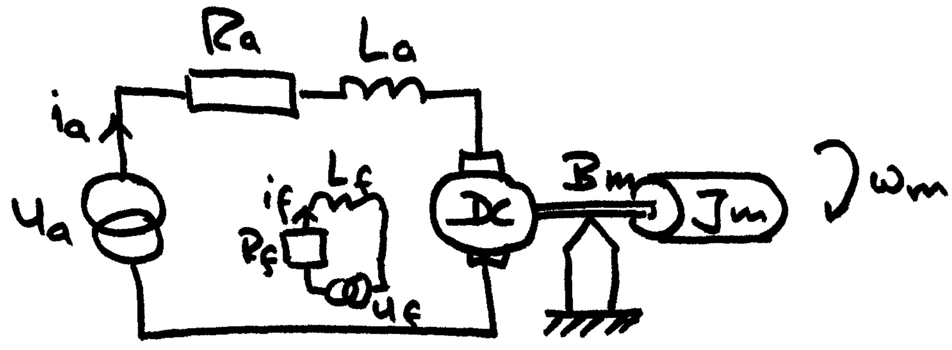
Model Reduction by Bond Graphs

Ref. Loucas S. Louca (1998), "An Energy-based Model Reduction Methodology for Automated Modeling," Ph.D. Dissertation, University of Michigan

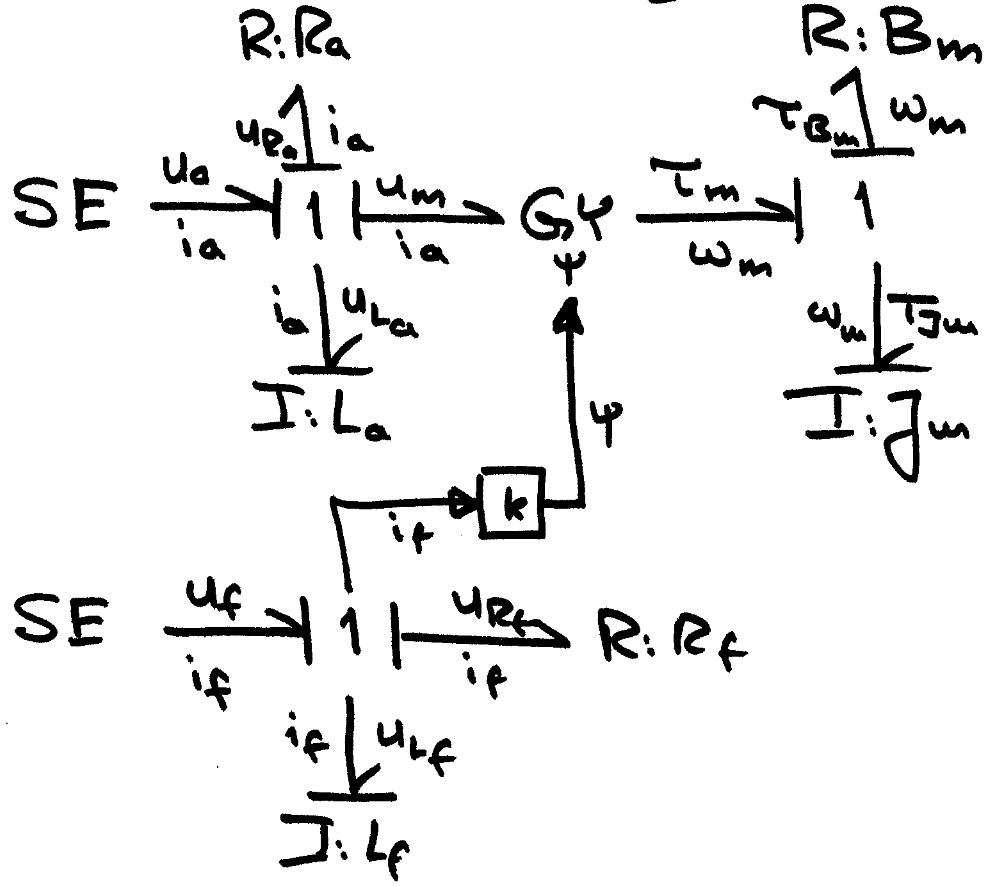
- works for arbitrarily nonlinear physical systems.

