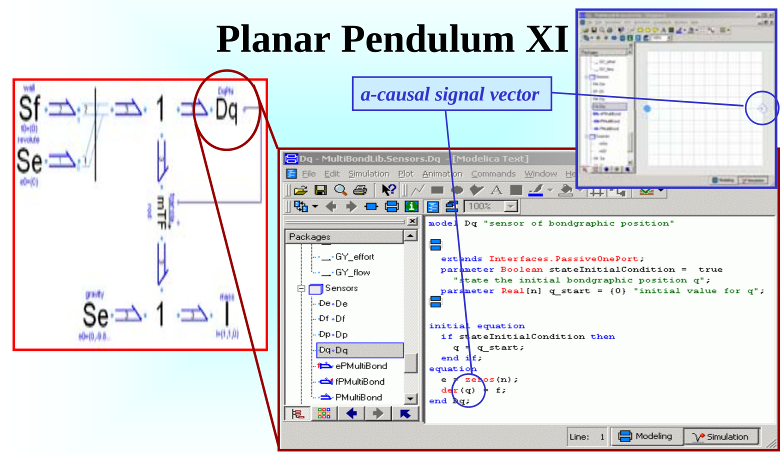
Planar Pendulum XI



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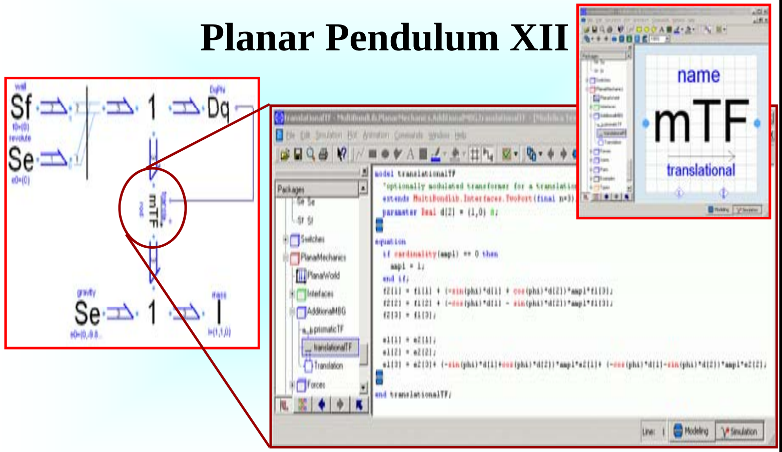
Planar Pendulum XII



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Multi-bond Graph Basics

- The basic multi-bond graph models contain little that is surprising. They represent essentially natural extensions of the regular bond graph models.
- A few points are worth mentioning though. First, there is the ***defaults model*** that must be included in each multi-bond graph model. It contains only a single parameter, the dimensional parameter, ***n***, that specifies, how many bonds each multi-bond contains by default.
- The defaults model must be referenced in each multi-bond graph model as an ***outer model***.

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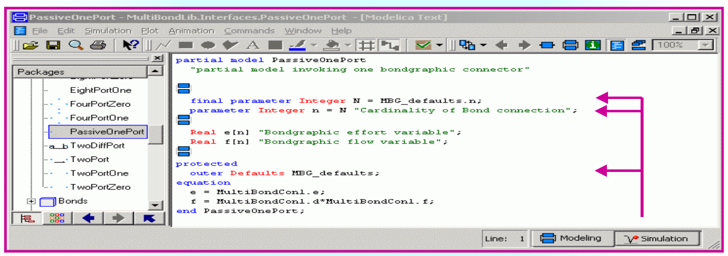
Multi-bond Graph Basics II



- If the multi-bond graph model inherits one of the partial models, this has already been taken care of.

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Multi-bond Graph Basics III

- A second difference concerns the use of junctions. Whereas the general bond graph library provides separate junction models for 2..6 bond connections, the multi-bond graph library offers only junctions with either 4 or 8 connectors. Yet, individual connectors may be left unconnected as needed.
- A third difference is in the use of transformers and gyrators. The multi-bond graph library offers a much larger variety of different transformer and gyrator models when compared to the regular bond graph library.

