Algebraic Loops and Structural Singularities

- The sorting algorithm, as it was demonstrated so far, does not always work correctly. All of the examples shown to this point had been chosen carefully to hide these problems.
- The aim of the lecture is to generalize the algorithms to systems containing ***algebraic loops*** and/or ***singular structures***.

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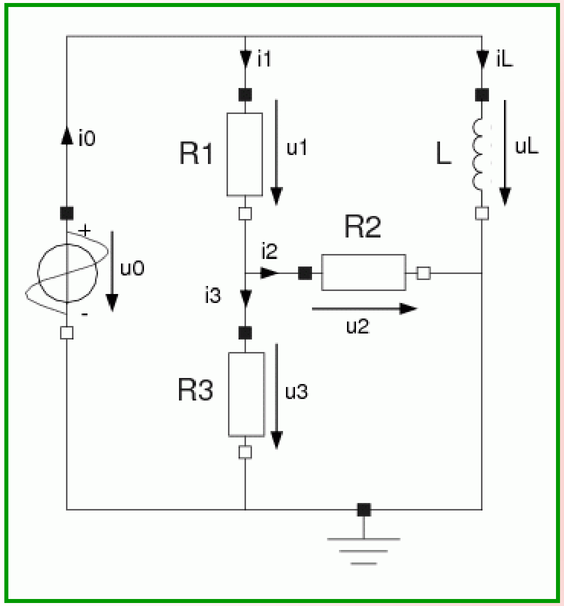
Algebraic Loops: An Example



The circuit contains 5 components

⇒ We require 10 equations in 10 unknowns

Component equations:

$$U_0 = f(t) \qquad u_3 = R_3 \cdot i_3$$

$$u_1 = R_1 \cdot i_1 \qquad u_L = L \cdot di_L/dt$$

$$u_2 = R_2 \cdot i_2$$

Node equations:

$$i_0 = i_1 + i_L \qquad i_1 = i_2 + i_3$$

Mesh equations:

$$U_0 = u_1 + u_3 \qquad u_L = u_1 + u_2$$

$$u_3 = u_2$$

Horizontal Sorting I

1.

$U_0 = f(t)$	$i_0 = i_1 + i_L$
$u_1 = R_1 \cdot i_1$	$i_1 = i_2 + i_3$
$u_2 = R_2 \cdot i_2$	$U_0 = u_1 + u_3$
$u_3 = R_3 \cdot i_3$	$u_3 = u_2$
$u_L = L \cdot di_L/dt$	$u_L = u_1 + u_2$

2.

$U_0 = f(t)$	$i_0 = i_1 + i_L$
$u_1 = R_1 \cdot i_1$	$i_1 = i_2 + i_3$
$u_2 = R_2 \cdot i_2$	$U_0 = u_1 + u_3$
$u_3 = R_3 \cdot i_3$	$u_3 = u_2$
$u_L = L \cdot di_L/dt$	$u_L = u_1 + u_2$

3.

$U_0 = f(t)$	$i_0 = i_1 + i_L$
$u_1 = R_1 \cdot i_1$	$i_1 = i_2 + i_3$
$u_2 = R_2 \cdot i_2$	$U_0 = u_1 + u_3$
$u_3 = R_3 \cdot i_3$	$u_3 = u_2$
$u_L = L \cdot di_L/dt$	$u_L = u_1 + u_2$

4.

$U_0 = f(t)$	$i_0 = i_1 + i_L$
$u_1 = R_1 \cdot i_1$	$i_1 = i_2 + i_3$
$u_2 = R_2 \cdot i_2$	$U_0 = u_1 + u_3$
$u_3 = R_3 \cdot i_3$	$u_3 = u_2$
$u_L = L \cdot di_L/dt$	$u_L = u_1 + u_2$

Horizontal Sorting II

$U_0 = f(t)$	$i_0 = i_1 + i_L$
$u_1 = R_1 \cdot i_1$	$i_1 = i_2 + i_3$
$u_2 = R_2 \cdot i_2$	$U_0 = u_1 + u_3$
$u_3 = R_3 \cdot i_3$	$u_3 = u_2$
$u_L = L \cdot di_L/dt$	$u_L = u_1 + u_2$

Of the six equations that are still a-causal (i.e., the equations not containing a red variable), every one contains at least two unknowns. Furthermore, every one of the unknowns shows up in at least two of the a-causal equations.

$\Rightarrow$ **Such a situation indicates the existence of algebraic loops.**

Algebraic Loops I

1. $u_1 = R_1 \cdot i_1$	4. $i_1 = i_2 + i_3$
2. $u_2 = R_2 \cdot i_2$	5. $U_0 = u_1 + u_3$
3. $u_3 = R_3 \cdot i_3$	6. $u_3 = u_2$

We choose one unknown from one equation, e.g. variable i_1 from equation *4.* We assume this variable to be known and continue as before.

