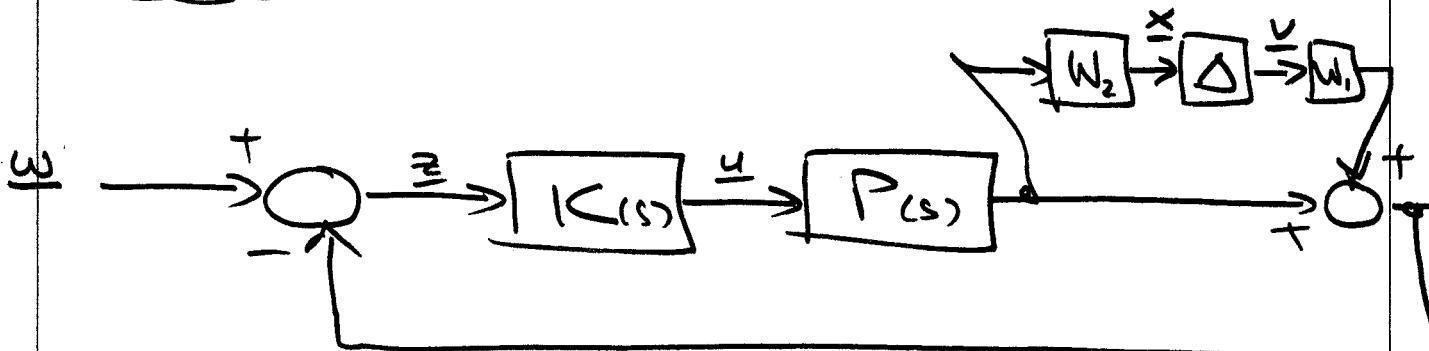


It is always possible to pull out the controllers from below and the normalized uncertainties from above.

National®Brand  
13-782  
42-381  
50 SHEETS FILLER 5 SQUARE  
50 SHEETS EYE-EASE® 5 SQUARE  
100 SHEETS EYE-EASE® 5 SQUARE  
200 SHEETS EYE-EASE® 5 SQUARE  
42-389  
42-382  
100 RECYCLED WHITE 5 SQUARE  
42-388  
200 RECYCLED WHITE 5 SQUARE  
Atene in U.S.A.

Example:



$$\text{i.e., } \overline{H(s)} = (I + W_1(s) \cdot \Delta \cdot W_e(s)) P(s)$$

$$\begin{aligned} x &= W_2 \cdot P \cdot u \\ y &= W_1 \cdot v + P \cdot u \\ z &= w - W_1 \cdot v - P \cdot u \end{aligned}$$

