# Bond Graphs I

- Until this point in the class, we have concerned ourselves with the symbolic manipulation of sets of differential and algebraic equations (DAE's). We have not yet considered the question, where the equations come from that describe the physics of the systems to be analyzed.
- For this reason, we had to limit our discussion to the analysis of very simple systems, such as linear electrical circuits, for which we already know from other classes, what equations are needed to describe their dynamics.
- We shall now touch on the question of ***modeling systems*** in the sense of deriving "correct" mathematical descriptions of the dynamics of systems whose underlying equations are hitherto unknown to us.

# Table of Contents

# Energy and Power

- All physical systems have in common their ***conservation laws for energy and mass***.
- Bond graphs concern themselves intimately with the ***conservation of energy*** in a physical system.
- Since energy in a closed system is being conserved, it can only be affected by any one of three mechanisms :
  - Energy can be ***stored***.
  - Energy can be ***transported*** from one place to another.
  - Energy can be ***converted*** from one form to another.

# Energy and Power II

- The amount of ***energy (E)*** present at any one place can only change, if either additional energy is flowing in, or if energy is flowing off.
- In both cases, we therefore require ***energy flows*** that can be defined as the derivatives of energy with respect to time.

$$P = dE/dt$$

- ***P*** is also referred to as ***Power***.
- The energy is measured in ***Joule [J]***, whereas the power is measured in ***Watt [W]***.

# Energy and Power III

- In all physical systems, energy flows can be written as products of two different physical variables, one of which is ***extensive*** (i.e., proportional to the amount), whereas the other is ***intensive*** (i.e., independent of the amount).
- In the case of coupled energy flows, it may be necessary to describe a single energy flow as the ***sum of products*** of such ***adjugate variables***.

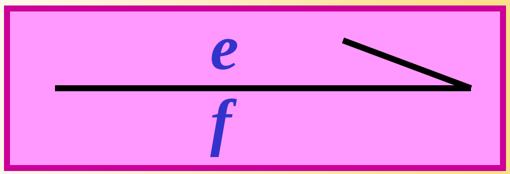
***Examples:***

$$P_{el} = u \cdot i$$
$$P_{mech} = f \cdot v$$

$$[W] = [V] \cdot [A] = [N] \cdot [m/s] = [kg \cdot m^2 \cdot s^{-3}]$$

# Energy Flow

- The modeling of physical systems by means of bond graphs operates on a graphical description of energy flows.