1.

1. $u_1 = R_1 \cdot i_1$	4. $i_1 = i_2 + i_3$
2. $u_2 = R_2 \cdot i_2$	5. $U_0 = u_1 + u_3$
3. $u_3 = R_3 \cdot i_3$	6. $u_3 = u_2$

2.

1. $u_1 = R_1 \cdot i_1$	4. $i_1 = i_2 + i_3$
2. $u_2 = R_2 \cdot i_2$	5. $U_0 = u_1 + u_3$
3. $u_3 = R_3 \cdot i_3$	6. $u_3 = u_2$

3.

1. $u_1 = R_1 \cdot i_1$	4. $i_1 = i_2 + i_3$
2. $u_2 = R_2 \cdot i_2$	5. $U_0 = u_1 + u_3$
3. $u_3 = R_3 \cdot i_3$	6. $u_3 = u_2$

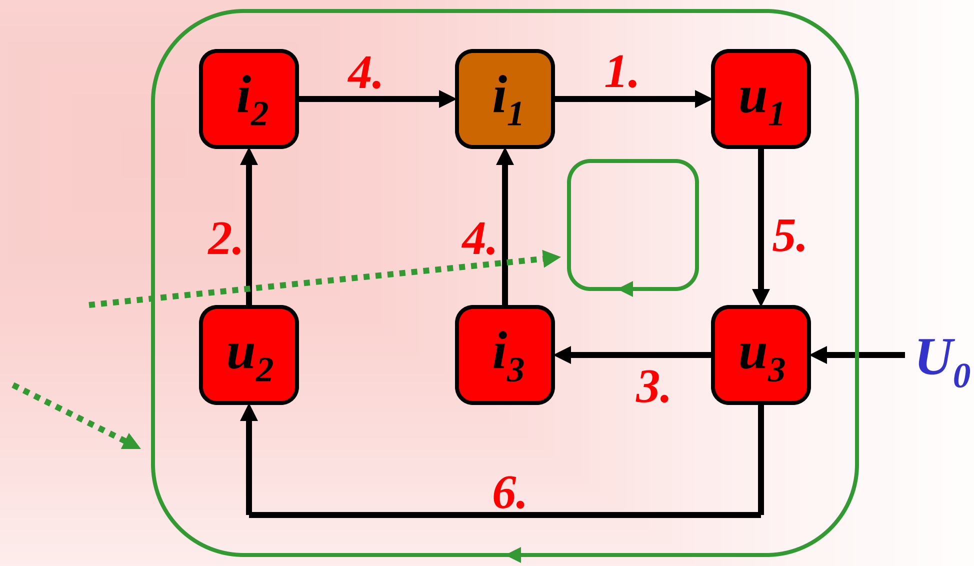
4.

1. $u_1 = R_1 \cdot i_1$	4. $i_1 = i_2 + i_3$
2. $u_2 = R_2 \cdot i_2$	5. $U_0 = u_1 + u_3$
3. $u_3 = R_3 \cdot i_3$	6. $u_3 = u_2$

Algebraic Loops II

1. $u_1 = R_1 \cdot i_1$
2. $u_2 = R_2 \cdot i_2$
3. $u_3 = R_3 \cdot i_3$
4. $i_1 = i_2 + i_3$
5. $U_0 = u_1 + u_3$
6. $u_3 = u_2$

Structure diagram



Algebraic Loops

Solution of Algebraic Loops I

1. $u_1 = R_1 \cdot i_1$	4. $i_1 = i_2 + i_3$
2. $u_2 = R_2 \cdot i_2$	5. $U_0 = u_1 + u_3$
3. $u_3 = R_3 \cdot i_3$	6. $u_3 = u_2$

$\Rightarrow$

1. $u_1 = R_1 \cdot i_1$	4. $i_1 = i_2 + i_3$
2. $i_2 = u_2 / R_2$	5. $u_3 = U_0 - u_1$
3. $i_3 = u_3 / R_3$	6. $u_2 = u_3$

$$
\begin{aligned}
i_1 &= i_2 + i_3 \\
&= u_2 / R_2 + u_3 / R_3 \\
&= u_3 / R_2 + u_3 / R_3 \\
&= ((R_2 + R_3) / (R_2 \cdot R_3)) \cdot u_3 \\
&= ((R_2 + R_3) / (R_2 \cdot R_3)) \cdot (U_0 - u_1) \\
&= ((R_2 + R_3) / (R_2 \cdot R_3)) \cdot (U_0 - R_1 \cdot i_1)
\end{aligned}
$$

$\Rightarrow$

$$
i_1 = \frac{R_2 + R_3}{R_1 R_2 + R_1 R_3 + R_2 R_3} \cdot U_0
$$

Equation 4. is replaced by the new equation.