Y

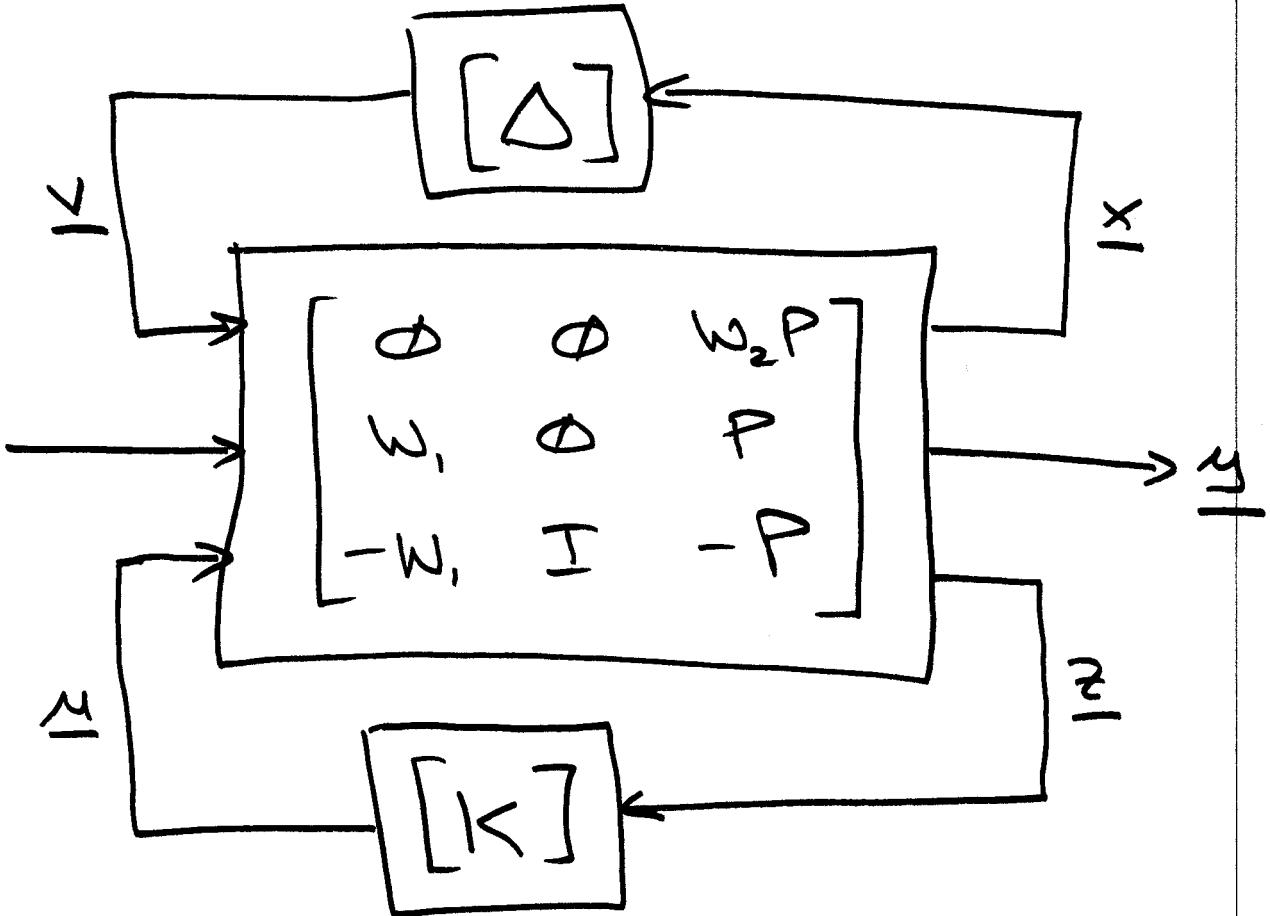
$$\begin{bmatrix} \mathbf{w} \\ \mathbf{u} \end{bmatrix} = \begin{bmatrix} -\mathbf{z}, \mathbf{e}, \emptyset \\ \mathbf{H} \emptyset \emptyset \end{bmatrix} \cdot \begin{bmatrix} -\mathbf{P} & \mathbf{P}_2 \mathbf{z} \mathbf{P} \\ \mathbf{P} & \end{bmatrix} \cdot \begin{bmatrix} \mathbf{z} \\ \mathbf{e} \end{bmatrix}$$

$$\mathbf{z} = \mathbf{K} \mathbf{u}$$

$$\mathbf{u} = \begin{bmatrix} \Delta \end{bmatrix} \mathbf{x}$$

$$\hat{\mathbf{P}}_{ss}$$

$\mathbf{z}$



is  
on an LFT parametrization  
of the system.

500 SHEETS FILLER 5 SQUARE  
50 SHEETS EYE-EASE® 5 SQUARE  
100 SHEETS EYE-EASE® 5 SQUARE  
12-382 200 SHEETS EYE-EASE® 5 SQUARE  
12-383 200 RECYCLED WHITE 5 SQUARE  
12-389 200 RECYCLED WHITE 5 SQUARE  
12-390 200 RECYCLED WHITE 5 SQUARE  
12-399 200 RECYCLED WHITE 5 SQUARE  
NATIONAL BRAND  
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$$\rightarrow \bar{T}_{wy}(s) = \bar{F}_u(\bar{F}_e(\hat{P}, K), \Delta)$$
$$= \bar{F}_e(\bar{F}_u(\hat{P}, \Delta), K)$$

- $\hat{P}(s)$  contains everything that is known about the plant.
- $K(s)$  contains all the controllers.
- $\Delta(s)$  contains all the normalized uncertainties about the plant.

