Convective Mass Flows III

- In this lecture, we shall concern ourselves once more with convective mass and heat flows, as we still have not gained a comprehensive understanding of the physics behind such phenomena.
- We shall start by looking once more at the ***capacitive field***.
- We shall then study the ***internal energy*** of matter.
- Finally, we shall look at ***general energy transport phenomena***, which by now include mass flows as an integral aspect of general energy flows.

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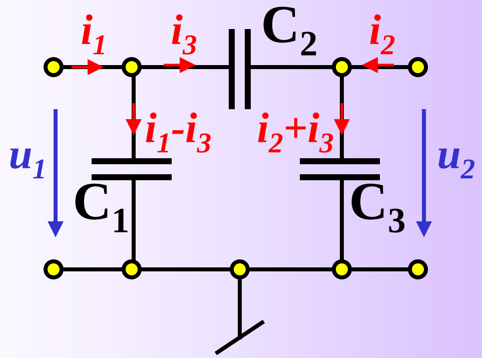
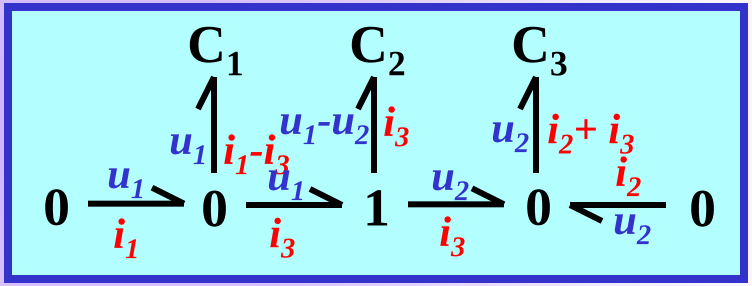
Capacitive Fields III

- Let us briefly consider the following electrical circuit:





$$i_1 - i_3 = C_1 \cdot du_1/dt$$
$$i_2 + i_3 = C_3 \cdot du_2/dt$$
$$i_3 = C_2 \cdot (du_1/dt - du_2/dt)$$

$$\Rightarrow$$

$$i_1 = (C_1 + C_2) \cdot du_1/dt - C_2 \cdot du_2/dt$$
$$i_2 = -C_2 \cdot du_1/dt + (C_2 + C_3) \cdot du_2/dt$$

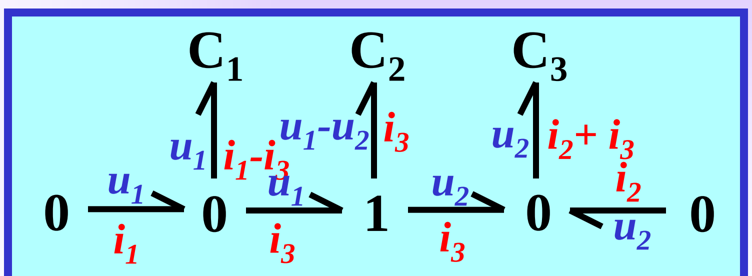
Capacitive Fields IV

$$i_1 = (C_1 + C_2)\cdot du_1/dt - C_2 \cdot du_2/dt$$
$$i_2 = -C_2 \cdot du_1/dt + (C_2 + C_3)\cdot du_2/dt$$

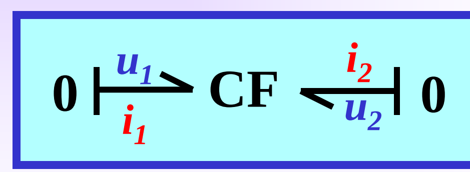
Symmetric capacity matrix

$$\Rightarrow \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} (C_1 + C_2) & -C_2 \\ -C_2 & (C_2 + C_3) \end{bmatrix} \cdot \begin{bmatrix} du_1/dt \\ du_2/dt \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} du_1/dt \\ du_2/dt \end{bmatrix} = \frac{\begin{bmatrix} (C_2 + C_3) & C_2 \\ C_2 & (C_1 + C_2) \end{bmatrix}}{C_1 C_2 + C_1 C_3 + C_2 C_3} \cdot \begin{bmatrix} i_1 \\ i_2 \end{bmatrix}$$

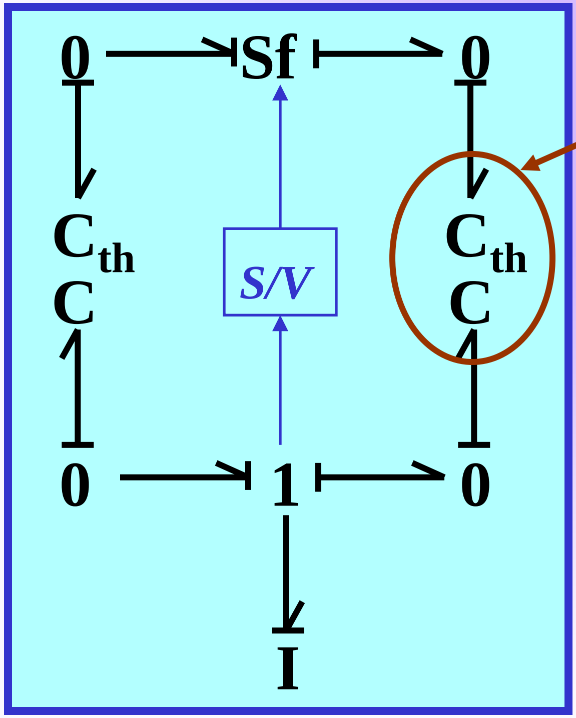


$\Rightarrow$



Volume and Entropy Storage

- Let us consider once more the situation discussed in the previous lecture.



It was no accident that I drew the two capacitors so close to each other. In reality, the two capacitors together form a two-port capacitive field. After all, heat and volume are only two different properties of one and the same material.

The Internal Energy of Matter I

- As we have already seen, there are three different (though inseparable) storages of matter:
    - Mass
    - Volume
    - Heat
- These three storage elements represent different storage properties of one and the same material.
- Consequently, we are dealing with a ***storage field***.
- This storage field is of a capacitive nature.
- The capacitive field stores the ***internal energy of matter***.

The Internal Energy of Matter II

- Change of the internal energy in a system, i.e. the total power flow into or out of the capacitive field, can be described as follows :

Chemical potential

Flow of internal energy

$$\dot{U} = T \cdot \dot{S} - p \cdot \dot{V} + \sum_{\forall i} \mu_i \cdot \dot{N}_i$$

Molar mass flow

Heat flow

Mass flow

Volume flow

- This is the ***Gibbs equation***.