Example:

Given a DC-motor:



We can easily model the power flow through the system using a bond graph.



From the model, we can read out the set of equations describing the system:

$$u_a = f(t)$$
$$u_{Ra} = R_a \cdot i_a$$
$$\frac{di_a}{dt} = u_{La}/L_a$$
$$u_{La} = u_a - u_{Ra} - u_m$$
$$u_m = \psi \cdot \omega_m$$
$$\tau_m = \psi \cdot i_a$$
$$\psi = k \cdot i_f$$

$$u_f = g(t)$$
$$u_{Rf} = R_f \cdot i_f$$
$$\frac{di_f}{dt} = u_{Lf}/L_f$$
$$\tau_{Bm} = B_m \cdot \omega_m$$
$$\frac{d\omega_m}{dt} = \tau_{Jm}/J_m$$
$$\tau_{Jm} = \tau_m - \tau_{Bm}$$

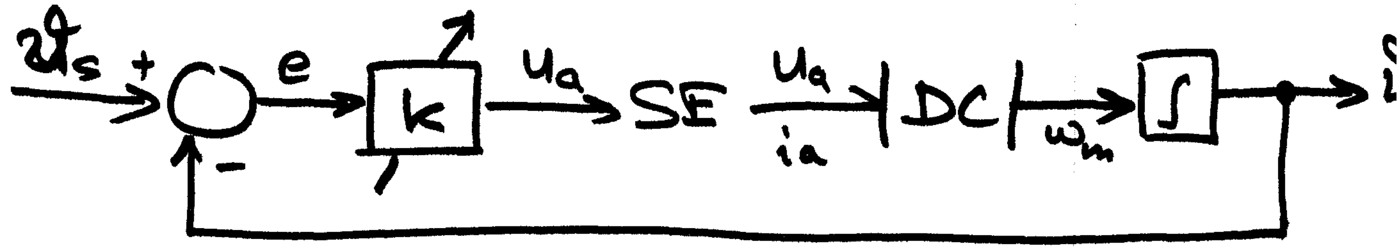
The typical question is whether L_a and L_f should be included in the model.

To decide this question, we model the DC-motor in its controlled environment.

Let us say, we want to perform a position control. We create a macro-element for the DC-motor:

$$SE \xrightarrow[i_a]{u_0} | DC | \xrightarrow[\omega_m]{}$$

and embed it within the control architecture, e.g.



Now, we exert the system in the same way that we plan to use during regular operation. We measure the power:

$$P = e \cdot f = u \cdot i = \tau \cdot \omega$$

along each bond as a function of time, and compute its norm:

$$\|P\|_{\circ} = \|P\|_2 \leftarrow \underline{\text{realistic}}$$

$$\text{or} = \|P\|_{\infty} \leftarrow \underline{\text{pessimistic}}$$

If the $\|P\|_{\circ}$ in any bond is sufficiently small, e.g. less than 1% of the largest norm:

e.g. $\underline{P} = [P_1, P_2, \ldots, P_n]$

is the vector of powers in all bonds.

