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Mathematical Modeling of Physical Systems

Economic Modeling

- In this lecture, we shall deal with an application of *Fuzzy Inductive Reasoning (FIR)*: making economic predictions.
- The presentation demonstrates how *FIR* can be used to improve the *System Dynamics (SD)* approach to soft-science modeling.
- It shows furthermore how *hierarchical modeling* can be used in the context of *FIR*, and demonstrates that by means of hierarchical modeling, the quality of economic predictions can dramatically be improved.

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Using FIR for Identifying Laundry Lists I

- One of the most daring (and dubious!) assumptions made by *Forrester* in his *system dynamics* approach to modeling soft-science systems was that a function of multiple variables can be written as a product of functions of a single variable each:

$$\text{Birth_rate} = \text{Population} \cdot f(\text{Pollution, Food, Crowding, Material_Standard_of_Living})$$

⇓

$$\text{Birth_rate} = \text{Population} \cdot f_1(\text{Pollution}) \cdot f_2(\text{Food}) \cdot f_3(\text{Crowding}) \cdot f_4(\text{Material_Standard_of_Living})$$

- This is obviously not generally true, and *Forrester* of course knew it. He made this assumption simply because he didn't know how else to proceed.

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Using FIR for Identifying Laundry Lists II

- An alternative might be to make use of *FIR* instead of table look-up functions for identifying any one of these unknown relationships among variables forming a laundry list.
- This is what we shall attempt in this lecture.
- Since *FIR* models are by themselves usually *dynamic* (since the *optimal mask* usually spans several rows), the functional relationships of each laundry list may furthermore be dynamic rather than static.

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Modeling in the Agricultural Sector I

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Modeling in the Agricultural Sector II

- In general, *specific economic variables*, such as *food consumption* patterns, depend on the *general state of the economy*.
- If the economy is doing well, Americans are more likely to consume steak, whereas otherwise, they may choose to buy hamburgers instead.
- The *general state of the economy* can in a first instance be viewed as depending primarily on two variables: availability of *jobs*, and availability of *money*.
- If people don't have savings, they can't buy much, and if they don't have jobs, they are less likely to spend money, even if they have some savings.
- The *general state of the economy* is heavily influenced by *population dynamics*.
- It takes people to produce goods, and it takes customers to buy them.

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Modeling in the Agricultural Sector III

Demographic layer
Food consumption is modeled in a hierarchical fashion.

Generic economic layer
Three distinct layers are identified. The *specific economic layer* depends on a *generic economic layer*, which in turn depends on a *demographic layer*.

Specific economic layer
Each of the *rate variables* has a local *delay block* associated with it. This delay block represents the fact that the rates are modeled using *FIR*, which allows to obtain *dynamic models of laundry lists*.

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Population Dynamics I

U.S. population below 5 years of age

Contraceptives

The Great Depression

Food Supply
Food Demand
Macro-economy
Population Dynamics

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Prediction of Growth Functions I

- One of the major difficulties with (and greatest strengths of) *FIR* modeling is its *inability to extrapolate*.
- Thus, if a variable is growing, such as the population, *FIR* has no means of predicting this directly.
- A simple trick solves this dilemma.
- *Economists* have known about this problem for a long time, since many other, mostly statistical, approaches to making predictions share *FIR*'s inability to extrapolate.
- When economists wish to make predictions about the value of a stock, *x*, they make use of the so-called *daily return* variable.

$$\text{daily return} = \frac{x(\text{end of day}) - x(\text{end of previous day})}{x(\text{end of day})}$$

- Whereas *x* may be increasing or decreasing, the *daily return* is mostly stationary.

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Prediction of Growth Functions II

$$\frac{dP}{dt} = k(t) \cdot P \Rightarrow \frac{dP}{dt} \approx \frac{P(n) - P(n-1)}{\Delta t} \Rightarrow k(n) \approx \frac{P(n) - P(n-1)}{P(n)}$$

If *P(t)* is growing exponentially, *k(t)* is constant.

$$\Rightarrow k(n+1) = \text{FIR} [k(n), P(n), k(n-1), P(n-1), \dots]$$

$$\Downarrow$$

$$P(n+1) \approx \frac{P(n)}{1.0 - k(n+1)}$$

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Population Dynamics II

Forecast period in number of years

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Macro-economy I

Forecast period in number of years

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Macro-economy II

The unemployment rate is a controlled variable. It is influenced by the lending rate. For many years, the feds tried to keep it around 6%. The variations around this value are difficult to predict accurately.

Food Supply
Food Demand
Macro-economy
Population Dynamics

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Food Demand/Supply I

Food Supply
Food Demand
Macro-economy
Population Dynamics

Average error incurred when making use of own past in predictions only

Average error found when making use of population dynamics and economy forecasts as well

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Discussion I

- The models have shown that making use of predictions already made for the more generic layers of the architecture helps in improving the predictions of variables associated with the more specific layers.
- In most cases, the prediction errors are reduced by approximately a factor of three in this fashion.
- Notice that the best prediction techniques available were used in all cases. In particular, the confidence measure has been heavily exploited by always making several predictions in parallel, preserving in every step the one with the highest confidence value.

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Refined Model

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Age Groups

Food Supply
Food Demand
Macro-economy
Population Dynamics

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Macro-economy III

Food Supply
Food Demand
Macro-economy
Population Dynamics

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Food Demand/Supply II

Food Supply
Food Demand
Macro-economy
Population Dynamics

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Population Dynamics III

Food Supply
Food Demand
Macro-economy
Population Dynamics

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Population Dynamics IV

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Macro-economy IV

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Food Demand

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Food Supply

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Discussion II

- Making use of the more generic layers of the multi-layer architecture in making predictions has consistently helped in reducing the average prediction error.
- The same architecture can be applied to any segment of the economy, i.e., if the application changes, only the application layer needs to be re-identified. The more generic layers of the architecture are invariant to the application at hand.

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Conclusions

- **Fuzzy Inductive Reasoning** offers an exciting alternative to **neural networks** for modeling systems from observations of behavior.
- **Fuzzy Inductive Reasoning** is highly robust when used correctly.
- **Fuzzy Inductive Reasoning** features a **model synthesis** capability rather than a **model learning** approach. It is therefore quite fast in setting up the model.
- **Fuzzy Inductive Reasoning** offers a **self-assessment** feature, which is easily the most important characteristic of the methodology.
- **Fuzzy Inductive Reasoning** is a practical tool with many industrial applications. Contrary to most other qualitative modeling techniques, FIR does not suffer in major ways from scale-up problems.

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