The Internal Energy of Matter III

- The internal energy is proportional to the the total mass ***n***.
- By normalizing with ***n***, all extensive variables can be made intensive.

$$u = \frac{U}{n} \qquad s = \frac{S}{n} \qquad v = \frac{V}{n} \qquad n_i = \frac{N_i}{n}$$

- Therefore:

$$\frac{d}{dt}(n \cdot u) = T \cdot \frac{d}{dt}(n \cdot s) - p \cdot \frac{d}{dt}(n \cdot v) + \sum_{\forall i} \mu_i \cdot \frac{d}{dt}(n \cdot n_i)$$

$$\Rightarrow \boxed{\frac{d}{dt}(n \cdot u) - T \cdot \frac{d}{dt}(n \cdot s) + p \cdot \frac{d}{dt}(n \cdot v) - \sum_{\forall i} \mu_i \cdot \frac{d}{dt}(n \cdot n_i) = 0}$$

The Internal Energy of Matter IV

$$\frac{d}{dt}(n \cdot u) - T \cdot \frac{d}{dt}(n \cdot s) + p \cdot \frac{d}{dt}(n \cdot v) - \sum_{\forall i} \mu_i \cdot \frac{d}{dt}(n \cdot n_i) = 0$$

$$\Rightarrow \quad n \cdot \left[\frac{du}{dt} - T \cdot \frac{ds}{dt} + p \cdot \frac{dv}{dt} - \sum_{\forall i} \mu_i \cdot \frac{dn_i}{dt}\right] + \frac{dn}{dt} \cdot \left[u - T \cdot s + p \cdot v - \sum_{\forall i} \mu_i \cdot n_i\right] = 0$$

This equation must be valid independently of the amount n, therefore:

Finally, here is an explanation, why it was okay to compute with funny derivatives.

$$\frac{du}{dt} - T \cdot \frac{ds}{dt} + p \cdot \frac{dv}{dt} - \sum_{\forall i} \mu_i \cdot \frac{dn_i}{dt} = 0$$

Flow of internal energy

$$u - T \cdot s + p \cdot v - \sum_{\forall i} \mu_i \cdot n_i = 0$$

Internal energy

The Internal Energy of Matter V

$$\boxed{U = T \cdot S - p \cdot V + \sum_{\forall i} \mu_i \cdot N_i}$$

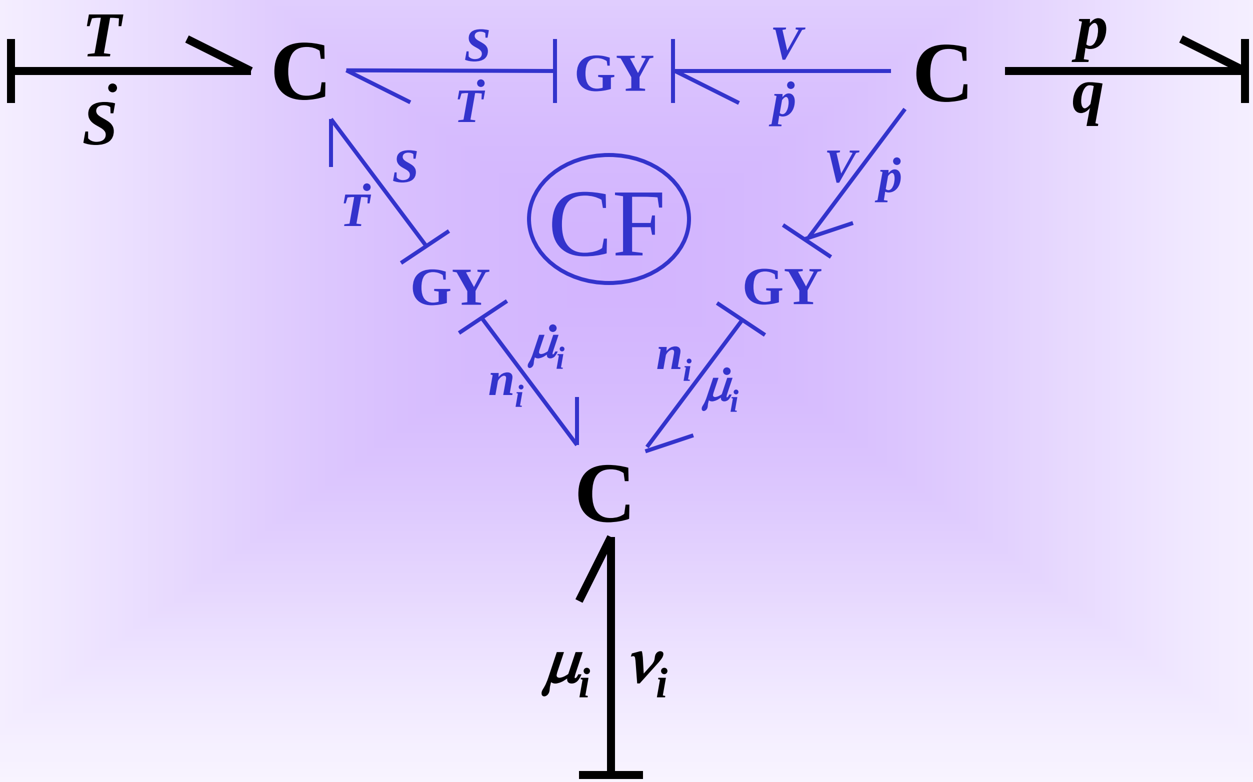
$$\Rightarrow \quad \dot{U} = T \cdot \dot{S} - p \cdot \dot{V} + \sum_{\forall i} \mu_i \cdot \dot{N}_i + \dot{T} \cdot S - \dot{p} \cdot V + \sum_{\forall i} \dot{\mu}_i \cdot N_i$$

$$= T \cdot \dot{S} - p \cdot \dot{V} + \sum_{\forall i} \mu_i \cdot \dot{N}_i$$

$$\Rightarrow \quad \boxed{\dot{T} \cdot S - \dot{p} \cdot V + \sum \dot{\mu}_i \cdot N_i = 0}$$

- This is the ***Gibbs-Duhem equation***.

