Numerical Simulation of Dynamic Systems: Hw11 - Problem

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[H9.1] Runge-Kutta-Fehlberg with Root Solver

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where x_{c0} is a column vector containing the initial values of the continuous state variables; x_{d0} is a column vector containing the initial values of the discrete state variables; t is a row vector of communication instants in time; and tol is the desired absolute error bound on the states and also on the zero-crossing functions.

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The function returns y, a matrix of output values, where each row denotes one output variable, and each column denotes one time instant, at which the output variables were recorded; x_c is the matrix of continuous state variables; x_d is the matrix of discrete state variables; and $tout$ is the vector of time instants, at which the states and outputs were recorded.

[H9.1] Runge-Kutta-Fehlberg with Root Solver II

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Function *rkf45rt* calls upon a number of internal functions:

- A single step of the Runge-Kutta-Fehlberg algorithm is being computed by the function:

  ```
  function [xc4, xc5] = rkf45rt_step(xc, xd, t, h)
  ```

  which looks essentially like the routine you coded earlier. x_d is treated like a parameter vector, since the discrete state variables don't change their values except at event times.

[H9.1] Runge-Kutta-Fehlberg with Root Solver III

- We check on zero-crossings using the function:

  function $[iter] \;= \text{zc\_iter}(f, tol)$

  where f is a matrix with two column vectors. The first column vector contains the values of the zero-crossing functions at the beginning of the interval, and the second column vector contains the values of the zero-crossing functions at the end of the interval. tol is the largest distance from zero, for which the iteration will terminate.

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  The variable $iter$ returns 0, if no zero crossing occurred in the interval; it returns $+1$, if either multiple zero crossings took place inside the interval, or if a single zero crossing took place that hasn't converged yet; it returns $-i$, if one zero crossing took place and has converged. The index i is the index of the zero-crossing function that triggered the state event.

[H9.1] Runge-Kutta-Fehlberg with Root Solver IV

- If *iter* $= 1$, we wish to perform one iteration step of *regula falsi*. To this end, we code the function:

  function [*tnew*] = reg_falsi(t, f)

  where t is a row vector of length two containing the time values corresponding to the beginning and the end of the interval, respectively, and f is the same matrix used also by function *zc_iter*.

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  The *reg_falsi* routine needs to take care of intervals containing a single triggered zero-crossing function or multiple triggered zero-crossing functions.

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Since this class concerns itself with *continuous systems simulation* and not with *discrete event simulation*, we shall implement the event calendar in a simple straight-forward manner as a matrix, rather than as a linear forward and backward linked list.

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The event calendar is maintained by three functions: *push_evt*, *pull_evt*, and *query_evt*.

[H9.1] Runge-Kutta-Fehlberg with Root Solver VI

- The function:

  function push_evt(*t*, *evt_nbr*)

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- The function:

  function [*tnext*, *evt_nbr*] = query_evt()

  returns the event information of the next time event without removing the event from the event calendar.

[H9.1] Runge-Kutta-Fehlberg with Root Solver VII

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- The function:

  function $[xcdot] = \text{cst\_eq}(xc, xd, t)$

  assumes the same role that the function *st_eq* had assumed earlier. It computes the continuous state derivatives at time t. Since the discrete states x_d are constant during each continuous simulation segment, this vector assumes the role of a parameter vector.

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  function [*y*] = out_eq(*xc*, *xd*, *t*)

  assumes the same role as earlier.

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- The function:

  function $[y]$ $=$ out_eq(xc, xd, t)

  assumes the same role as earlier.

- The new function:

  function $[f]$ $=$ zcf(xc, xd, t)

  returns the current values of the zero-crossing functions as a column vector.

[H9.1] Runge-Kutta-Fehlberg with Root Solver VIII

- The new function:

  function $[xdnew] = \text{dst\_eq}(xc, xd, t, evt\_nbr)$

  returns the new discrete state vector after an event has taken place.