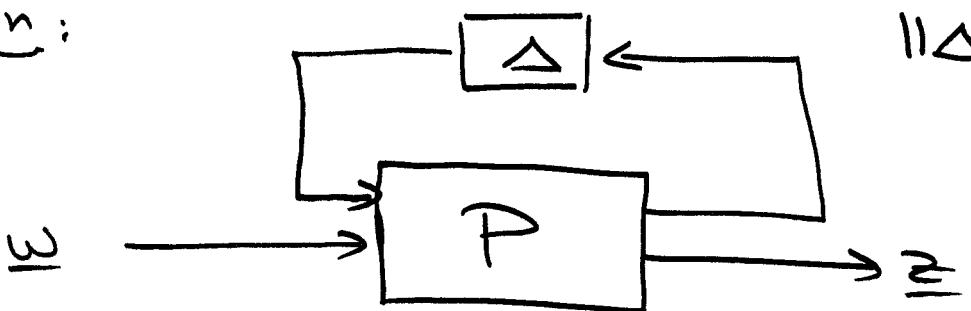
$$\|\Delta\|_\infty = 1.$$

13-732 500 SHEETS, FILLER 5 SQUARE  
42-381 50 SHEETS EYE-EASE® 5 SQUARE  
45-362 100 SHEETS EYE-EASE® 5 SQUARE  
42-382 200 RECYCLED WHITE 5 SQUARE  
42-389 200 RECYCLED WHITE 5 SQUARE  
MFG. IN U.S.A.

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## Small Gain Theorem:

Given:



$$\|\Delta\|_\infty = 1$$

$$\Rightarrow T_{wz} = \tilde{f}_u(P, \Delta) = P_{22} + P_{21}\Delta [I - P_{11}\Delta]^{-1} \cdot P_{12}$$

Assume  $P$  is stable

$\Rightarrow T_{wz}$  is stable and well-posed

$$\text{iff } \|P_{11}\Delta\|_\infty < 1$$

$$\Rightarrow \cancel{\|P_{11}\|_\infty < 1}$$

## Robust Stability:

Let us assume:

$$\bar{T} = (I + W_1 \Delta W_2) P$$

$$\Rightarrow P_{11} = W_2 \bar{T}_0 W_1$$

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42-381 50 SHEETS EYE-EASED 5 SQUARE  
42-392 100 SHEETS EYE-EASED 5 SQUARE  
42-389 200 SHEETS EYE-EASED 5 SQUARE  
42-382 100 RECYCLED WHITE 5 SQUARE  
42-393 200 RECYCLED WHITE 5 SQUARE  
National® U.S.A.

ANSWER National® Brand

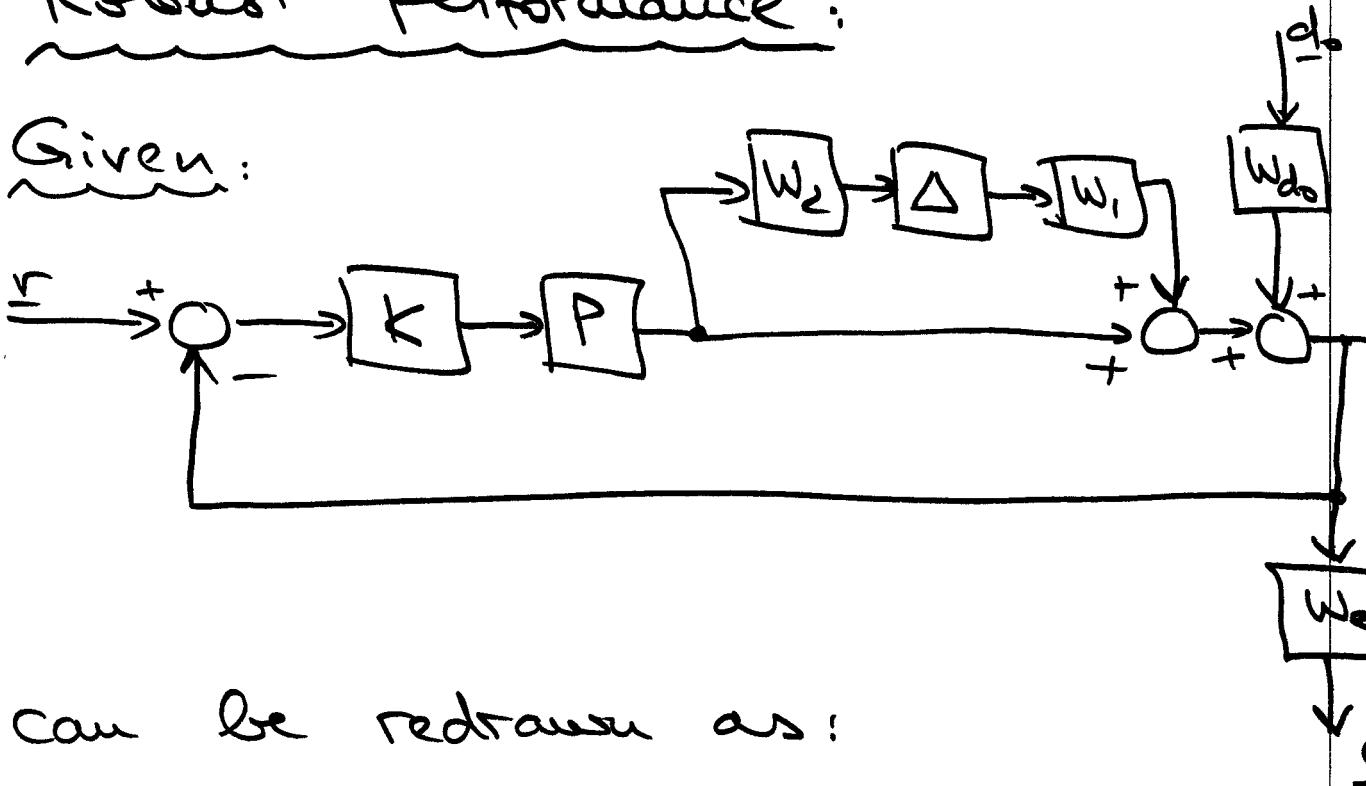
and robust stability implies:

$$\|P_{ii}\|_\infty = \|W_2 T_0 W_i\|_\infty < 1$$

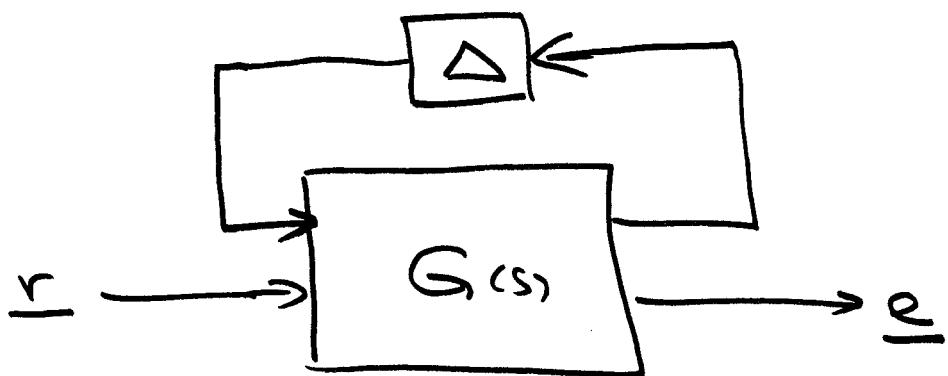
as shown earlier.

Robust performance:

Given:



can be redrawn as:



with:

$$G(s) = \begin{bmatrix} -w_2 T_0 w_1 & -w_2 T_0 w_{d0} \\ w_e s_0 w_1 & w_e s_0 w_{d0} \end{bmatrix}$$

Robust stability requires:

$$\|w_2 T_0 w_1\|_\infty < 1$$

Robust performance requires:

$$\|F_u(G, \Delta)\|_\infty < 1$$

This can be rewritten as :

$$\|G\|_M = \inf_{d_w \in \mathbb{R}_+} \overline{\sigma} \left( \begin{bmatrix} -W_2^T \omega W, & -d_w \cdot W_2^T \omega W_d, \\ \frac{1}{d_w} \cdot W_e S_0 W, & W_e S_0 W_d, \end{bmatrix} \right)$$

⇒ The robust performance problem can be reinterpreted as a robust stability problem with a second  $\Delta$  fed back from  $e$  to  $d_0$ .

The  $\| \cdot \|_u$  gives us less conservative results than the  $\| \cdot \|_\infty$  norm, as shall be shown shortly.

Let us assume a system with  $S$  structured sources of uncertainty:

$$\Delta_i = \delta_i I ; i = 1, \dots, S$$

and  $F$  unstructured sources of uncertainty:

$$\Delta_j ; j = 1, \dots, F$$

Pulling out all the  $\Delta$ s, leaves us with:

500 SHEETS FULLER'S SQUARE  
500 SHEETS FULLER'S SQUARE  
100 SHEETS EVERLAST  
200 SHEETS EVERLAST  
100 RECYCLED WHITE 5 SQUARE  
200 RECYCLED WHITE 5 SQUARE  
Made in U.S.A.