The Capacitive Field of Matter



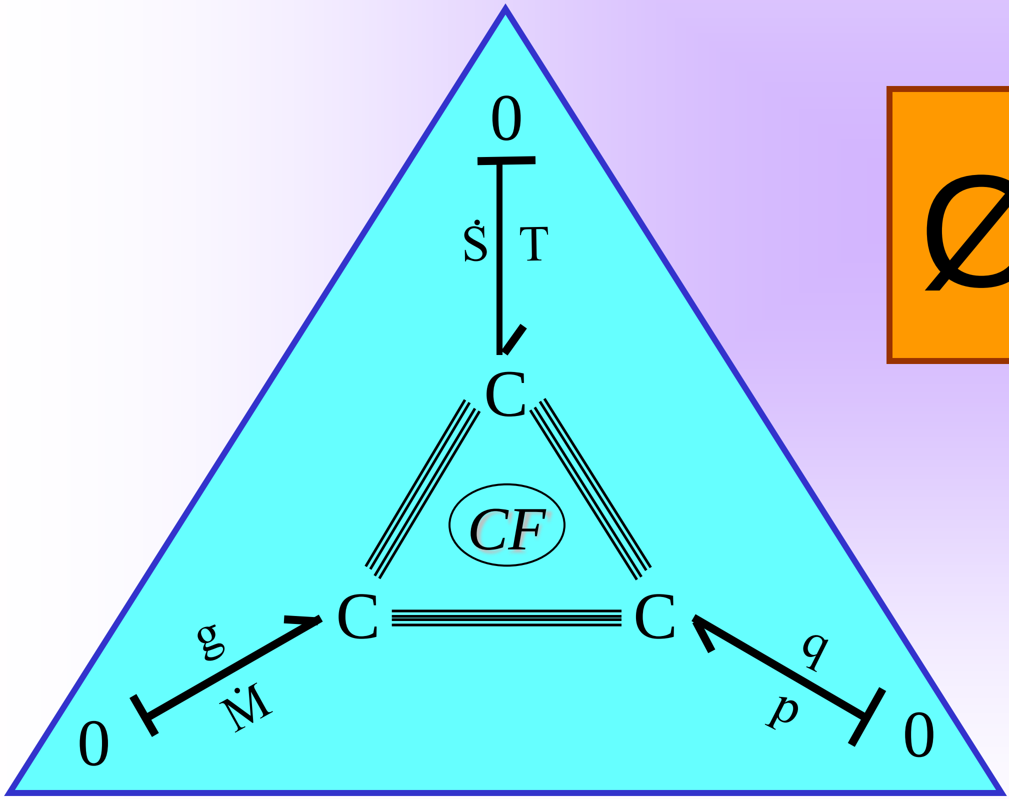
Simplifications

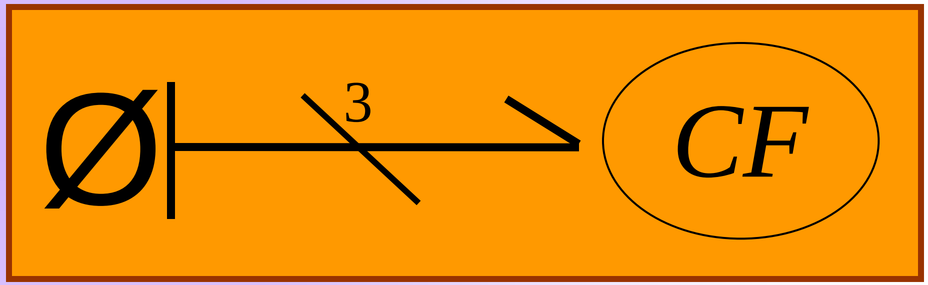
- In the case that no chemical reactions take place, it is possible to replace the ***molar mass flows*** by conventional ***mass flows***.
- In this case, the ***chemical potential*** is replaced by the ***Gibbs potential***.

$$\frac{dU}{dt} = T \cdot \dot{S} - p \cdot \dot{V} + g \cdot \dot{M}$$

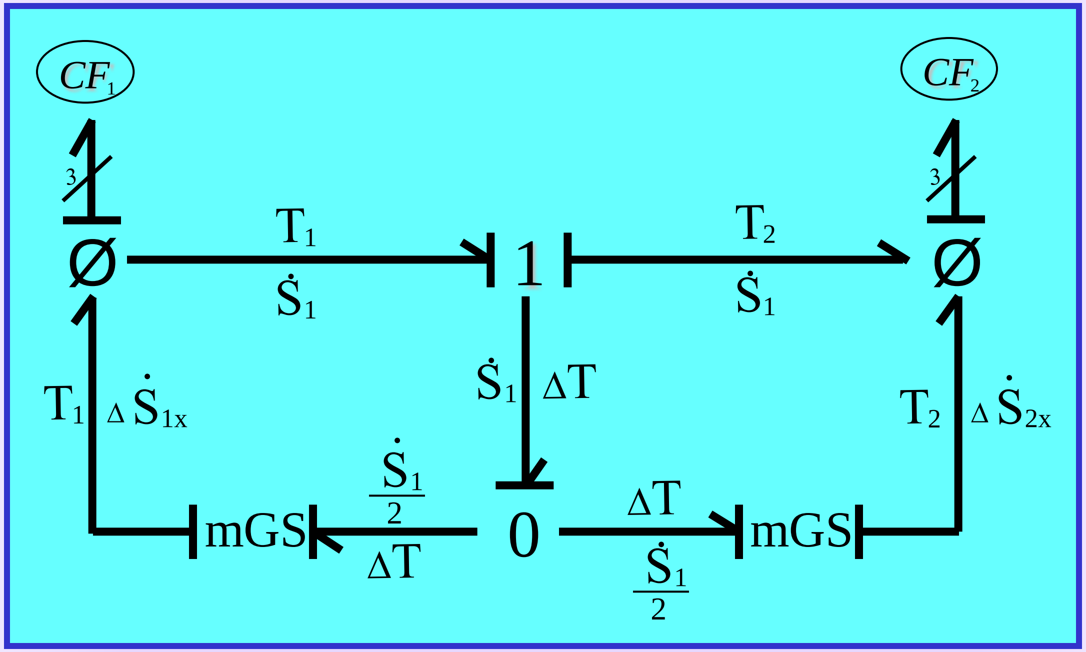
Bus-Bond and Bus-0-Junction

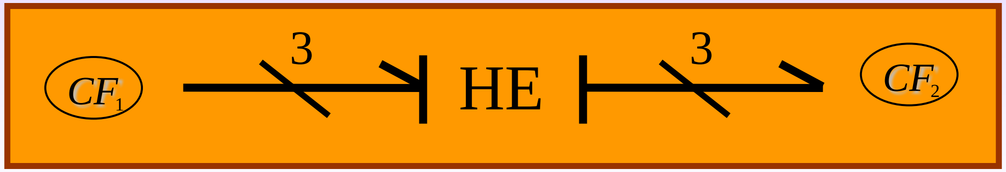
- The three outer legs of the CF-element can be grouped together.