Solution of Algebraic Loops II

$U_0 = f(t)$	$i_0 = i_1 + i_L$
$u_1 = R_1 \cdot i_1$	$i_1 = \dfrac{R_2 + R_3}{R_1 R_2 + R_1 R_3 + R_2 R_3} \cdot U_0$
$u_2 = R_2 \cdot i_2$	$U_0 = u_1 + u_3$
$u_3 = R_3 \cdot i_3$	$u_3 = u_2$
$u_L = L \cdot di_L/dt$	$u_L = u_1 + u_2$

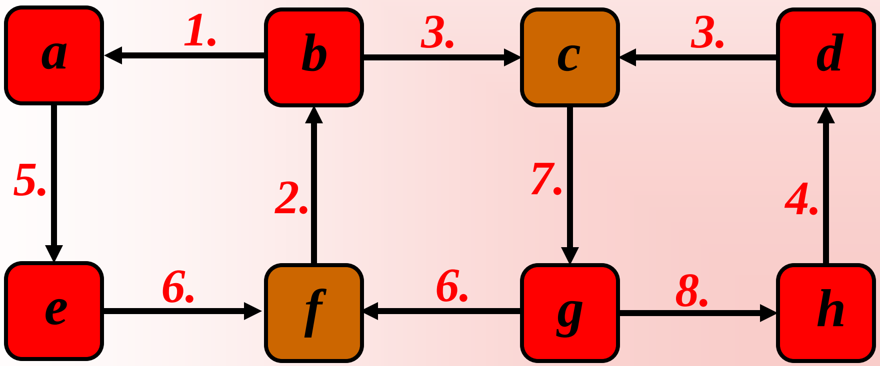
$\Rightarrow$ The algebraic loop has now been solved, and we can continue with the sorting algorithm as before.

Horizontal Sorting III

$U_0 = f(t)$	$i_0 = i_1 + i_L$
$u_1 = R_1 \cdot i_1$	$i_1 = \dfrac{R_2 + R_3}{R_1 R_2 + R_1 R_3 + R_2 R_3} \cdot U_0$
$u_2 = R_2 \cdot i_2$	$U_0 = u_1 + u_3$
$u_3 = R_3 \cdot i_3$	$u_3 = u_2$
$u_L = L \cdot di_L/dt$	$u_L = u_1 + u_2$

$U_0 = f(t)$	$i_0 = i_1 + i_L$
$u_1 = R_1 \cdot i_1$	$i_1 = \dfrac{R_2 + R_3}{R_1 R_2 + R_1 R_3 + R_2 R_3} \cdot U_0$
$u_2 = R_2 \cdot i_2$	$U_0 = u_1 + u_3$
$u_3 = R_3 \cdot i_3$	$u_3 = u_2$
$u_L = L \cdot di_L/dt$	$u_L = u_1 + u_2$

Multiple Coupled Algebraic Loops



$$c = b + d = 3 \cdot f + h = 3 \cdot f + g = 3 \cdot f + 2 \cdot c$$

$$f = e + g = a + 2 \cdot c = b + 2 \cdot c + 1 = 3 \cdot f + 2 \cdot c + 1$$

1. $a = b + 1$
2. $b = 3 \cdot f$
3. $c = b + d$
4. $d = h$
5. $e = a$
6. $f = e + g$
7. $g = 2 \cdot c$
8. $h = g$

$\Rightarrow$

1. $a = b + 1$
2. $b = 3 \cdot f$
3. $c = b + d$
4. $d = h$
5. $e = a$
6. $f = e + g$
7. $g = 2 \cdot c$
8. $h = g$