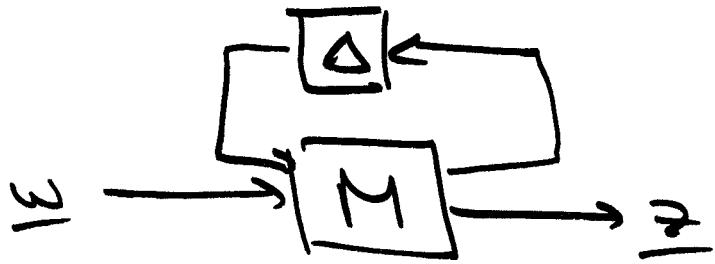
$$\Delta = \begin{bmatrix} \delta_1 I & & & & \\ & \ddots & & & \\ & & \delta_s I & & \\ & & & \emptyset & \\ & & & & \Delta_1 \\ & & & & \ddots \\ & & & & & \Delta_F \end{bmatrix}$$

$\Delta$  is block-diagonal, with

$$\|\delta_i\|_\infty = 1 ; \quad \|\Delta_j\|_\infty = 1$$

$$\rightarrow \|\Delta\|_\infty = 1 .$$

The small gain theorem suggests that :



$$\|M\|_\infty < 1$$

for robust stability. The problem is that there may

12-782 500 SHEETS, FULLER, 5" SQUARE  
12-351 50 SHEETS, EYE, FEASER, 3" SQUARE  
12-352 100 SHEETS, EYE, FEASER, 3" SQUARE  
12-359 200 SHEETS, EYE, FEASER, 3" SQUARE  
12-352 100 RECYCLED, WHITE, 5" SQUARE  
12-359 200 RECYCLED, WHITE, 5" SQUARE  
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not exist a  $M_{ii}$  with  $\|M_{ii}\|_\infty < 1$   
if all we assume is  $\|\Delta\|_\infty = 1$ .  
 $\Rightarrow$  The design may be too  
conservative.

We have thrown away  
valuable information about  
the internal structure of  
 $\Delta$ , i.e., the fact that it is  
block-diagonal.

- We can exploit this knowledge  
by reducing our requirement  
to:

$$\|M_{ii}\|_\mu < 1$$

which may have a solution,  
even if  $\|M_{ii}\|_\infty < 1$  does not.

For robust performance, we simply look at  $M$  as a whole, and request:

$$\|M\|_\mu < 1$$

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500 SHEETS, FILLER, 5" SQUARE  
42-381 50 SHEETS, EYE-FASES, 5" SQUARE  
12-389 100 SHEETS, EYE-FASES, 5" SQUARE  
12-392 100 RECYCLED, 5" SQUARE  
42-389 200 RECYCLED, WHITE, 5" SQUARE  
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I already gave you an algorithm to calculate  $\|M\|_\mu$ . However, we still may have a BAD search problem, because  $M$  is parameterized in the set of all stabilizing controllers, either using pole-Kacera parameterization or the technique that follows now.

## General decomposition theorem:

Given a nominal plant

P :

$$\dot{x} = Ax + Bu$$

We can find an optimal controller with:

$$\min_{\Sigma} \int_0^{\infty} \begin{bmatrix} x \\ u \end{bmatrix}^* \begin{bmatrix} Q & R \\ Z^* & PZ \end{bmatrix} \begin{bmatrix} x \\ u \end{bmatrix} dt$$

where:

$$\begin{bmatrix} Q & RZ \\ Z^* & PZ \end{bmatrix} \geq \emptyset$$

We must assume:  $R \succ 0$ .

We can always normalize  $R$  to  $I$ :

$$\Rightarrow \begin{bmatrix} Q \\ Z^* H \end{bmatrix} \geq \phi$$

If this matrix doesn't have full rank, we can decompose it as:

$$\begin{bmatrix} Q \\ Z^* H \end{bmatrix} = \begin{bmatrix} C_1^* \\ D_{12}^* \end{bmatrix} \cdot \begin{bmatrix} C_1 & D_{12} \end{bmatrix}$$

and reformulate the optimization problem as:

$$\min_u \|C_1 \underline{x} + D_{12} \underline{u}\|_2$$

$\Rightarrow$  We can reinterpret this as an output equation:

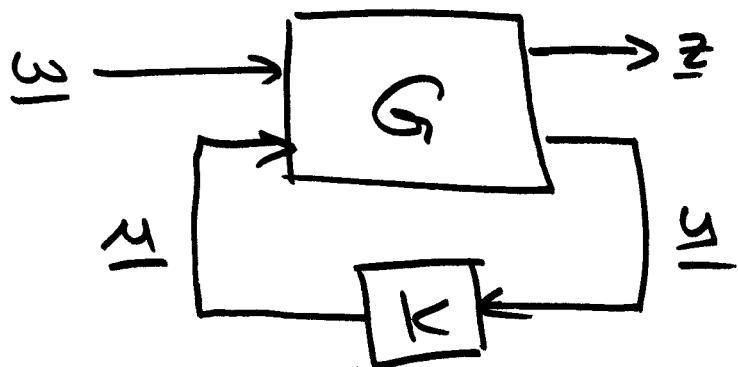
$$\left| \begin{array}{l} \dot{\underline{x}} = A \underline{x} + B_2 \underline{u} \\ \underline{y} = C_1 \underline{x} + D_{12} \underline{u} \\ \min_u \|\underline{y}\|_2 \end{array} \right|$$

50 SHEETS, FILLER, 5 SQUARE  
42-382 50 SHEETS, NYLON, 5 SQUARE  
42-389 100 SHEETS CYAN, 5 SQUARE  
42-392 100 RECYCLED WHITE, 5 SQUARE  
42-389 200 RECYCLED WHITE, 5 SQUARE  
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Can be solved by Riccati equations.

Generalization:



$$G(s) = \left[ \begin{array}{c|cc} A & B & B_2 \\ \hline C_1 & \emptyset & D_{12} \\ C_2 & D_{21} & \emptyset \end{array} \right]$$

⇒ We formulate the two Hamiltonians:

$$H_2 = \begin{bmatrix} A & \emptyset \\ -C_1^*, -A^* \end{bmatrix} - \begin{bmatrix} B_2 \\ -C_1^* D_{12} \end{bmatrix} \cdot \begin{bmatrix} D_{12}^* C_1 & B_2^* \end{bmatrix}$$

$$J_2 = \begin{bmatrix} A^* & \emptyset \\ -B_1 B_1^* & -A \end{bmatrix} - \begin{bmatrix} C_2^* \\ -B_1 D_{21}^* \end{bmatrix} \cdot \begin{bmatrix} D_{21} B_1^* & C_2 \end{bmatrix}$$

We solve :

$$X_2 = R_{iC}(H_2) \geq 0$$

$$Y_2 = R_{iC}(J_2) \geq 0$$

$$\Rightarrow F_2 = -(B_2^* X_2 + D_2^* C_1)$$

$$L_2 = -(Y_2 C_2^* + B_1 D_2^*)$$

$$\Rightarrow A_{F_2} = A + B_2 F_2$$

$$C_{F_2} = C_1 + D_1 L_2$$

$$A_{L_2} = A + L_2 C_2$$

$$B_{1L_2} = B_1 + L_2 D_2$$

$$A_K = A + B_2 F_2 + L_2 C_2$$

$$\Rightarrow G_C(s) = \left[ \begin{array}{c|c} A_{F_2} & H \\ \hline C_{F_2} & \emptyset \end{array} \right];$$

$$G_L(s) = \left[ \begin{array}{c|c} A_{L_2} & B_{1L_2} \\ \hline H & \emptyset \end{array} \right];$$

$$K_{\text{nom}}(s) = \begin{bmatrix} A_k & -L_2 \\ F_2 & \emptyset \end{bmatrix}$$

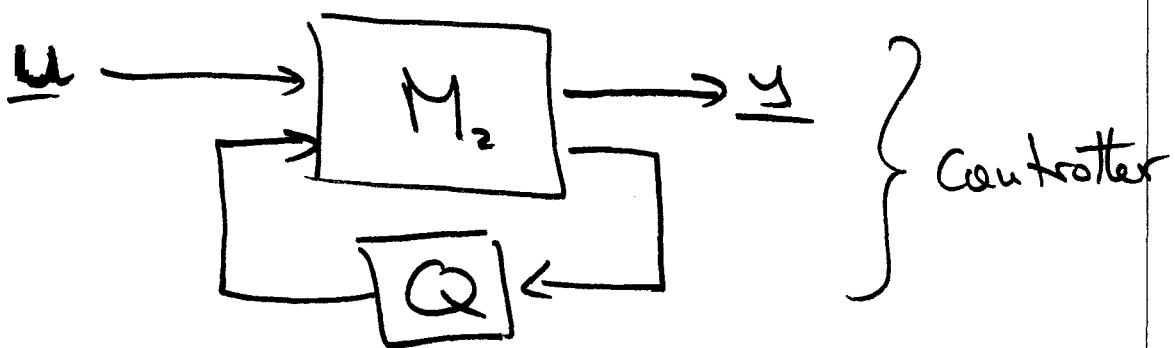
Also:

$$\begin{aligned} \|T_{wz}\|_2^2 &= \|G_c B_1\|_2^2 + \|F_2 G_L\|_2^2 \\ &= \|G_c L_2\|_2^2 + \|C_1 G_L\|_2^2 \end{aligned}$$

