

## Convective Mass Flows I

- In this lecture, we shall begin looking at the problem of convective mass flows.
- Irreversible thermodynamics* concerns itself with purely thermal phenomena, as well as the conversion of free energy into heat.
- In *reversible thermodynamics*, the situation is complicated by the fact that it often concerns itself with mass flows next to energy flows.

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## Mass Flows vs. Entropy Flows

- Although there exist phenomena that take place purely in the thermal domain, there can be no mass flows that occur without heat flows accompanying them.
- The problem has to do with the fact that *mass flows* always carry their *volume* and *stored heat* with them. It is therefore not meaningful to consider these quantities independently of each other.
- The water circulation within *Biosphere 2* may serve as an excellent example. The thermal phenomena of the water cycle cannot be properly described without taking into account its mass flow (or at least its volume flow) as well.

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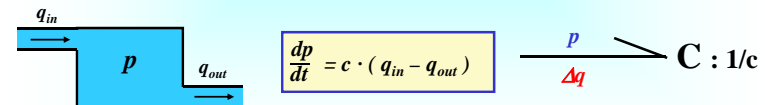
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## Fluid Moving in a Pipe I

- We shall start by modeling the flow of fluids (liquids or gases) in a pipe.
- The pipe can be subdivided into segments of length  $\Delta x$ .
- If more fluid enters a segment than leaves it, the pressure of the fluid in the segment must evidently grow. The fluid is being *compressed*.



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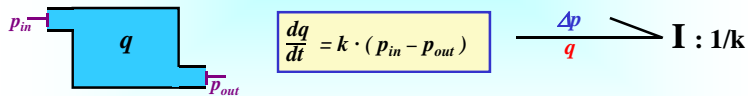
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## Fluid Moving in a Pipe II

- When the pressure at the entrance of a segment is higher than at the exit, the speed of the fluid must increase. This effect is caused by the mechanical nature of the fluid. The pressure is proportional to a force, and the volume is proportional to the mass. Consequently, this is an inductive phenomenon. It describes the *inertia* of the moving mass.



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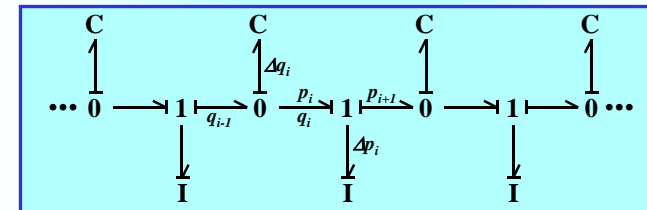
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## Fluid Moving in a Pipe III

- Consequently, the following bond graph may be proposed:



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## Capacitors and Inductors

- Although the hydraulic/pneumatic inductor describes the same physical phenomenon as the mechanical inductor, its measurement units are nevertheless different.

$$\Delta q = C \cdot \frac{dp}{dt} \quad [C] = [\Delta q] / [dp/dt] = \frac{m^3 \cdot s^{-1}}{N \cdot m^{-2} \cdot s^{-1}} = m^4 \cdot s^2 \cdot kg^{-1}$$

$$\Delta p = L \cdot \frac{dq}{dt} \quad [L] = [\Delta p] / [dq/dt] = \frac{N \cdot m^{-2}}{m^3 \cdot s^{-2}} = kg \cdot m^{-4}$$

$$\Rightarrow [\sqrt{L \cdot C}] = s \quad \text{is still a time constant}$$

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## The Wave Equation I

- Any text book of physics teaches us that moving fluid in a pipe satisfies the wave equation.

$$\frac{\partial^2 p}{\partial t^2} = c^2 \cdot \frac{\partial^2 p}{\partial x^2}$$

- Discretization in space leads to:

$$\frac{d^2 p_k}{dt^2} = \frac{c^2}{\Delta x^2} \cdot (p_{k+1} - 2 \cdot p_k + p_{k-1})$$


$$\Rightarrow \left( \frac{\Delta x}{c} \right)^2 \cdot \frac{d^2 p_k}{dt^2} = p_{k+1} - 2 \cdot p_k + p_{k-1}$$