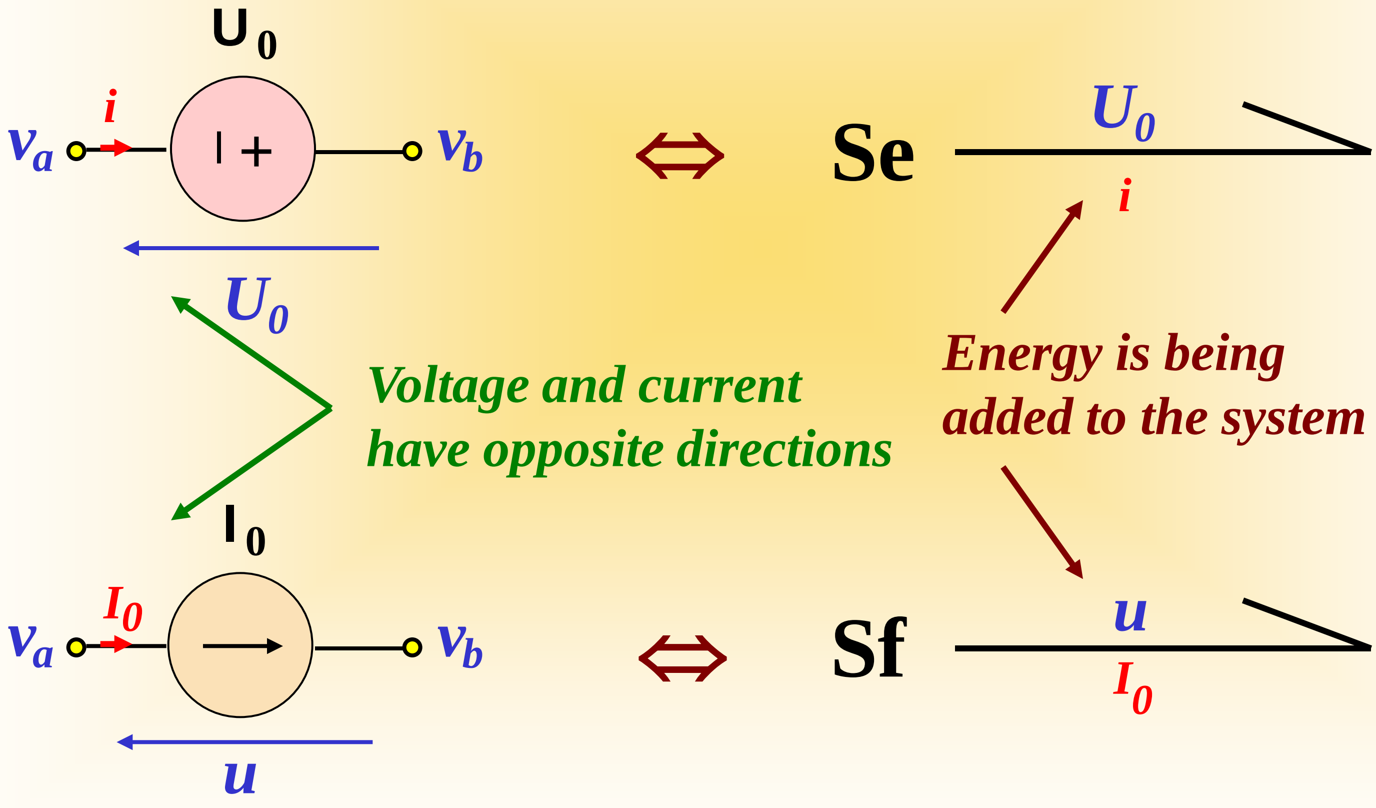



$$P = e \cdot f$$

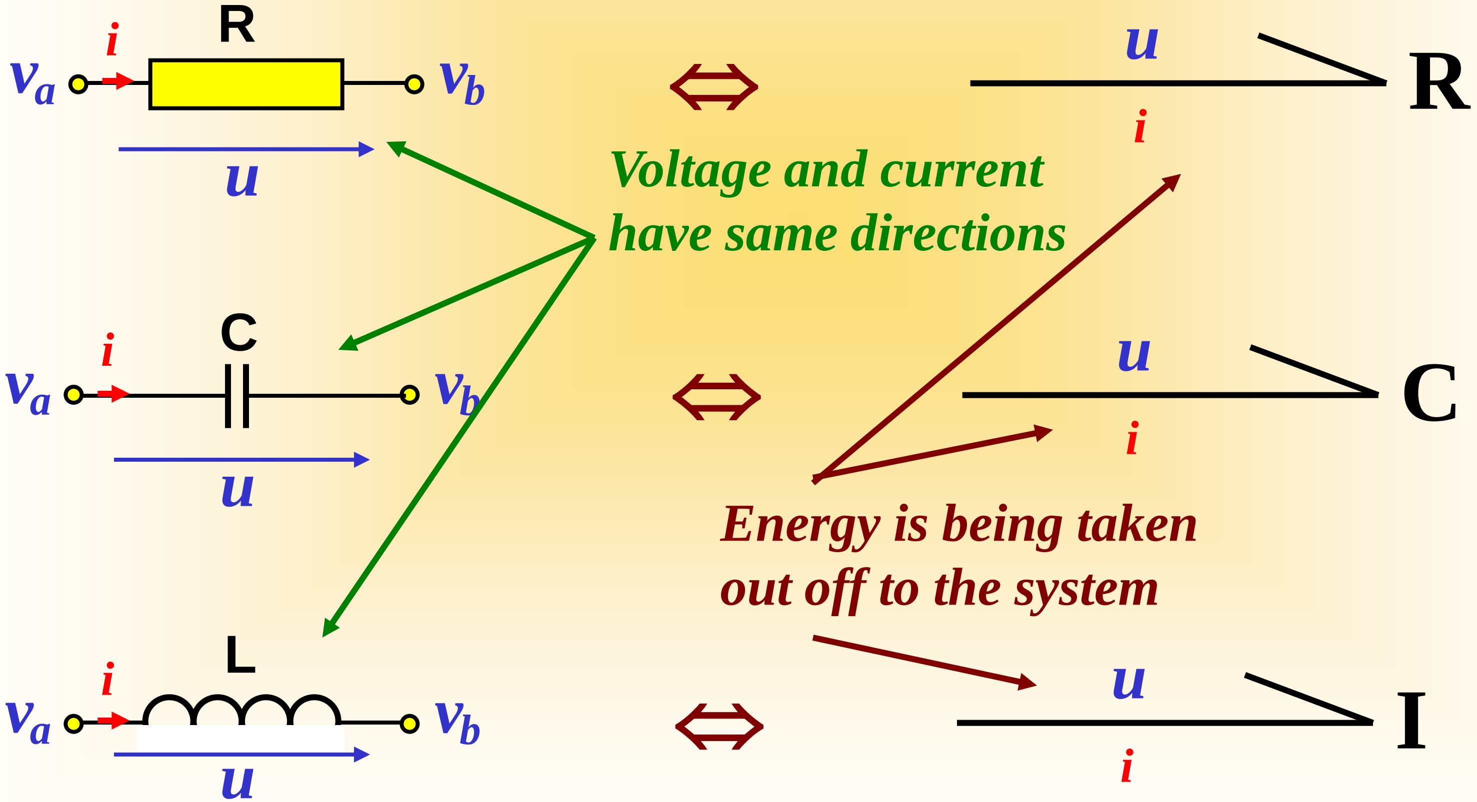
***e*: Effort**
***f*: Flow**

- The energy flows are represented as ***directed harpoons***. The two ***adjugate variables***, which are responsible for the energy flow, are annotated ***above*** ***(intensive: potential variable, "e")*** and ***below*** ***(extensive: flow variable, "f")*** the harpoon.