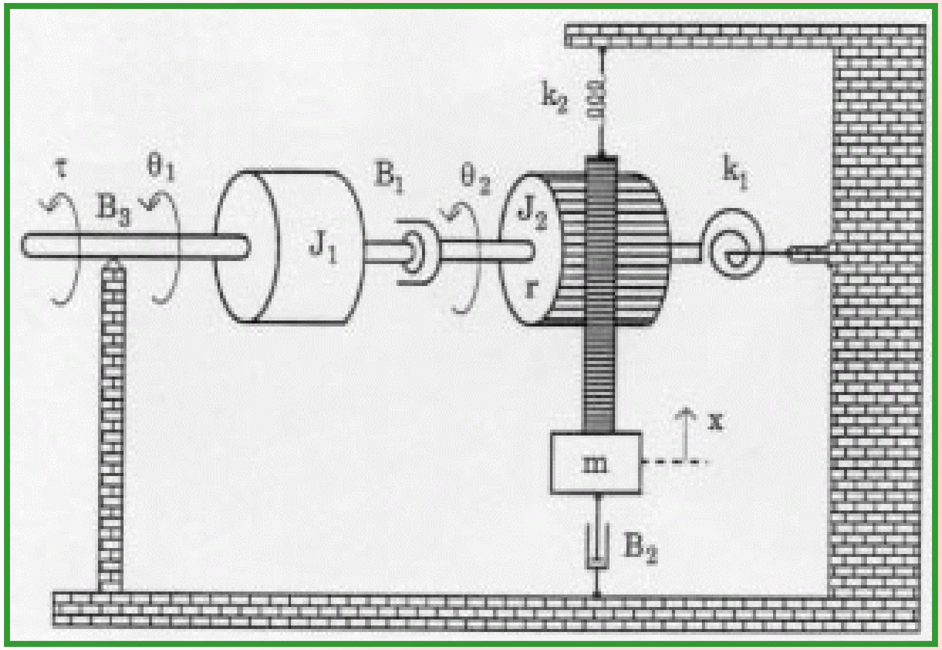
$\Rightarrow$

1. $a = b + 1$
2. $b = 3 \cdot f$
3. $c = b + d$
4. $d = h$
5. $e = a$
6. $f = e + g$
7. $g = 2 \cdot c$
8. $h = g$

$$c + 3 \cdot f = 0$$
$$2 \cdot c + 2 \cdot f = -1$$

$$c = -\,0.75$$
$$f = +\,0.25$$

Structural Singularities: An Example

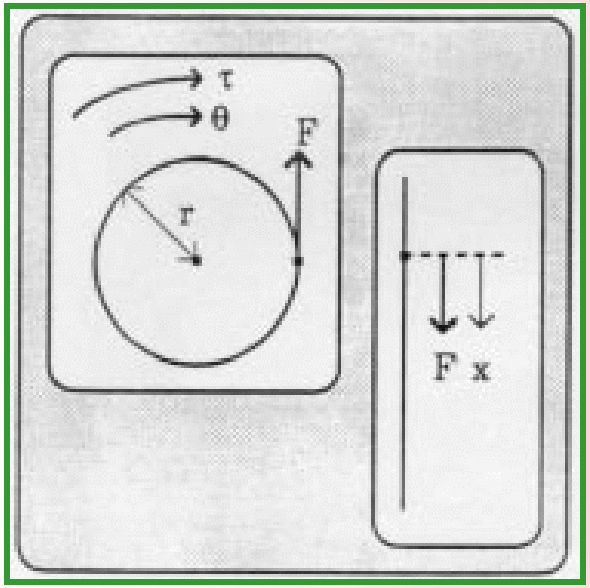


The mixed rotational and translational system exhibits three bodies: the inertiae J_1 and J_2 as well as the mass m. Therefore, we would expect the system to be of 6^{th} *order*.

3 bodies ⇒ 6 differential equations + 3 algebraic equations (D'Alembert)
3 frictions ⇒ 3 algebraic equations (friction forces)
2 springs ⇒ 2 algebraic equations (spring forces)
1 gear ⇒ 2 algebraic equations (transmission)

⇒ *16 equations*
16 unknowns

Modeling of the Gear

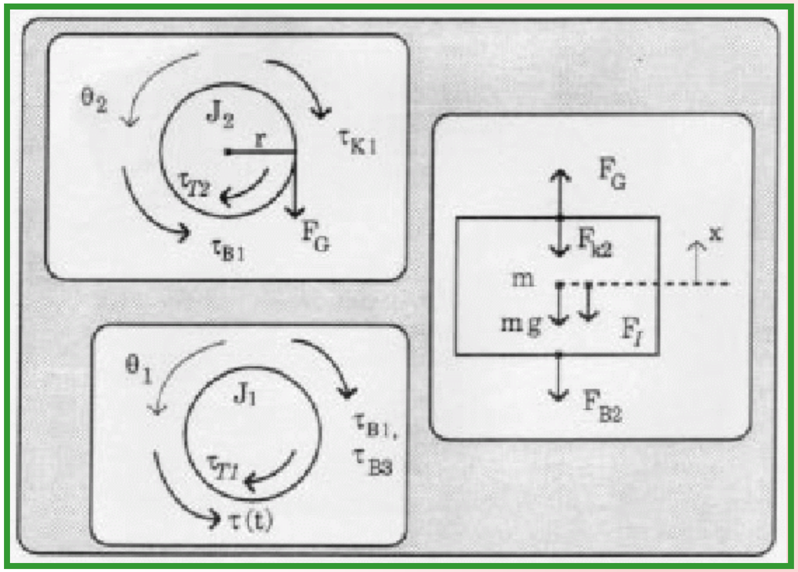


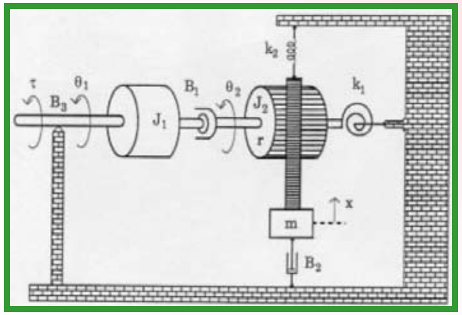
We cut through the gear. The effect of the other body is replaced by a *cutting force*.