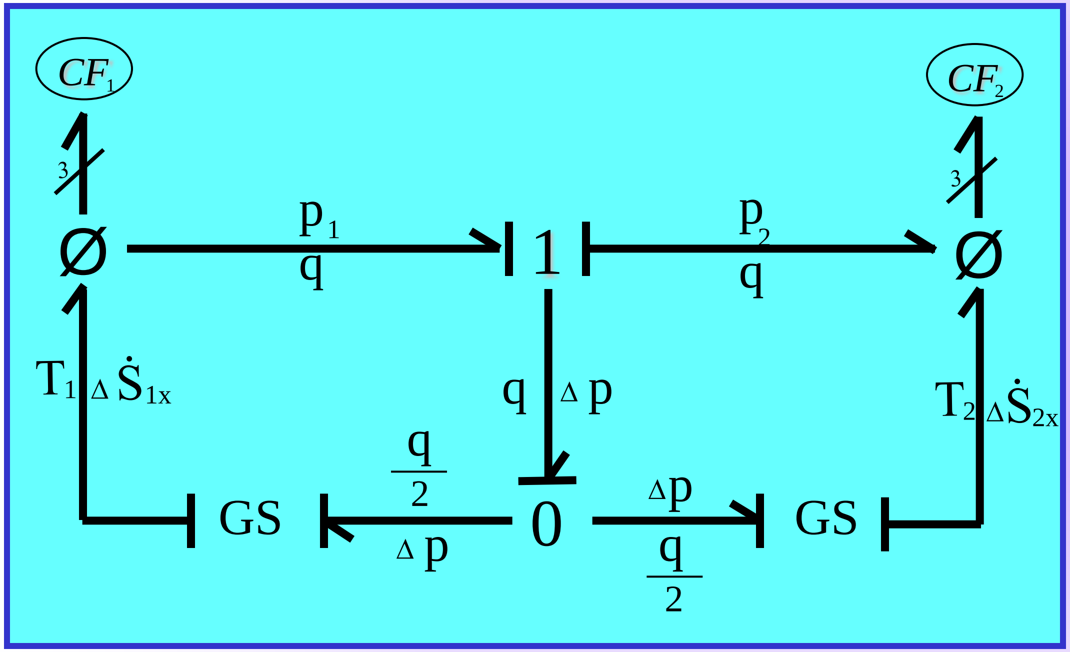


Once Again Heat Conduction





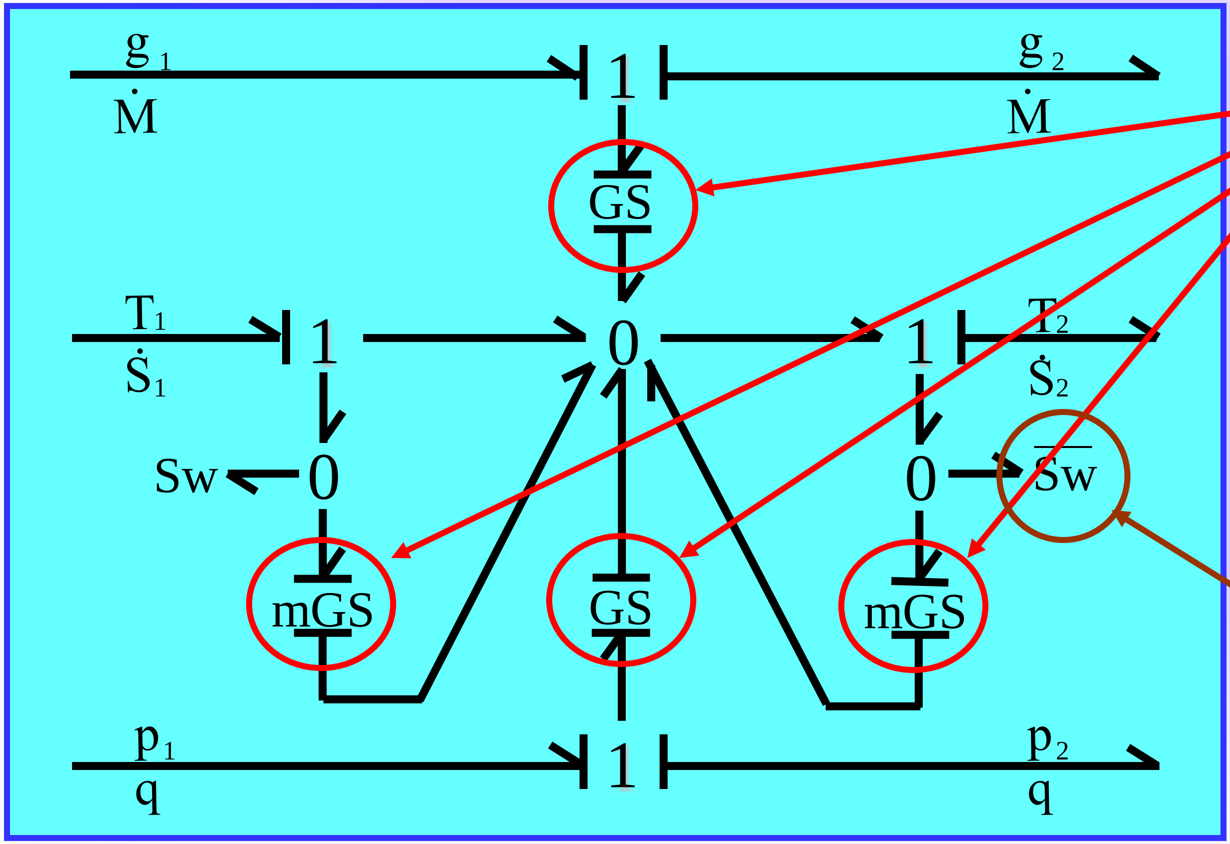
Volume Pressure Exchange



Pressure is being equilibrated just like temperature. It is assumed that the inertia of the mass may be neglected (relatively small masses and/or velocities), and that the equilibration occurs without friction.

The model makes sense if the exchange occurs locally, and if not too large masses get moved in the process.

General Exchange Element I

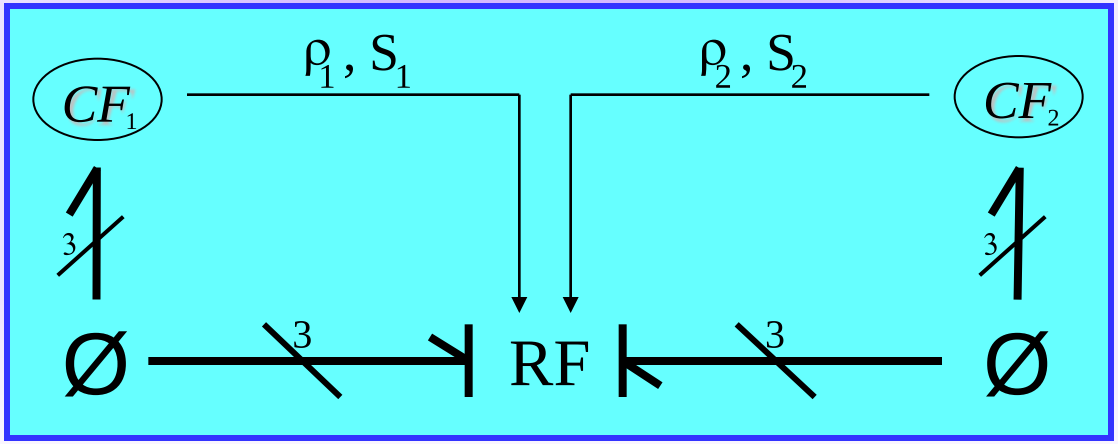


The three flows are coupled through RS-elements.