- The hook of the harpoon always points to the left, and the term ***"above"*** refers to the side with the hook.

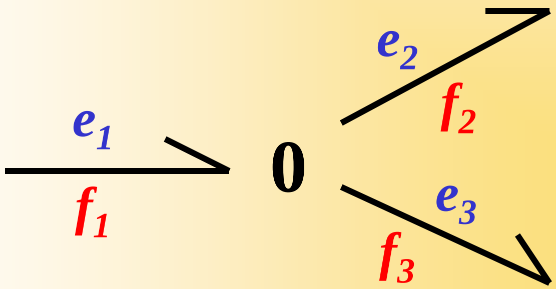
# A-causal Bond Graphs



# Passive Electrical Elements in Bond Graph Representation

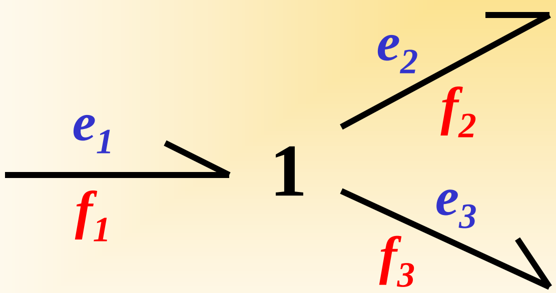


# Junctions




$\Leftrightarrow$

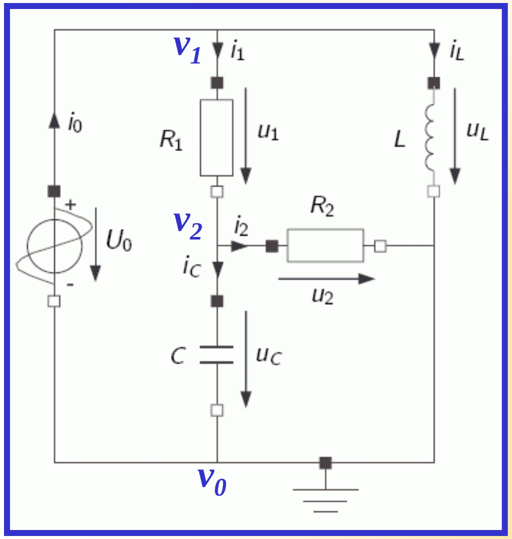
$$e_1 = e_2$$
$$e_2 = e_3$$
$$f_1 - f_2 - f_3 = 0$$