$\Rightarrow$ **The torque τ is proportional to the cutting force F, and the displacement x is proportional to the angle θ.**

$$\tau = r \cdot F$$
$$x = r \cdot \theta$$

Cutting the System





⇒ *16 equations*
16 unknowns

$$\tau(t) = \tau_{T1} + \tau_{B1} + \tau_{B3}$$

$$\tau_{B1} = \tau_{T2} + \tau_{k1} + \tau_{G}$$

$$F_G = F_I + F_{k2} + F_{B2} + m \cdot g$$

$$\tau_{T1} = J_1 \cdot \frac{d\omega_1}{dt}$$

$$\frac{d\theta_1}{dt} = \omega_1$$

$$\tau_{T2} = J_2 \cdot \frac{d\omega_2}{dt}$$

$$\frac{d\theta_2}{dt} = \omega_2$$

$$F_I = m \cdot \frac{dv}{dt}$$

$$\frac{dx}{dt} = v$$

$$\tau_G = r \cdot F_G$$

$$x = r \cdot \theta_2$$

$$\tau_{B1} = B_1 \cdot (\omega_1 - \omega_2)$$

$$\tau_{B3} = B_3 \cdot \omega_1$$

$$F_{B2} = B_2 \cdot v$$

$$\tau_{k1} = k_1 \cdot \theta_2$$

$$F_{k2} = k_2 \cdot x$$

Horizontal Sorting I

$$\tau(t) = \tau_{T1} + \tau_{B1} + \tau_{B3}$$
$$\tau_{B1} = \tau_{T2} + \tau_{k1} + \tau_G$$
$$F_G = F_I + F_{k2} + F_{B2} + m \cdot g$$

$$\tau_{T1} = J_1 \cdot \frac{d\omega_1}{dt}$$
$$\frac{d\theta_1}{dt} = \omega_1$$
$$\tau_{T2} = J_2 \cdot \frac{d\omega_2}{dt}$$
$$\frac{d\theta_2}{dt} = \omega_2$$
$$F_I = m \cdot \frac{dv}{dt}$$
$$\frac{dx}{dt} = v$$

$$\tau_G = r \cdot F_G$$
$$x = r \cdot \theta_2$$

$$\tau_{B1} = B_1 \cdot (\omega_1 - \omega_2)$$
$$\tau_{B3} = B_3 \cdot \omega_1$$
$$F_{B2} = B_2 \cdot v$$
$$\tau_{k1} = k_1 \cdot \theta_2$$
$$F_{k2} = k_2 \cdot x$$

This equation cannot be used since it contains no unknown.

Idea: *If an equation holds true for all times, then every derivative of that equation holds true as well.*

$\Rightarrow$ *Replace the unusable equation by its time derivative.*

Differentiation I

$$\tau(t) = \tau_{T1} + \tau_{B1} + \tau_{B3}$$
$$\tau_{B1} = \tau_{T2} + \tau_{k1} + \tau_G$$
$$F_G = F_I + F_{k2} + F_{B2} + m \cdot g$$

$$\tau_{T1} = J_1 \cdot \frac{d\omega_1}{dt}$$
$$\frac{d\theta_1}{dt} = \omega_1$$
$$\tau_{T2} = J_2 \cdot \frac{d\omega_2}{dt}$$
$$\frac{d\theta_2}{dt} = \omega_2$$
$$F_I = m \cdot \frac{dv}{dt}$$
$$\frac{dx}{dt} = v$$

$$\tau_G = r \cdot F_G$$
$$v = r \cdot \omega_2$$

$$\tau_{B1} = B_1 \cdot (\omega_1 - \omega_2)$$
$$\tau_{B3} = B_3 \cdot \omega_1$$
$$F_{B2} = B_2 \cdot v$$
$$\tau_{k1} = k_1 \cdot \theta_2$$
$$F_{k2} = k_2 \cdot x$$

Unfortunately, the equation is still not usable, because it still does not contain any unknown.