This is a switching element used to encode the direction of positive flow.

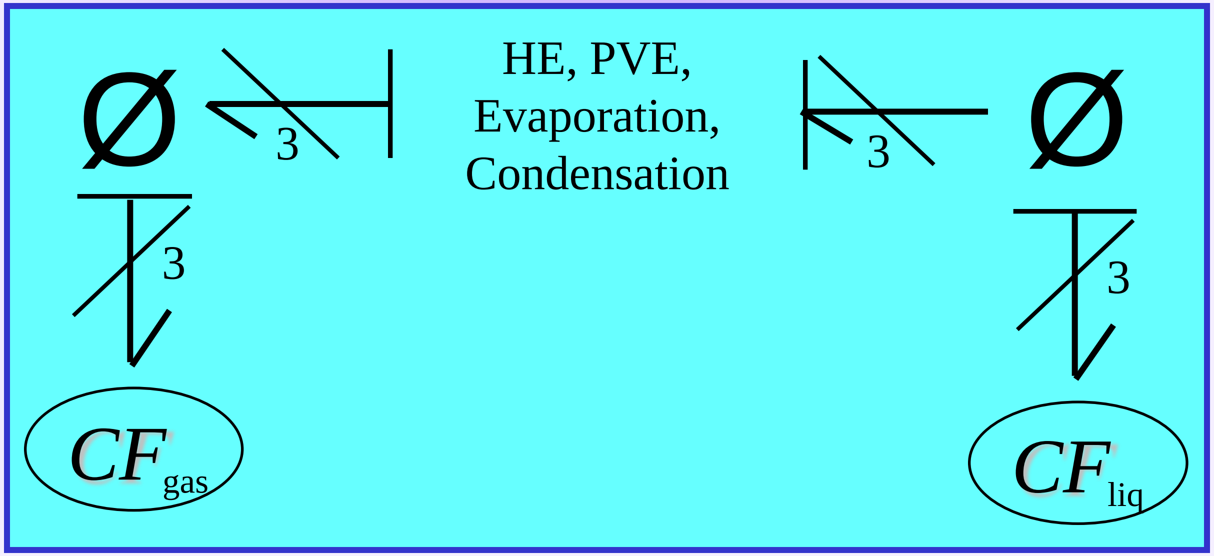
General Exchange Element II

- In the general exchange element, the temperatures, the pressures, and the Gibbs potentials of neighboring media are being equilibrated.
- This process can be interpreted as a ***resistive field***.



Multi-phase Systems

- We may also wish to study phenomena such as ***evaporation*** and ***condensation***.

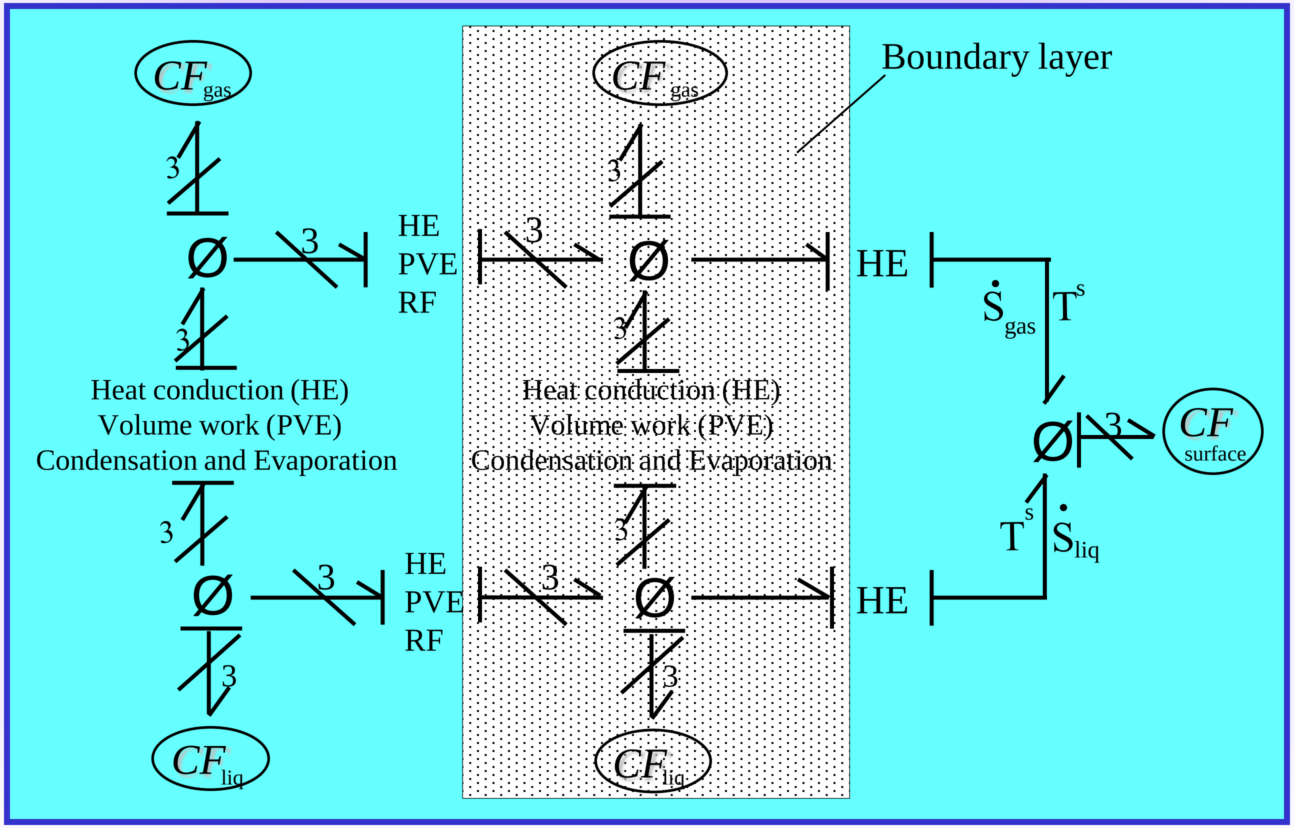


Evaporation (Boiling)

- Mass and energy exchange between capacitive storages of matter (***CF-elements***) representing different ***phases*** is accomplished by means of special resistive fields (***RF-elements***).
- The mass flows are calculated as functions of the pressure and the corresponding saturation pressure.
- The volume flows are computed as the product of the mass flows with the saturation volume at the given temperature.
- The entropy flows are superposed with the enthalpy of evaporation (in the process of evaporation, the thermal domain loses heat → ***latent heat***).