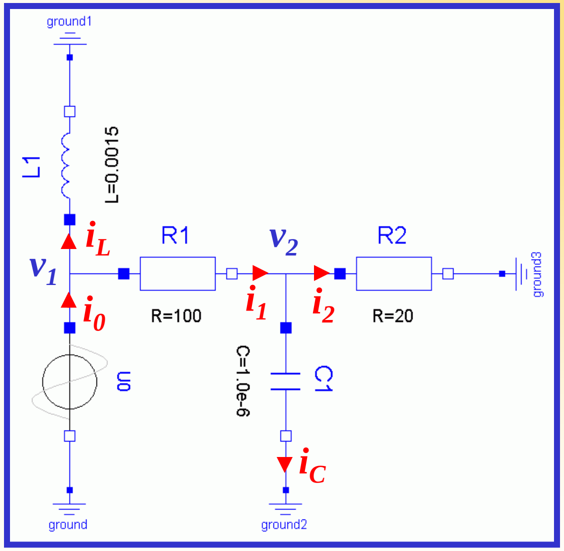


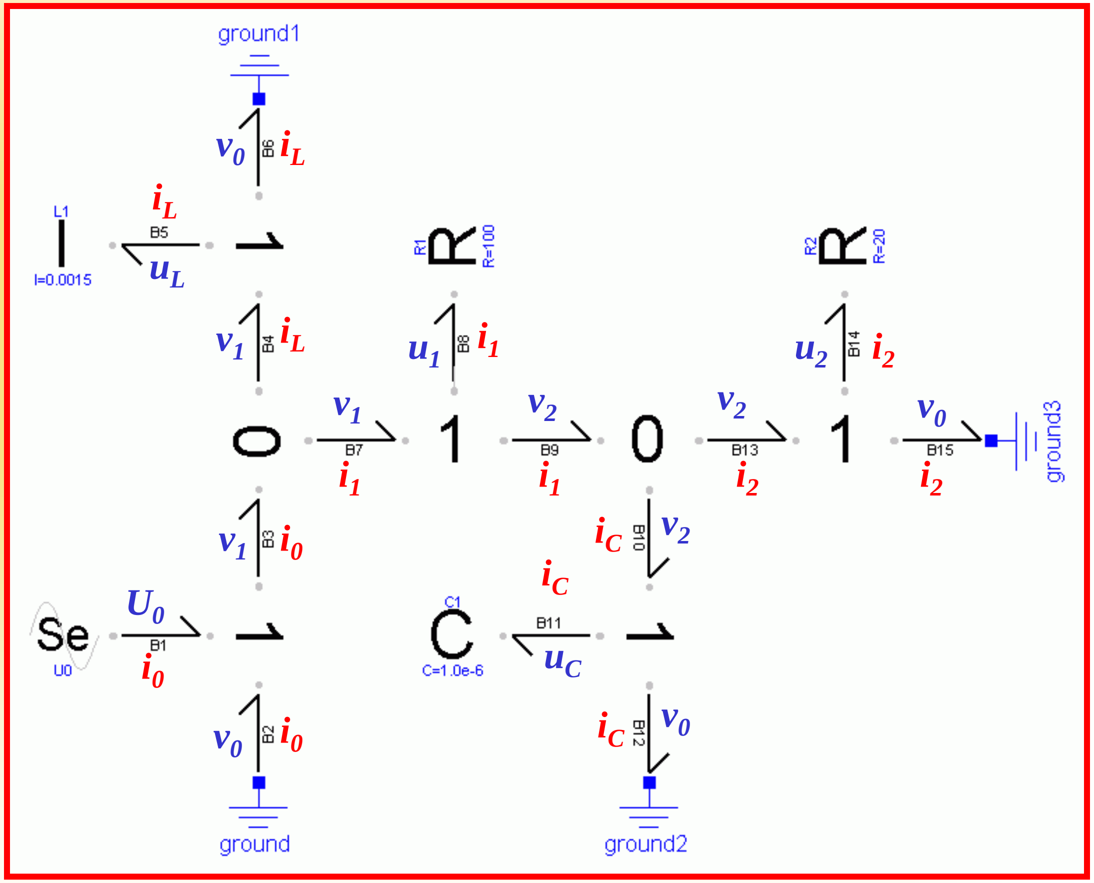

$\Leftrightarrow$

$$f_1 = f_2$$
$$f_2 = f_3$$
$$e_1 - e_2 - e_3 = 0$$

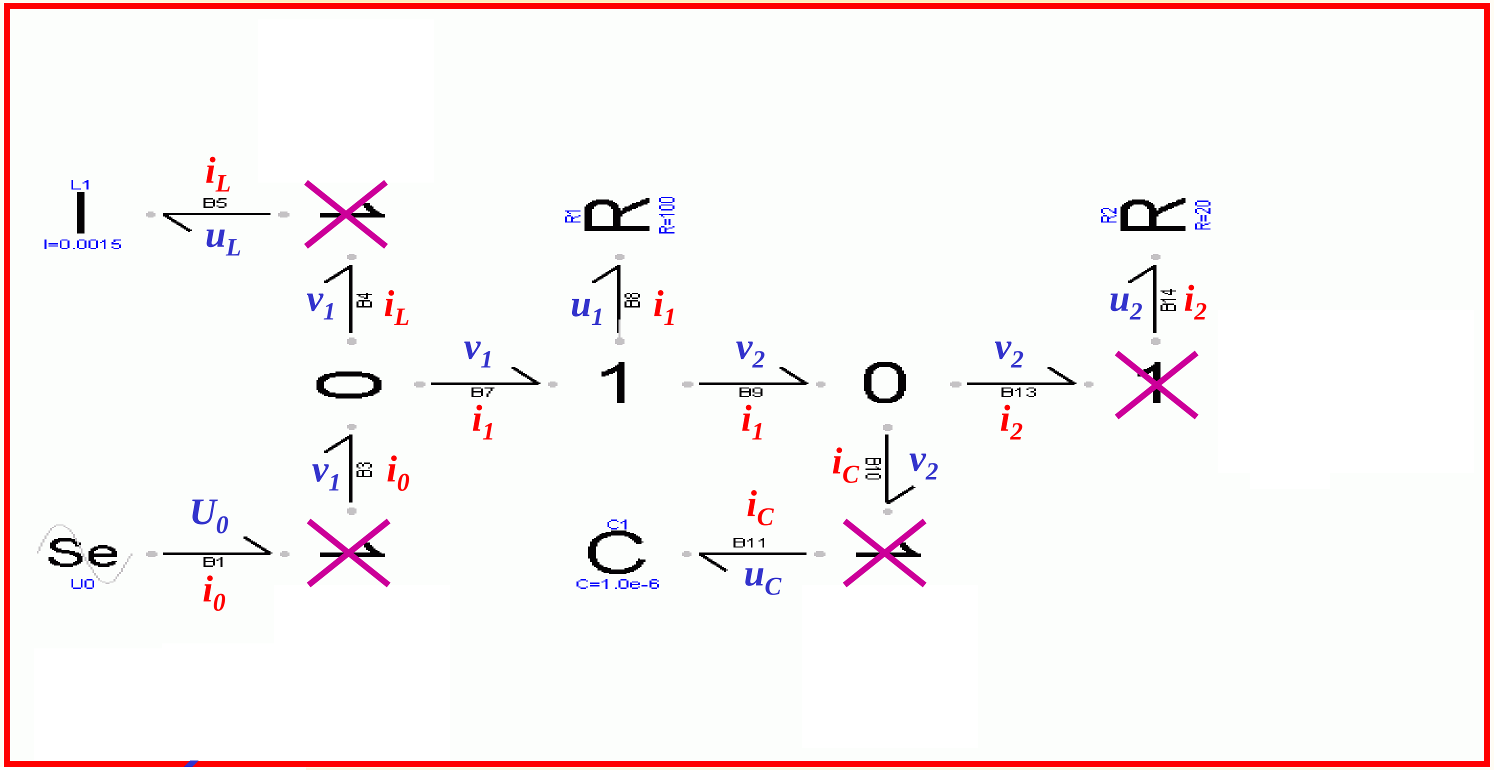
# An Example I


$v_1$
$i_1$
$i_L$
$i_0$