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  The routine handles both *time events* and *state events*. It is called with a positive event number for time events, and with a negative event number for state events.

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  In the case of a time event, the *rkf45rt* function logs the current states, then removes the time event from the event calendar, then calls function *dst_eq*, and finally logs the new states once again.

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  In the case of a time event, the *rkf45rt* function logs the current states, then removes the time event from the event calendar, then calls function *dst_eq*, and finally logs the new states once again.

  Consequently, the *dst_eq* function does not need to remove the current time event from the event calendar, but it needs to schedule future time events that are a consequence of the current event action.

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- The main program calculates the values of both the continuous and the discrete initial states, and it places the initial time events on the event calendar.
- It then calls routine *rkf45rt* to perform the simulation.
- It finally plots the simulation results.

[H9.7] Thyristor

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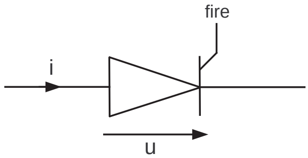

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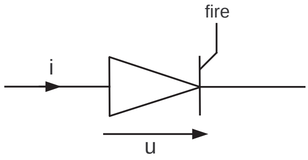

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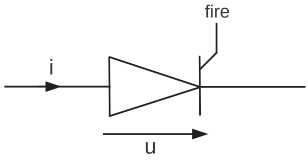

The thyristor is a diode with a modified firing logic. The diode can only close when the external Boolean variable *fire* has a value of *true*. The opening logic is the same as for the regular diode.

Since the thyristor is a diode, we can use the same *parameterized curve description* that we used for the regular diode. Only the switching condition is modified.

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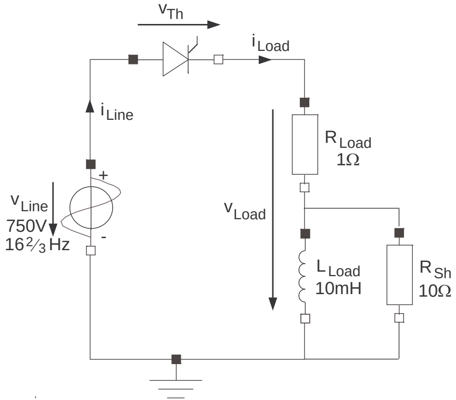

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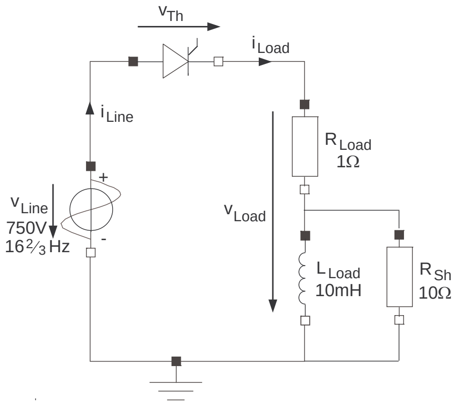

A shunt resistor was added to avoid having to deal with a *variable structure model*.

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Choose a suitable tearing structure, and solve the equations both horizontally and vertically using the variable substitution technique.

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The external control variable of the thyristor, *fire*, is to be assigned a value of *true* from the angle of 30^o until the angle of 45^o, and from the angle of 210^o until the angle of 225^o during each period of the line voltage, v_{Line}. During all other times, it is set to *false*.

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Plot the load voltage, v_{Load}, as well as the load current, i_{Load}, as functions of time.

[H9.7] Thyristor V

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In **Matlab**, Booleans are represented by integers, whereby *true* $\Rightarrow 1$ and *false* $\Rightarrow 0$.

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In **Matlab**, Boolean operators have been defined for the pseudo-Boolean variables in the form of functions. Thus, toggling a Boolean variable can be written as:

$ms = \text{not}(ms);$

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m_0 needs to be updated at the end of every discrete event.