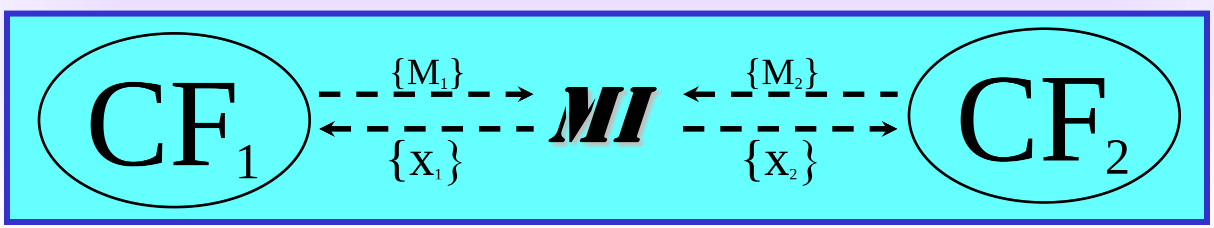
Condensation On Cold Surfaces

- Here, a boundary layer must be introduced.



Thermodynamics of Mixtures

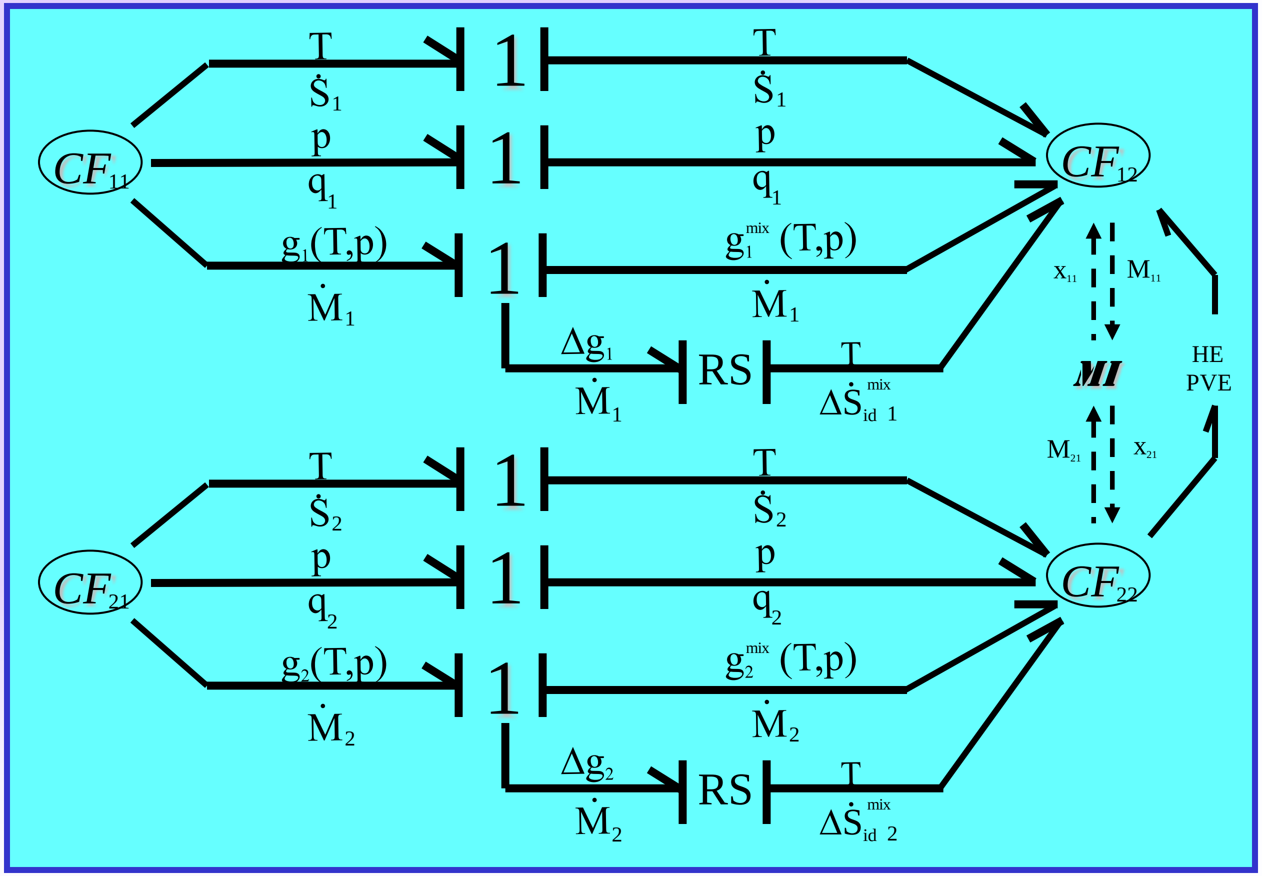
- When fluids (gases or liquids) are being mixed, additional entropy is generated.
- This ***mixing entropy*** must be distributed among the participating component fluids.
- The distribution is a function of the ***partial masses***.
- Usually, neighboring ***CF-elements*** are not supposed to know anything about each other. In the process of mixing, this rule cannot be maintained. The necessary information is being exchanged.