Let: $\|\underline{P}\|_\infty \stackrel{!}{=} \|[\|P_i\|_\infty]\|_\infty$

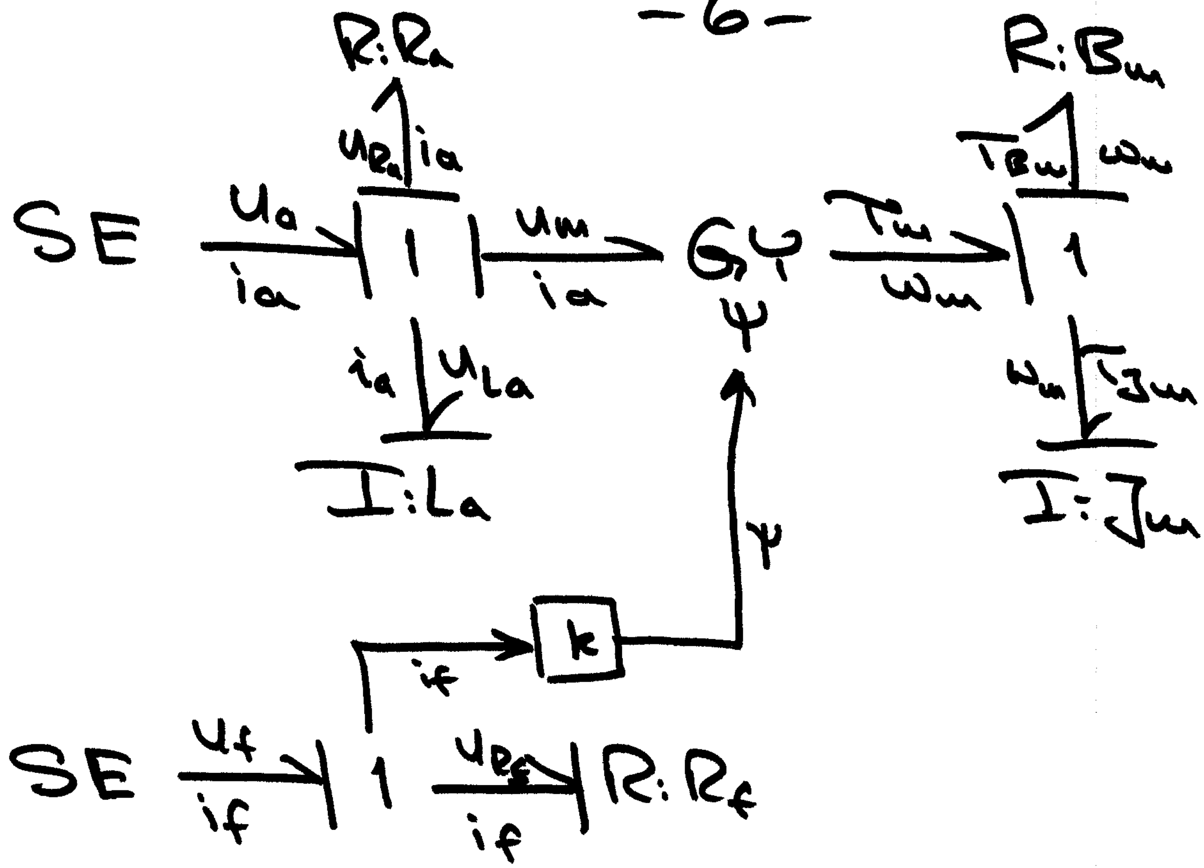
then, if for any i:

$$\frac{\|P_i\|_\infty}{\|\underline{P}\|_\infty} < 0.01$$

$\Rightarrow$ bond can be eliminated.

Let us assume we find that the bond leading to L_f can be eliminated.

The modified bond graph is:



with the equations:

$$u_a = f(t)$$
$$u_{Ra} = R_a \cdot i_a$$
$$\frac{di_a}{dt} = u_{La} / L_a$$
$$u_{La} = u_a - u_{Ra} - u_m$$
$$u_m = \psi \cdot \omega_m$$
$$T_m = \psi \cdot i_a$$
$$\psi = k \cdot i_f$$

$$u_f = g(t)$$
$$i_f = u_{Rf} / i_f$$
$$T_{Bm} = B_m \cdot \omega_m$$
$$\frac{d\omega_m}{dt} = T_{Jm} / J_m$$
$$T_{Jm} = T_m - T_{Bm}$$

The model order of the DC-motor itself got reduced from 3 to 2 (the controller adds one model order due to the integrator computing ϑ_m).

This is <u>not</u> a modal decomposition. The eigen-values may change in the process.

Works for arbitrarily nonlinear plants.