Parametrization of all stabilizing controllers:

We are interested in finding all stabilizing controllers with:

$$\|T_{wz}\|_2 < \gamma :$$



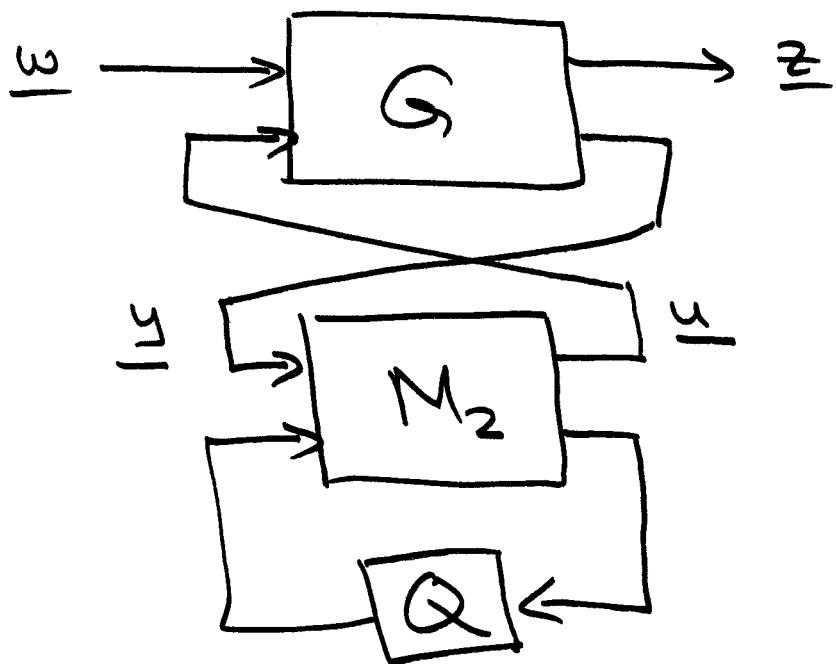
$$M_2^{(S)} = \left[ \begin{array}{c|cc} A_k & -L_2 & B_2 \\ \hline F_2 & \emptyset & I \\ -C_2 & I & \emptyset \end{array} \right]$$

$$Q > \phi ; \quad \|Q\|_2^2 \leq \gamma^2 = \left( \|G_c B_1\|_2^2 + \|F_2 G_L\|_2^2 \right)$$

is a set of suboptimal stabilizing controllers.

Clearly :  $Q = \emptyset \Leftrightarrow M_2(s) = K_{\text{now}}(s)$

Adding the plant:



$$\begin{array}{c} \Rightarrow \\ \left. \begin{array}{l} K(s) = F_L(M_2(s), Q(s)) \\ T_{w_2}(s) = F_L(G(s), K(s)) \end{array} \right| \end{array}$$

$\therefore T_{w_2}(s) = F_L(N(s), Q(s))$

With:

$$N(s) = \left[ \begin{array}{cc|cc} A_{F_2} & -B_2 \cdot F_2 & B_1 & B_2 \\ \emptyset & A_{L_2} & B_{1L_2} & \emptyset \\ \hline C_{1F_2} & -D_{12} \cdot F_2 & \emptyset & D_{12} \\ \emptyset & C_2 & D_{21} & \emptyset \end{array} \right]$$