$R_1$
$u_1$
$L$
$u_L$
$U_0$
$v_2$
$i_2$
$R_2$
$i_C$
$u_2$
$C$
$u_C$
$v_0$

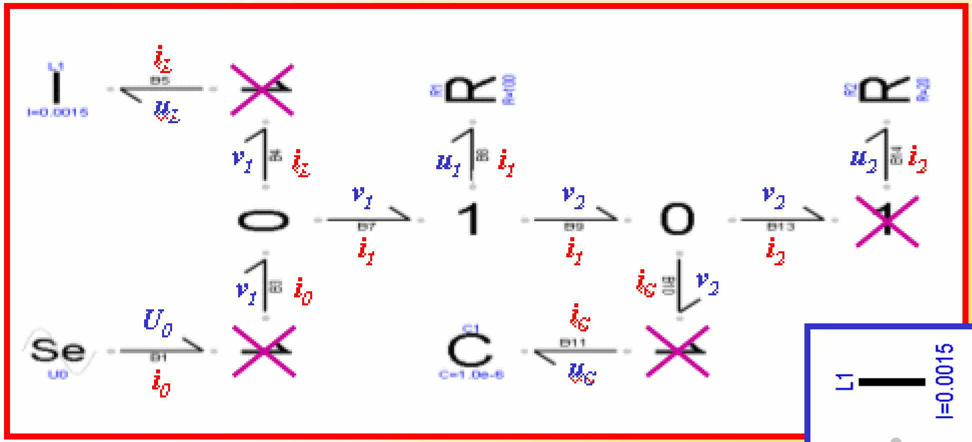

ground1
L1
L=0.0015
$i_L$
$v_1$
R1
$v_2$
R2
$i_0$
R=100
$i_1$
$i_2$
R=20
ground3
U0
C=1.0e-6
C1
$i_C$
ground
ground2