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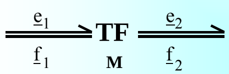
Multi-port Transformers



Transformation: $\underline{e}_1 = M \cdot \underline{e}_2$ *(1)*

Energy Conservation: $\underline{e}_1^T \cdot \underline{f}_1 = \underline{e}_2^T \cdot \underline{f}_2$ *(2)*

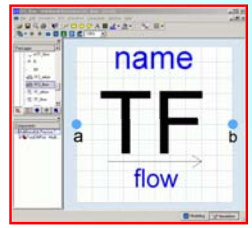
$\Rightarrow (M \cdot \underline{e}_2)^T \cdot \underline{f}_1 = \underline{e}_2^T \cdot \underline{f}_2$ *(3)*

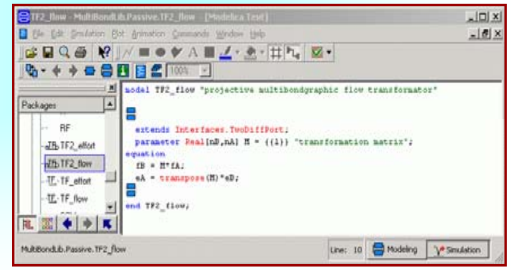
$\Rightarrow \underline{f}_2 = M^T \cdot \underline{f}_1$ *(4)*

$\Rightarrow$ *The transformer may either be described by means of equations (1) and (2) or using equations (1) and (4).*

Multi-port Transformers II

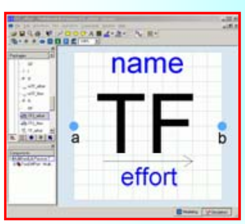
- The transformer that looks most similar to the TF element of the regular bond graph library is the flow multi-port transformer. The cardinality of the bonds on the two sides doesn't have to be identical.

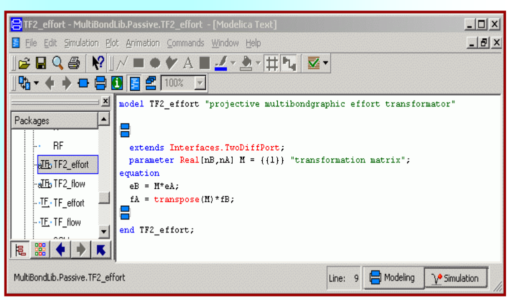




Multi-port Transformers III

- Yet, since ***M*** doesn't usually have an inverse, an effort transformer model must also be provided.

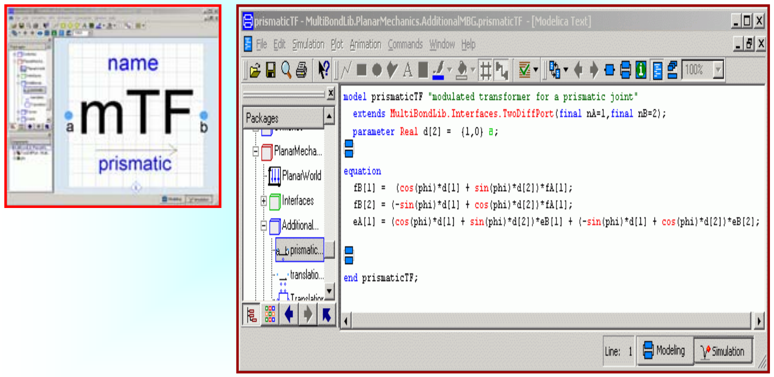




Multi-port Transformers IV

- Also offered are modulated versions of multi-port transformers and gyrators.
- Yet, this is still insufficient. Special transformers for particular purposes ought to be provided as well, since they are being used frequently in mechanics.
- We already met the ***translational transformer***.
- Also provided is a ***prismatic transformer***.
- The special transformers are contained in the 2D mechanics sub-library, since they are only useful in that context.

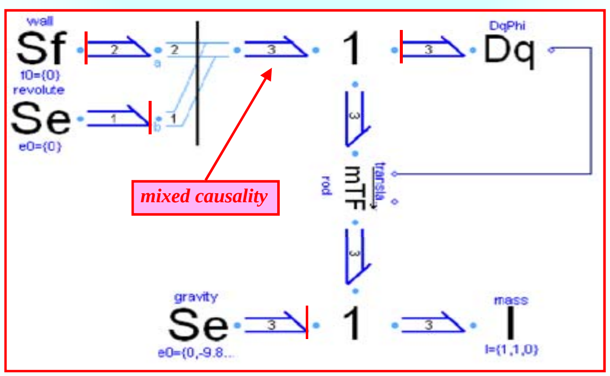
Multi-port Transformers V



Multi-bond Graph Basics IV

- Finally, although the library offers causal multi-bonds, these are much less useful than the causal regular bonds, because many multi-bonds have mixed computational causality. Hence causal multi-bonds are rarely used in practice.

Planar Pendulum XIII



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- Cellier, F.E. and D. Zimmer (2006), "Wrapping Multi-bond Graphs: A Structured Approach to Modeling Complex Multi-body Dynamics," *Proc. 20th European Conference on Modeling and Simulation*, Bonn, Germany, pp. 7-13.