$\Rightarrow$ *Differentiate the unusable equation once more.*

Differentiation II

$$\tau(t) = \tau_{T1} + \tau_{B1} + \tau_{B3}$$
$$\tau_{B1} = \tau_{T2} + \tau_{k1} + \tau_G$$
$$F_G = F_I + F_{k2} + F_{B2} + m \cdot g$$

$$\tau_{T1} = J_1 \cdot \frac{d\omega_1}{dt}$$
$$\frac{d\theta_1}{dt} = \omega_1$$
$$\tau_{T2} = J_2 \cdot \frac{d\omega_2}{dt}$$
$$\frac{d\theta_2}{dt} = \omega_2$$
$$F_I = m \cdot \frac{dv}{dt}$$
$$\frac{dx}{dt} = v$$

$$\tau_G = r \cdot F_G$$
$$\frac{dv}{dt} = r \cdot \frac{d\omega_2}{dt}$$

$$\tau_{B1} = B_1 \cdot (\omega_1 - \omega_2)$$
$$\tau_{B3} = B_3 \cdot \omega_1$$
$$F_{B2} = B_2 \cdot v$$
$$\tau_{k1} = k_1 \cdot \theta_2$$
$$F_{k2} = k_2 \cdot x$$

The equation has now become usable, since both of the variables contained in it are unknowns. The two derivatives had been red until now, because they appeared only once in the equation system. As they now appear twice, they need to be reset to black.

Horizontal Sorting II

$$\tau(t) = \tau_{T1} + \tau_{B1} + \tau_{B3}$$
$$\tau_{B1} = \tau_{T2} + \tau_{k1} + \tau_G$$
$$F_G = F_I + F_{k2} + F_{B2} + m \cdot g$$

$$\tau_{T1} = J_1 \cdot \frac{d\omega_1}{dt}$$
$$\frac{d\theta_1}{dt} = \omega_1$$
$$\tau_{T2} = J_2 \cdot \frac{d\omega_2}{dt}$$
$$\frac{d\theta_2}{dt} = \omega_2$$
$$F_I = m \cdot \frac{dv}{dt}$$
$$\frac{dx}{dt} = v$$

$$\tau_G = r \cdot F_G$$
$$\frac{dv}{dt} = r \cdot \frac{d\omega_2}{dt}$$

$$\tau_{B1} = B_1 \cdot (\omega_1 - \omega_2)$$
$$\tau_{B3} = B_3 \cdot \omega_1$$
$$F_{B2} = B_2 \cdot v$$
$$\tau_{k1} = k_1 \cdot \theta_2$$
$$F_{k2} = k_2 \cdot x$$

$$\Rightarrow$$

$$\tau(t) = \tau_{T1} + \tau_{B1} + \tau_{B3}$$
$$\tau_{B1} = \tau_{T2} + \tau_{k1} + \tau_G$$
$$F_G = F_I + F_{k2} + F_{B2} + m \cdot g$$

$$\tau_{T1} = J_1 \cdot \frac{d\omega_1}{dt}$$
$$\frac{d\theta_1}{dt} = \omega_1$$
$$\tau_{T2} = J_2 \cdot \frac{d\omega_2}{dt}$$
$$\frac{d\theta_2}{dt} = \omega_2$$
$$F_I = m \cdot \frac{dv}{dt}$$
$$\frac{dx}{dt} = v$$

$$\tau_G = r \cdot F_G$$
$$\frac{dv}{dt} = r \cdot \frac{d\omega_2}{dt}$$

$$\tau_{B1} = B_1 \cdot (\omega_1 - \omega_2)$$
$$\tau_{B3} = B_3 \cdot \omega_1$$
$$F_{B2} = B_2 \cdot v$$
$$\tau_{k1} = k_1 \cdot \theta_2$$
$$F_{k2} = k_2 \cdot x$$

Horizontal Sorting III

$$\tau(t) = \tau_{T1} + \tau_{B1} + \tau_{B3}$$
$$\tau_{B1} = \tau_{T2} + \tau_{k1} + \tau_G$$
$$F_G = F_I + F_{k2} + F_{B2} + m \cdot g$$

$$\tau_{T1} = J_1 \cdot \frac{d\omega_1}{dt}$$
$$\frac{d\theta_1}{dt} = \omega_1$$
$$\tau_{T2} = J_2 \cdot \frac{d\omega_2}{dt}$$
$$\frac{d\theta_2}{dt} = \omega_2$$
$$F_I = m \cdot \frac{dv}{dt}$$
$$\frac{dx}{dt} = v$$

$$\tau_G = r \cdot F_G$$
$$\frac{dv}{dt} = r \cdot \frac{d\omega_2}{dt}$$

$$\tau_{B1} = B_1 \cdot (\omega_1 - \omega_2)$$
$$\tau_{B3} = B_3 \cdot \omega_1$$
$$F_{B2} = B_2 \cdot v$$
$$\tau_{k1} = k_1 \cdot \theta_2$$
$$F_{k2} = k_2 \cdot x$$

There still remain 6 equations in 6 unknowns.