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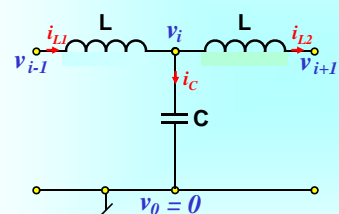


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## The Wave Equation II

- The following replacement circuit comes to mind:



$$\frac{dv_i}{dt} = i_C / C$$

$$i_C = i_{L1} - i_{L2}$$

$$v_{i-1} - v_i = L \cdot di_{L1} / dt$$

$$v_i - v_{i+1} = L \cdot di_{L2} / dt$$

$$C \cdot d^2 v_i / dt^2 = di_C / dt$$

$$di_C / dt = di_{L1} / dt - di_{L2} / dt$$


$$v_{i-1} - v_i = L \cdot di_{L1} / dt$$

$$v_i - v_{i+1} = L \cdot di_{L2} / dt$$

$$\Rightarrow (L \cdot C) \cdot \frac{d^2 v_i}{dt^2} = v_{i+1} - 2 \cdot v_i + v_{i-1}$$

$$\Rightarrow c = \frac{\Delta x}{\sqrt{L \cdot C}}$$

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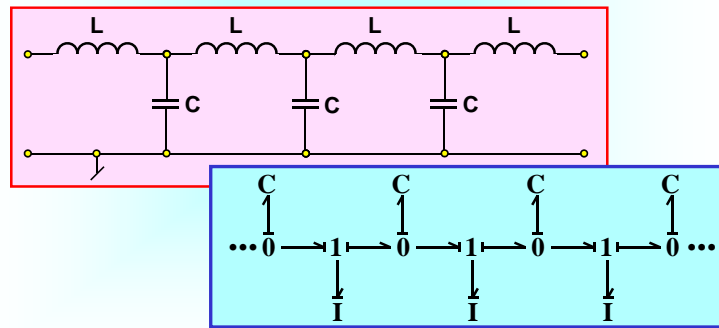


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## The Wave Equation III

- A chain of such links indeed corresponds to the proposed bond graph:



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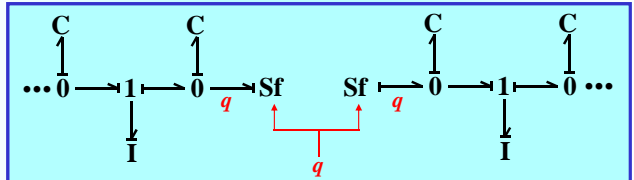


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## The Forced Flow I

- A forced flow can first be conceptualized as a modulated flow source.



What happens with the energy at the interrupted chain? As the flows to the left and to the right are identical (mass conservation), it makes sense to unite the two flow sources.

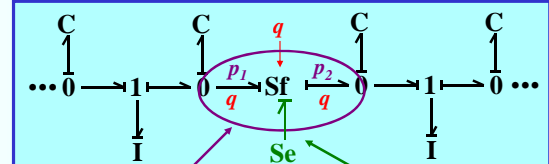
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## The Forced Flow II



Unfortunately, there is a small problem: the energy conservation is violated!

A flow cannot be forced without adding energy to the system.

It seems that we need to look at this problem a little more carefully ...

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## The Turbine I

- We shall check what happens, when the blade of a turbine is placed in the path of a flow that occurs for reasons external to the model.
- The pressure difference on the two sides of the blade generates a resulting force that produces a torque in the turbine.
- The generated torque is proportional to the pressure difference, and consequently, we recognize the effects of a bond-graphic **transformer** at work.
- If the turbine is designed optimally, the pressure difference is fully converted to a torque, i.e., no hydraulic energy is left to be stored in an inductor.

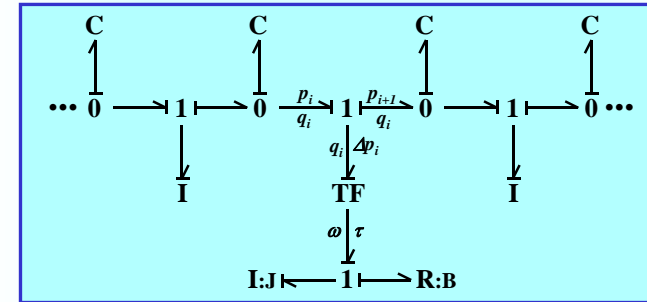
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## The Turbine II



The pressure difference  $\Delta p_i$  leads to a torque  $\tau$ . This in turn generates an angular velocity  $\omega$  at the turbine, which induces a flow  $q_i$  back on the hydraulic side.