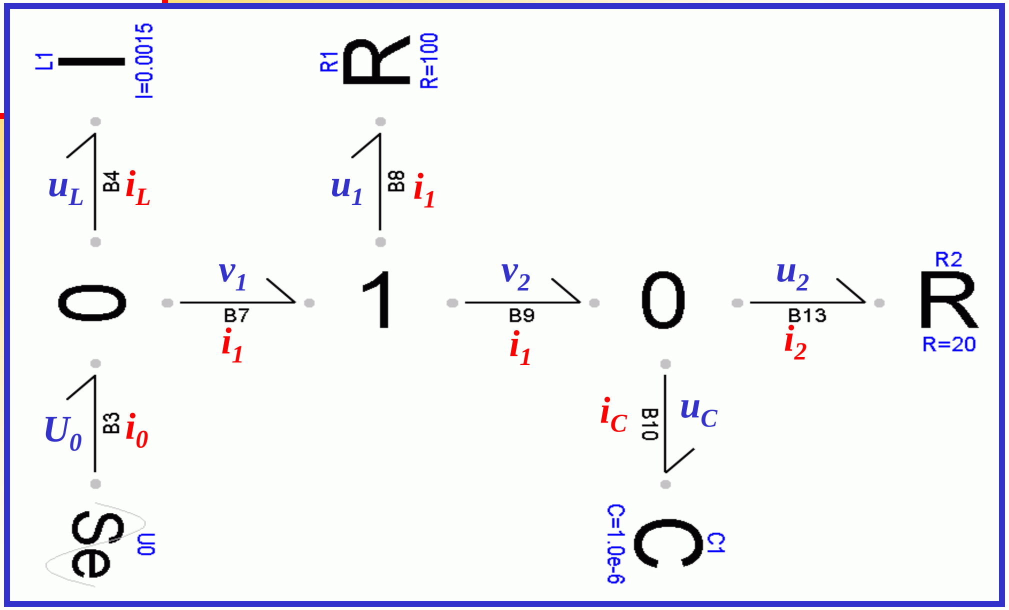

ground1
$v_0$
B6
$i_L$
L1
$i_L$
B5
$u_L$
I=0.0015
R1
R=100
R2
R=20
$v_1$
B4
$i_L$
$u_1$
B8
$i_1$
$u_2$
B14
$i_2$
$v_1$
B7
$i_1$
$v_2$
B9
$i_1$
$v_2$
B13
$i_2$
$v_0$
B15
$i_2$
ground3
$v_1$
B3
$i_0$
$i_C$
B10
$v_2$
$i_C$
C1
Se
U0
$U_0$
B1
$i_0$
B11
$u_C$
C=1.0e-6
$v_0$
B2
$i_0$
$i_C$
B12
$v_0$
ground
ground2