- *Every one of these equations contains at least two unknowns.*
- *Every one of the unknowns appears at least in two of the remaining equations.*

$\Rightarrow$ *We are again confronted with at least one algebraic loop.*

Algebraic Loop

$$\tau(t) = \tau_{T1} + \tau_{B1} + \tau_{B3}$$
$$\tau_{B1} = \tau_{T2} + \tau_{k1} + \tau_G$$
$$F_G = F_I + F_{k2} + F_{B2} + m \cdot g$$

$$\tau_{T1} = J_1 \cdot \frac{d\omega_1}{dt}$$
$$\frac{d\theta_1}{dt} = \omega_1$$
$$\tau_{T2} = J_2 \cdot \frac{d\omega_2}{dt}$$
$$\frac{d\theta_2}{dt} = \omega_2$$
$$F_I = m \cdot \frac{dv}{dt}$$
$$\frac{dx}{dt} = v$$

Choice

$$\tau_G = r \cdot F_G$$
$$\frac{dv}{dt} = r \cdot \frac{d\omega_2}{dt}$$

$$\tau_{B1} = B_1 \cdot (\omega_1 - \omega_2)$$
$$\tau_{B3} = B_3 \cdot \omega_1$$
$$F_{B2} = B_2 \cdot v$$
$$\tau_{k1} = k_1 \cdot \theta_2$$
$$F_{k2} = k_2 \cdot x$$

$\Rightarrow$

$$\tau(t) = \tau_{T1} + \tau_{B1} + \tau_{B3}$$
$$\tau_{B1} = \tau_{T2} + \tau_{k1} + \tau_G$$
$$F_G = F_I + F_{k2} + F_{B2} + m \cdot g$$

$$\tau_{T1} = J_1 \cdot \frac{d\omega_1}{dt}$$
$$\frac{d\theta_1}{dt} = \omega_1$$
$$\tau_{T2} = J_2 \cdot \frac{d\omega_2}{dt}$$
$$\frac{d\theta_2}{dt} = \omega_2$$
$$F_I = m \cdot \frac{dv}{dt}$$
$$\frac{dx}{dt} = v$$

$$\tau_G = r \cdot F_G$$
$$\frac{dv}{dt} = r \cdot \frac{d\omega_2}{dt}$$

$$\tau_{B1} = B_1 \cdot (\omega_1 - \omega_2)$$
$$\tau_{B3} = B_3 \cdot \omega_1$$
$$F_{B2} = B_2 \cdot v$$
$$\tau_{k1} = k_1 \cdot \theta_2$$
$$F_{k2} = k_2 \cdot x$$

Horizontal Sorting IV

$$\tau(t) = \tau_{T1} + \tau_{B1} + \tau_{B3}$$
$$\tau_{B1} = \tau_{T2} + \tau_{k1} + \tau_G$$
$$F_G = F_I + F_{k2} + F_{B2} + m \cdot g$$

$$\tau_{T1} = J_1 \cdot \frac{d\omega_1}{dt}$$
$$\frac{d\theta_1}{dt} = \omega_1$$
$$\tau_{T2} = J_2 \cdot \frac{d\omega_2}{dt}$$
$$\frac{d\theta_2}{dt} = \omega_2$$
$$F_I = m \cdot \frac{dv}{dt}$$
$$\frac{dx}{dt} = v$$

$$\tau_G = r \cdot F_G$$
$$\frac{dv}{dt} = r \cdot \frac{d\omega_2}{dt}$$

$$\tau_{B1} = B_1 \cdot (\omega_1 - \omega_2)$$
$$\tau_{B3} = B_3 \cdot \omega_1$$
$$F_{B2} = B_2 \cdot v$$
$$\tau_{k1} = k_1 \cdot \theta_2$$
$$F_{k2} = k_2 \cdot x$$

$\Rightarrow$

$$\tau_{T1} = \tau(t) - \tau_{B1} - \tau_{B3}$$
$$\tau_{T2} = \tau_{B1} - \tau_{k1} - \tau_G$$
$$F_G = F_I + F_{k2} + F_{B2} + m \cdot g$$

$$\frac{d\omega_1}{dt} = \tau_{T1} / J_1$$
$$\frac{d\theta_1}{dt} = \omega_1$$
$$\frac{d\omega_2}{dt} = \tau_{T2} / J_2$$
$$\frac{d\theta_2}{dt} = \omega_2$$
$$F_I = m \cdot \frac{dv}{dt}$$
$$\frac{dx}{dt} = v$$

$$\tau_G = r \cdot F_G$$
$$\frac{dv}{dt} = r \cdot \frac{d\omega_2}{dt}$$

$$\tau_{B1} = B_1 \cdot (\omega_1 - \omega_2)$$
$$\tau_{B3} = B_3 \cdot \omega_1$$
$$F_{B2} = B_2 \cdot v$$
$$\tau_{k1} = k_1 \cdot \theta_2$$
$$F_{k2} = k_2 \cdot x$$