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## Compressors and Pumps I

- Cause and effect can also be reversed. We can generate a torque at the turbine by means of a DC-motor sitting on the same axle. The turbine together with the motor is now called either **compressor** or **pump**. This device induces a flow  $q_i$  on the hydraulic side, which causes a corresponding pressure difference.

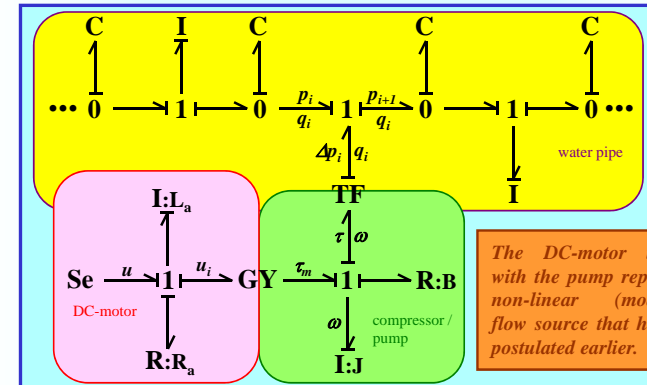
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## Compressors and Pumps II




The DC-motor together with the pump replace the non-linear (modulated) flow source that had been postulated earlier.

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## Heat Flow I

- We have meanwhile understood, how the *mass flow* needs to be modeled. However, the transported mass always carries its own *heat* along.
- When modeling thermal phenomena, it is therefore important, to correctly represent these heat flows, *which are not of a dissipative nature*.

Storage  
V, S

$\dot{S}$   
→  
Heat flow


$q$   
→  
Mass flow


$\Rightarrow$

$\dot{S} = \frac{q}{V}$

$\Rightarrow$

$\dot{S} = (S/V) \cdot q$

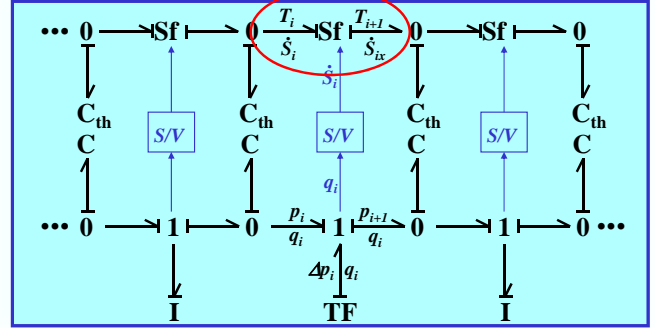
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## Heat Flow II

- The resulting heat flow can indeed be represented as a non-linear (modulated) flow source. The energy conservation law is not violated here, since entropy is not conserved.



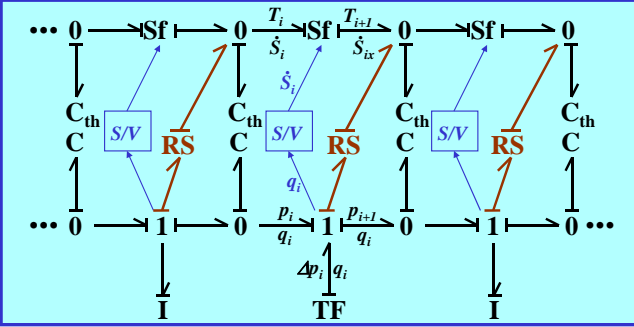
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## Mass Transport Losses

- Fluid transports are in reality always associated with losses due to friction.



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## Conclusions

- We have meanwhile understood that the *cause of convective heat flows* is to be found in the mass transport.
- The mass transport is captured by the wave equation, whereby forced flows (pumps, compressors) replace the inductors at the locations of flow-forcing devices.
- The heat flow is a consequence of the mass flow, and may be modeled using non-linear (modulated) internal flow sources.
- Frictional losses can be added to the model afterwards where and when needed.

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