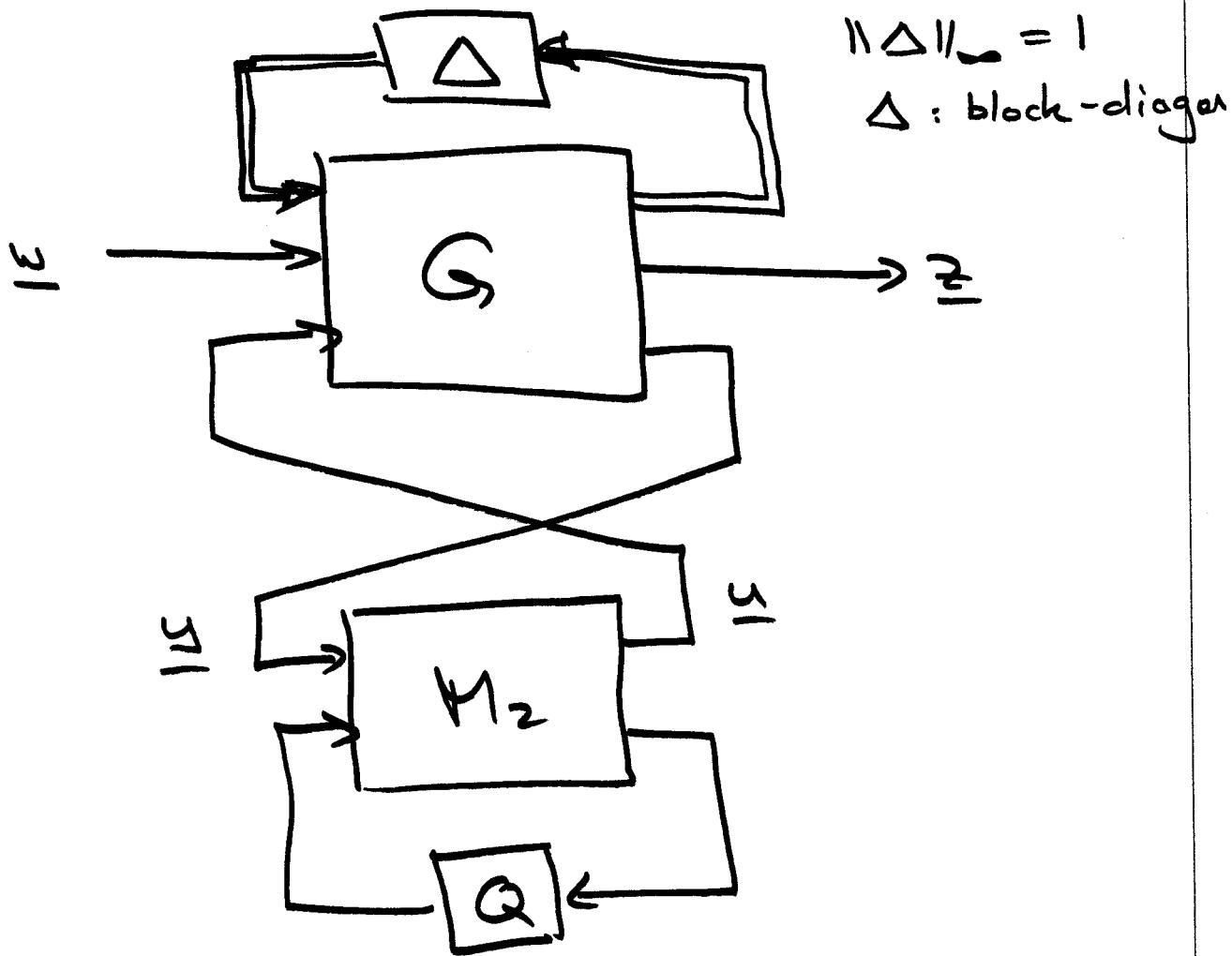
Let:

$$U(s) = \left[ \begin{array}{c|c} A_{F_2} & B_2 \\ \hline C_{1F_2} & D_{12} \end{array} \right]; V(s) = \left[ \begin{array}{c|c} A_{L_2} & B_{1L_2} \\ \hline C_2 & D_{21} \end{array} \right]$$

$$\Rightarrow T_{w_2}(s) = G_c B_1 - U F_2 G_c + U Q V$$

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42-381 50 SHEETS TRI-FOLD 5 SQUARE  
42-382 100 SHEETS 12" X 12" 5 SQUARE  
42-392 200 RECYCLED WHITE 5 SQUARE  
42-393 100 RECYCLED WHITE 5 SQUARE  
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Adding the plant uncertainty:



$$\Rightarrow K(s) = F_e(M_2, Q)$$

$$M(s) = F_e(G, K)$$

Choose:  $Q$  such that

$$\|M\|_\infty < 1$$

or:  $\min \|M\|_\infty$