Solution of the Algebraic Loop I

$$\tau_{T1} = \tau(t) - \tau_{B1} - \tau_{B3}$$
$$\tau_{T2} = \tau_{B1} - \tau_{k1} - \tau_G$$
$$F_G = F_I + F_{k2} + F_{B2} + m \cdot g$$

$$\frac{d\omega_1}{dt} = \tau_{T1} / J_1$$
$$\frac{d\theta_1}{dt} = \omega_1$$
$$\frac{d\omega_2}{dt} = \tau_{T2} / J_2$$
$$\frac{d\theta_2}{dt} = \omega_2$$
$$F_I = m \cdot \frac{dv}{dt}$$
$$\frac{dx}{dt} = v$$

$$\tau_G = r \cdot F_G$$
$$\frac{dv}{dt} = r \cdot \frac{d\omega_2}{dt}$$

$$\tau_{B1} = B_1 \cdot (\omega_1 - \omega_2)$$
$$\tau_{B3} = B_3 \cdot \omega_1$$
$$F_{B2} = B_2 \cdot v$$
$$\tau_{k1} = k_1 \cdot \theta_2$$
$$F_{k2} = k_2 \cdot x$$

$$\frac{d\omega_2}{dt} = \tau_{T2} / J_2$$
$$= (\tau_{B1} - \tau_{k1} - \tau_G) / J_2$$
$$= (\tau_{B1} - \tau_{k1}) / J_2 - \tau_G / J_2$$
$$= (\tau_{B1} - \tau_{k1}) / J_2 - (r / J_2) \cdot F_G$$
$$= (\tau_{B1} - \tau_{k1}) / J_2 - (r / J_2) \cdot (F_I + F_{k2} + F_{B2} + m \cdot g)$$

$$= (\tau_{B1} - \tau_{k1}) / J_2 - (r / J_2) \cdot (F_{k2} + F_{B2} + m \cdot g) - (r / J_2) \cdot F_I$$

$$= (\tau_{B1} - \tau_{k1}) / J_2 - (r / J_2) \cdot (F_{k2} + F_{B2} + m \cdot g) - (m \cdot r / J_2) \cdot dv/dt$$

$$= (\tau_{B1} - \tau_{k1}) / J_2 - (r / J_2) \cdot (F_{k2} + F_{B2} + m \cdot g) - (m \cdot r^2 / J_2) \cdot d\omega_2 / dt$$

Solution of the Algebraic Loop II

$$\tau_{T1} = \tau(t) - \tau_{B1} - \tau_{B3}$$
$$\tau_{T2} = \tau_{B1} - \tau_{k1} - \tau_G$$
$$F_G = F_I + F_{k2} + F_{B2} + m \cdot g$$

$$\frac{d\omega_1}{dt} = \tau_{T1} / J_1$$
$$\frac{d\theta_1}{dt} = \omega_1$$
$$\frac{d\omega_2}{dt} = \frac{\tau_{B1} - \tau_{k1} - r \cdot (F_{k2} + F_{B2}) - m \cdot g \cdot r}{J_2 + m \cdot r^2}$$
$$\frac{d\theta_2}{dt} = \omega_2$$
$$F_I = m \cdot \frac{dv}{dt}$$
$$\frac{dx}{dt} = v$$

$$\tau_G = r \cdot F_G$$
$$\frac{dv}{dt} = r \cdot \frac{d\omega_2}{dt}$$

$$\tau_{B1} = B_1 \cdot (\omega_1 - \omega_2)$$
$$\tau_{B3} = B_3 \cdot \omega_1$$
$$F_{B2} = B_2 \cdot v$$
$$\tau_{k1} = k_1 \cdot \theta_2$$
$$F_{k2} = k_2 \cdot x$$

Comments

- The problem of the ***structural singularity*** occurred, because the mass m and the inertia J_2 cannot be moved independently of each other.
- For this reason, it had to be possible to describe the system by only ***4 differential equations***.
- The solution approach presented here does not exploit that simplification directly.
- A better approach shall be explained in due course.

References

- Cellier, F.E. and H. Elmqvist (1993), "Automated formula manipulation supports object-oriented continuous-system modeling," *IEEE Control Systems*, **13**(2), pp. 28-38.