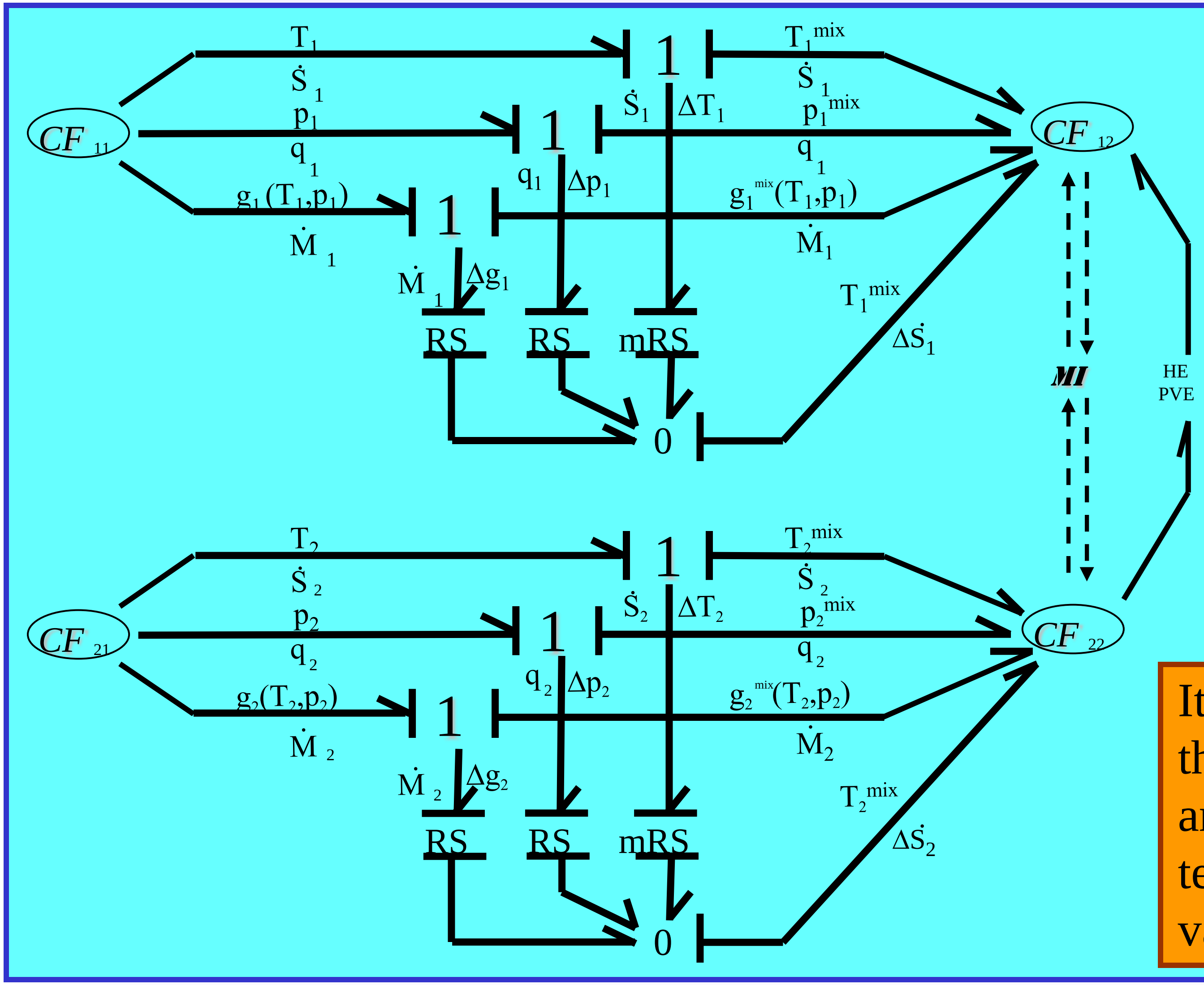


Entropy of Mixing

- The mixing entropy is taken out of the Gibbs potential.

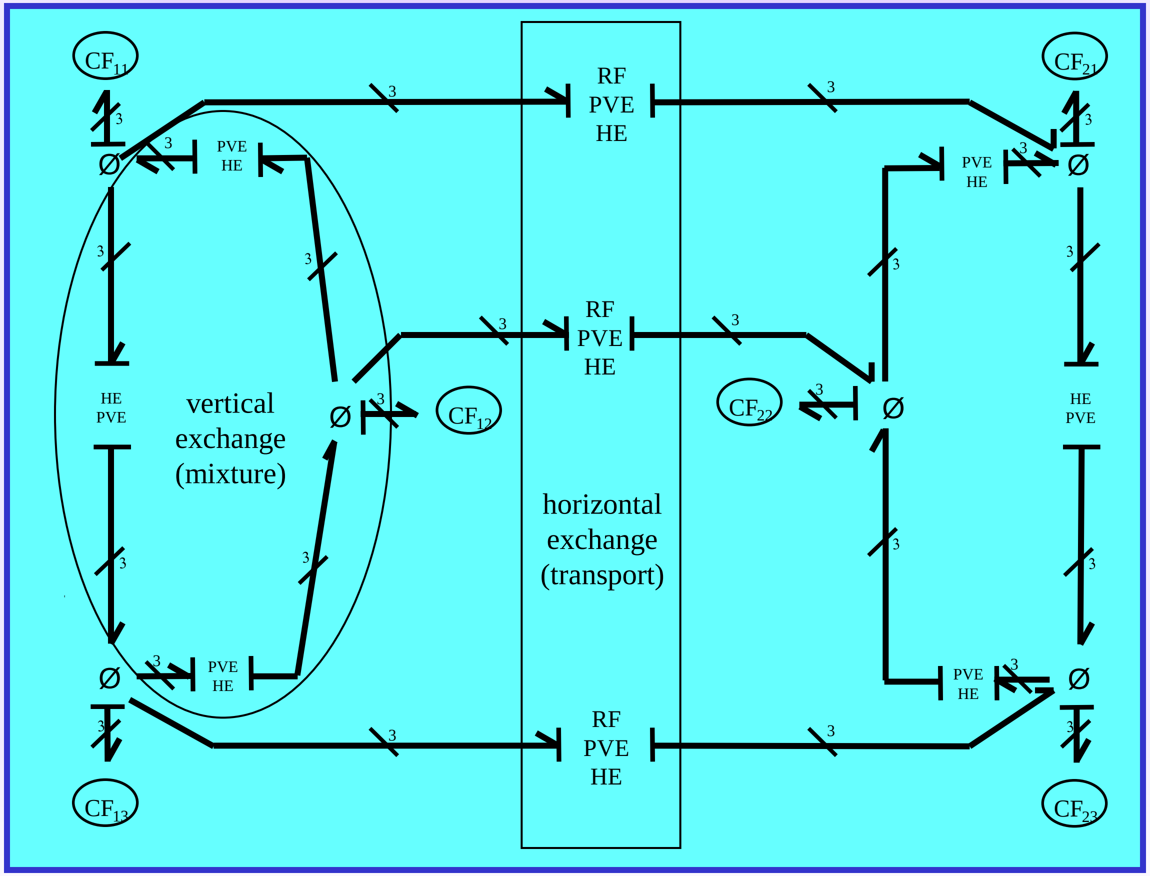
It was assumed here that the fluids to be mixed are at the same temperature and pressure.