# An Example II



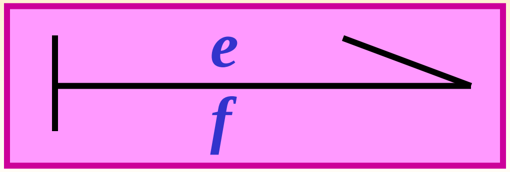
# An Example III



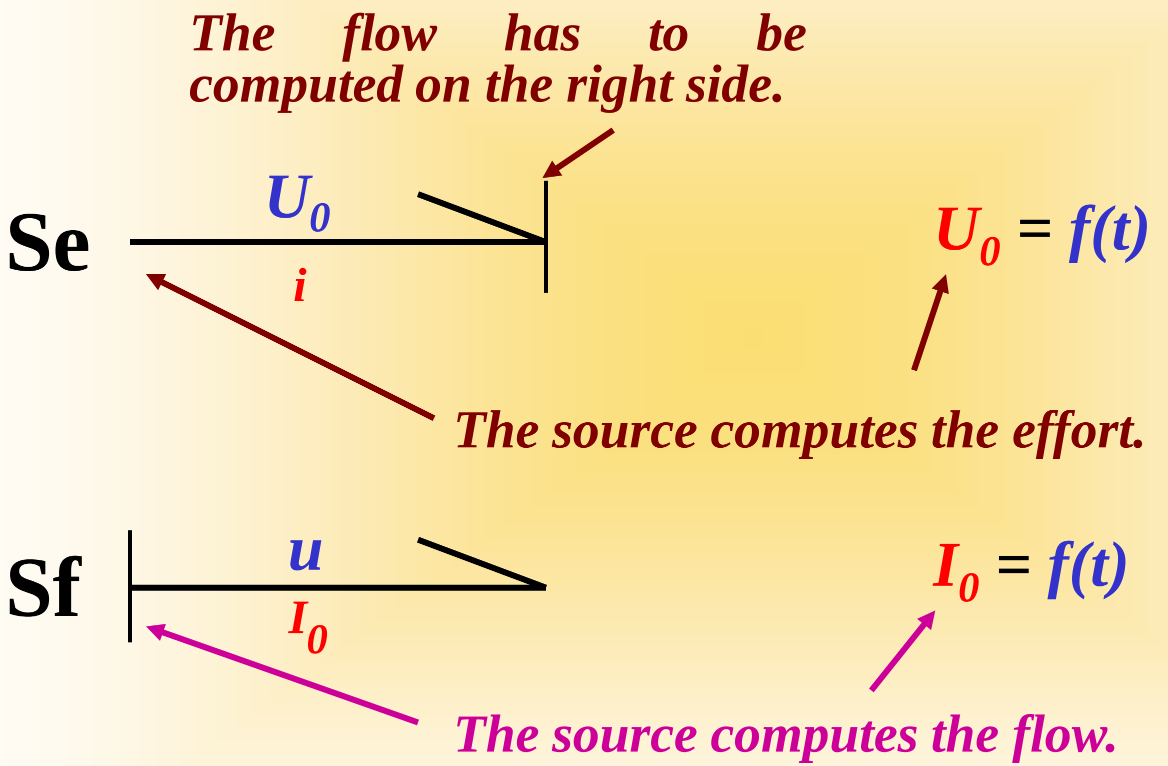


# Causal Bond Graphs

- Every bond defines two separate variables, the ***effort*** ***e*** and the ***flow*** ***f***.
- Consequently, we need two equations to compute values for these two variables.
- It turns out that it is always possible to compute one of the two variables at each side of the bond.
- A ***vertical bar*** symbolizes the side where the flow is being computed.

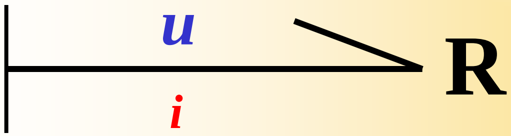


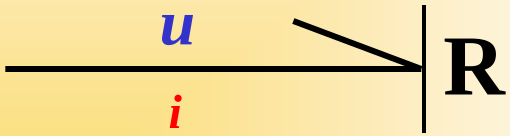
# "Causalization" of the Sources




⟹ *The causality of the sources is fixed.*

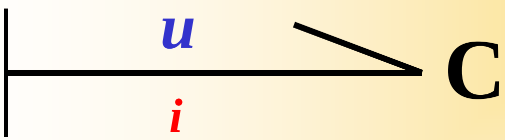
# "Causalization" of the Passive Elements

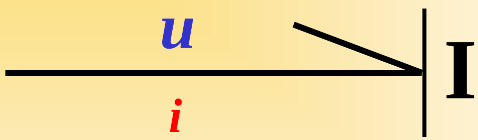



$u = R \cdot i$ $\qquad$ $i = u / R$

⇒ *The causality of resistors is free.*

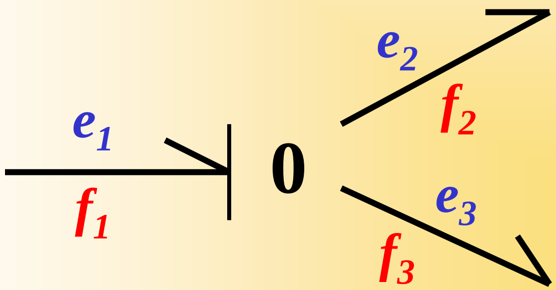



$du/dt = i \;/ C$ $\qquad$ $di/dt = u \;/ I$

⇒ ***The causality of the storage elements is determined by the desire to use integrators instead of differentiators.***