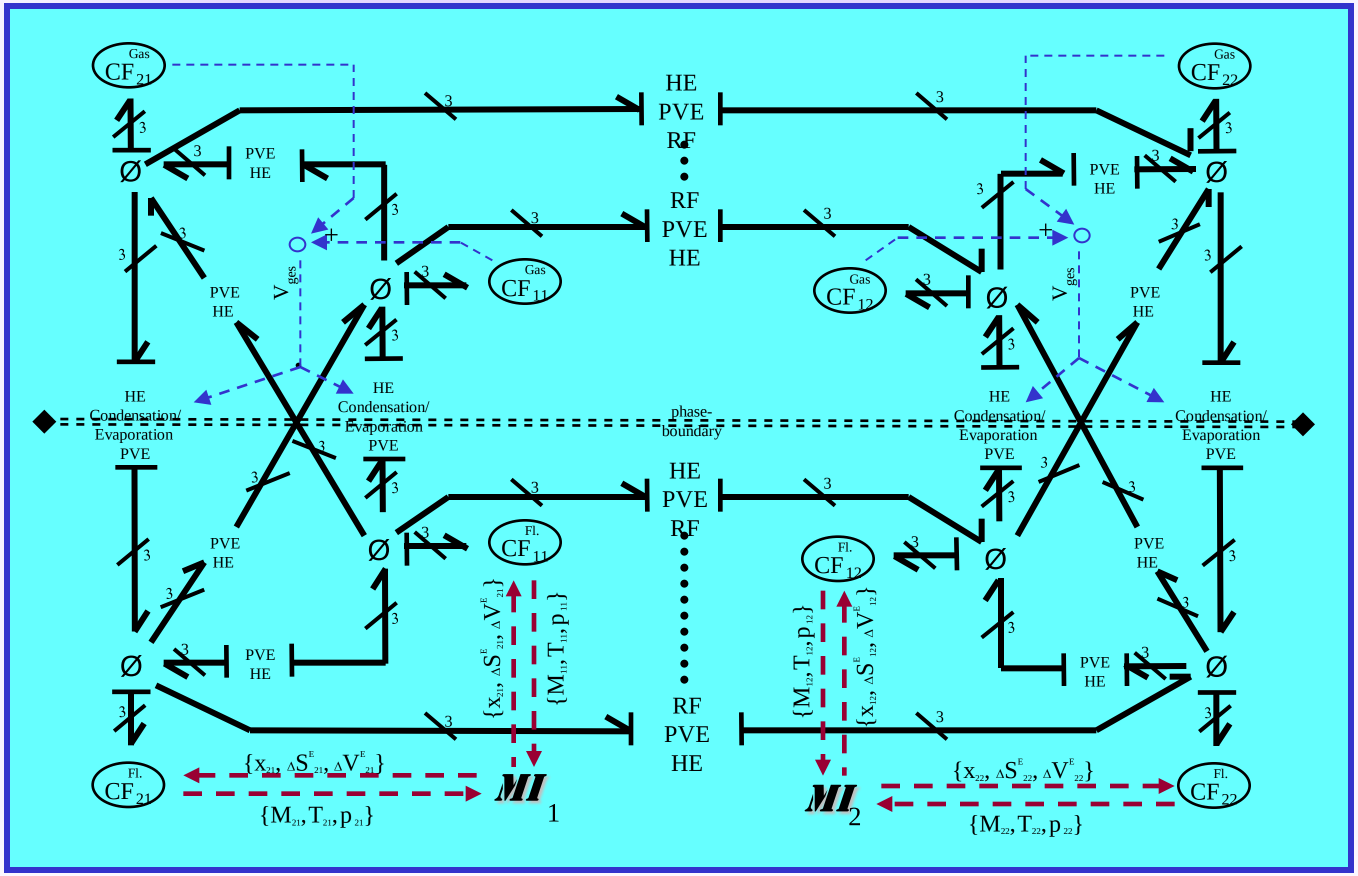


It is also possible that the fluids to be mixed are initially at different temperature or pressure values.

Convection in Multi-element Systems

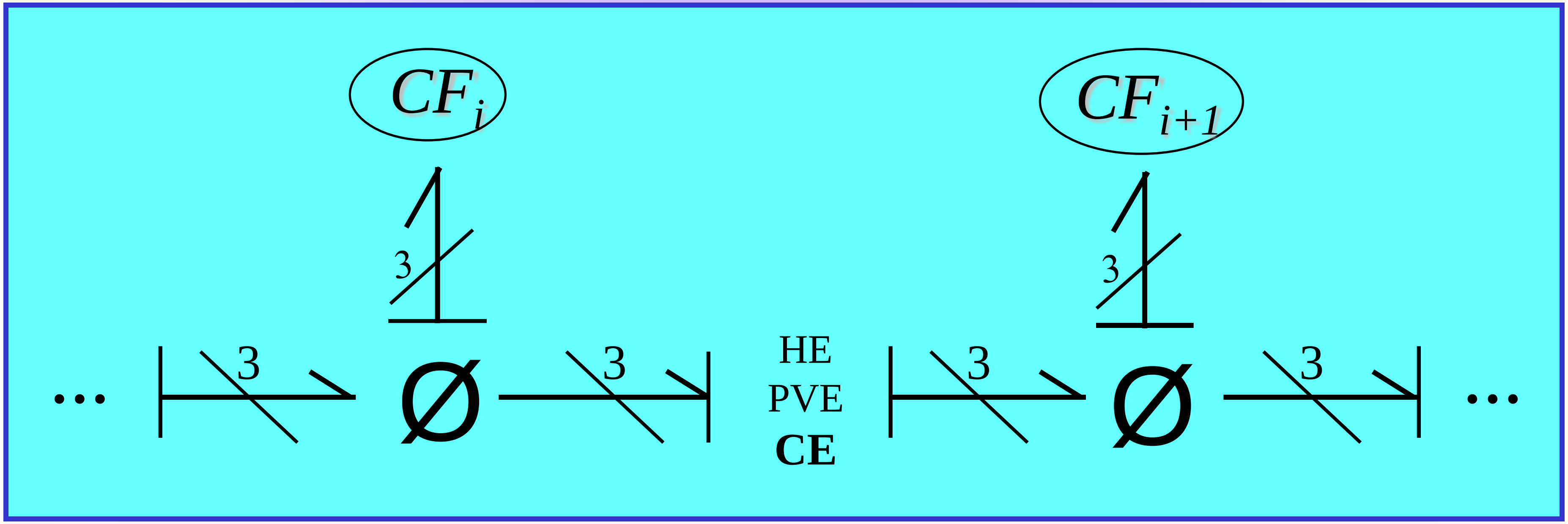


Two-element, Two-phase, Two-compartment Convective System



Concentration Exchange

- It may happen that neighboring compartments are not completely homogeneous. In that case, also the concentrations must be exchanged.



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- Greifeneder, J. and F.E. Cellier (2001), "Modeling convective flows using bond graphs," *Proc. ICBGM'01, Intl. Conference on Bond Graph Modeling and Simulation*, Phoenix, Arizona, pp. 276 – 284.
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