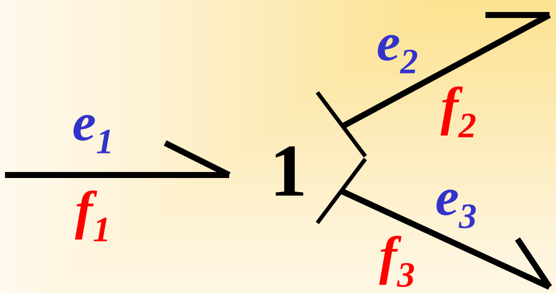
# "Causalization" of the Junctions




⇔

$$e_2 = e_1$$
$$e_3 = e_1$$
$$f_1 = f_2 + f_3$$

***Junctions of type 0 have only one flow equation, and therefore, they must have exactly one causality bar.***

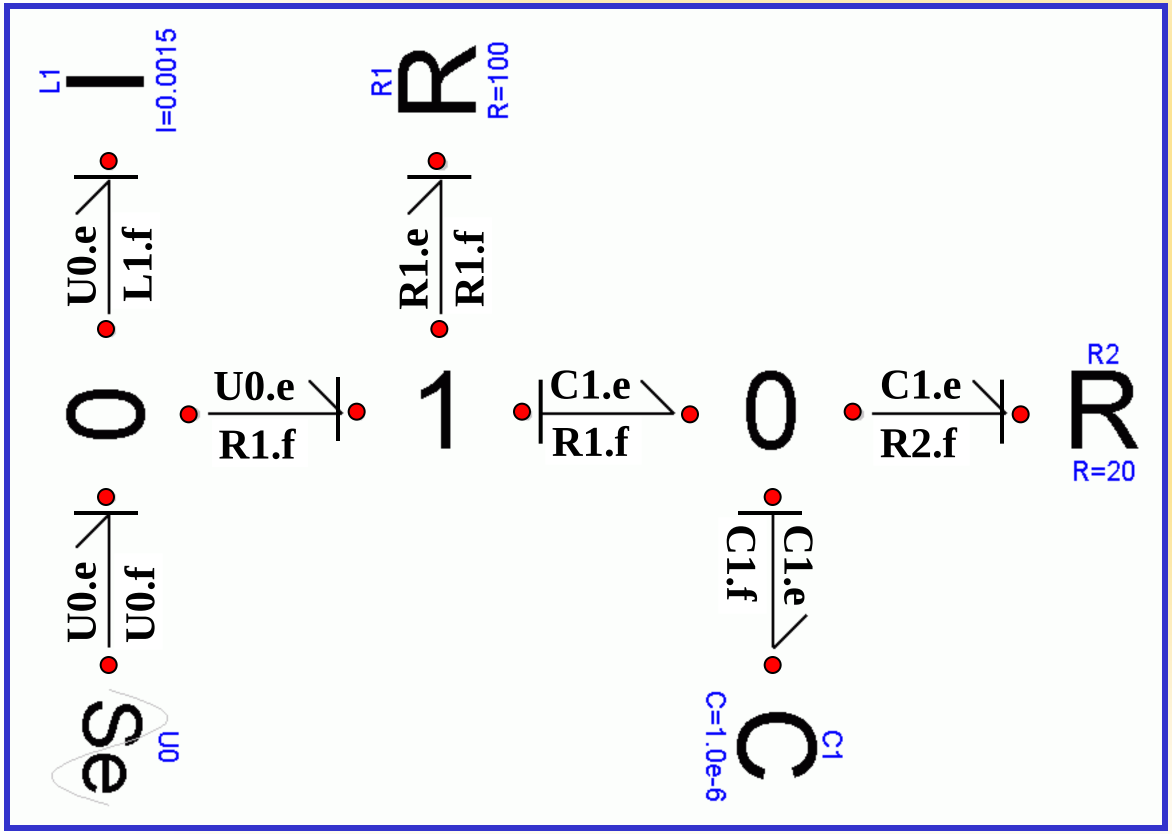



⇔

$$f_2 = f_1$$
$$f_3 = f_1$$
$$e_1 = e_2 + e_3$$

***Junctions of type 1 have only one effort equation, and therefore, they must have exactly (n-1) causality bars.***

# An Example IV




$$U0.e = f(t)$$
$$U0.f = L1.f + R1.f$$
$$dL1.f/dt = U0.e / L1$$
$$R1.e = U0.e - C1.e$$
$$R1.f = R1.e / R1$$
$$C1.f = R1.f - R2.f$$
$$dC1.e/dt = C1.f / C1$$
$$R2.f = C1.e / R2$$

# References I


- Cellier, F.E. (1991), *Continuous System Modeling*, Springer-Verlag, New York, Chapter 7.
- Cellier, F.E. (1992), "Hierarchical non-linear bond graphs: A unified methodology for modeling complex physical systems," *Simulation*, **58**(4), pp. 230-248.
- Cellier, F.E., H. Elmqvist, and M. Otter (1995), "Modeling from physical principles," *The Control Handbook* (W.S. Levine, ed.), CRC Press, Boca Raton, FL, pp. 99-108.

# References II

- Cellier, F.E. (1997), "World Wide Web - The Global Library: A Compendium of Knowledge About Bond Graph Research," *Proc. ICBGM'97, 3rd SCS Intl. Conf. on Bond Graph Modeling and Simulation*, Phoenix, AZ, pp.187-191.