Numerical Simulation of Dynamic Systems: Hw11 - Solution

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May 28, 2013

[H9.1] Runge-Kutta-Fehlberg with Root Solver

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where x_{c0} is a column vector containing the initial values of the continuous state variables; x_{d0} is a column vector containing the initial values of the discrete state variables; t is a row vector of communication instants in time; and tol is the desired absolute error bound on the states and also on the zero-crossing functions.

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The function returns y, a matrix of output values, where each row denotes one output variable, and each column denotes one time instant, at which the output variables were recorded; x_c is the matrix of continuous state variables; x_d is the matrix of discrete state variables; and $tout$ is the vector of time instants, at which the states and outputs were recorded.

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Function *rkf45rt* calls upon a number of internal functions:

- A single step of the Runge-Kutta-Fehlberg algorithm is being computed by the function:

  function [*xc*4, *xc*5] = rkf45rt_step(*xc*, *xd*, *t*, *h*)

  which looks essentially like the routine you coded earlier. x_d is treated like a parameter vector, since the discrete state variables don't change their values except at event times.

[H9.1] Runge-Kutta-Fehlberg with Root Solver III

- We check on zero-crossings using the function:

  function [*iter*] = zc_iter(*f*, *tol*)

  where *f* is a matrix with two column vectors. The first column vector contains the values of the zero-crossing functions at the beginning of the interval, and the second column vector contains the values of the zero-crossing functions at the end of the interval. *tol* is the largest distance from zero, for which the iteration will terminate.

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- We check on zero-crossings using the function:

  function [*iter*] = zc_iter(f, *tol*)

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  The variable *iter* returns 0, if no zero crossing occurred in the interval; it returns $+1$, if either multiple zero crossings took place inside the interval, or if a single zero crossing took place that hasn't converged yet; it returns $-i$, if one zero crossing took place and has converged. The index i is the index of the zero-crossing function that triggered the state event.

[H9.1] Runge-Kutta-Fehlberg with Root Solver IV

- If $iter = 1$, we wish to perform one iteration step of *regula falsi*. To this end, we code the function:

  function $[tnew] = \text{reg\_falsi}(t, f)$

  where t is a row vector of length two containing the time values corresponding to the beginning and the end of the interval, respectively, and f is the same matrix used also by function *zc_iter*.

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  The *reg_falsi* routine needs to take care of intervals containing a single triggered zero-crossing function or multiple triggered zero-crossing functions.

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Since this class concerns itself with *continuous systems simulation* and not with *discrete event simulation*, we shall implement the event calendar in a simple straight-forward manner as a matrix, rather than as a linear forward and backward linked list.

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The event calendar is maintained by three functions: *push_evt*, *pull_evt*, and *query_evt*.

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- The function:

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- The function:

  function [$tnext$, evt_nbr] = query_evt()

  returns the event information of the next time event without removing the event from the event calendar.

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- The function:

  function $[xcdot] = \text{cst\_eq}(xc, xd, t)$

  assumes the same role that the function *st_eq* had assumed earlier. It computes the continuous state derivatives at time t. Since the discrete states x_d are constant during each continuous simulation segment, this vector assumes the role of a parameter vector.

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- The new function:

  function $[f] = \text{zcf}(xc, xd, t)$

  returns the current values of the zero-crossing functions as a column vector.

[H9.1] Runge-Kutta-Fehlberg with Root Solver VIII

- The new function:

  function $[xdnew] = \text{dst\_eq}(xc, xd, t, evt\_nbr)$

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  In the case of a time event, the *rkf45rt* function logs the current states, then removes the time event from the event calendar, then calls function *dst_eq*, and finally logs the new states once again.

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  In the case of a time event, the *rkf45rt* function logs the current states, then removes the time event from the event calendar, then calls function *dst_eq*, and finally logs the new states once again.

  Consequently, the *dst_eq* function does not need to remove the current time event from the event calendar, but it needs to schedule future time events that are a consequence of the current event action.

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- The main program calculates the values of both the continuous and the discrete initial states, and it places the initial time events on the event calendar.
- It then calls routine *rkf45rt* to perform the simulation.
- It finally plots the simulation results.

[H9.1] Runge-Kutta-Fehlberg with Root Solver X

The code is self-documentary. Since its parts have been explained in much detail already, there is no need to offer more explanations here.

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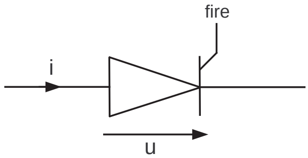

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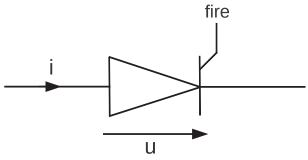

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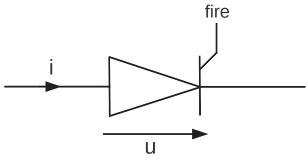

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Since the thyristor is a diode, we can use the same *parameterized curve description* that we used for the regular diode. Only the switching condition is modified.

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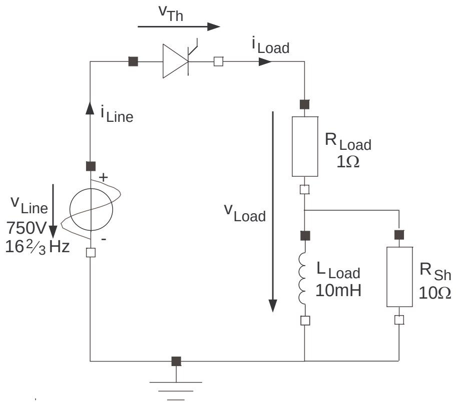

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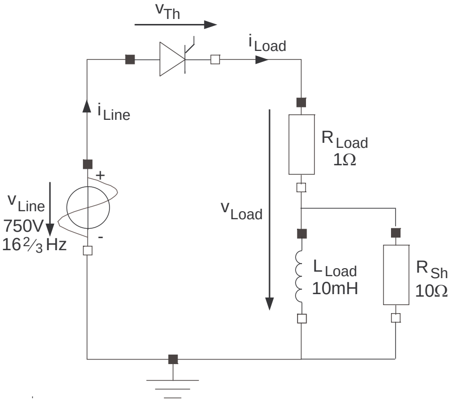

A shunt resistor was added to avoid having to deal with a *variable structure model*.

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Choose a suitable tearing structure, and solve the equations both horizontally and vertically using the variable substitution technique.

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The external control variable of the thyristor, *fire*, is to be assigned a value of *true* from the angle of 30^o until the angle of 45^o, and from the angle of 210^o until the angle of 225^o during each period of the line voltage, v_{Line}. During all other times, it is set to *false*.

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Plot the load voltage, v_{Load}, as well as the load current, i_{Load}, as functions of time.

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In **Matlab**, Booleans are represented by integers, whereby *true* $\Rightarrow 1$ and *false* $\Rightarrow 0$.

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In **Matlab**, Boolean operators have been defined for the pseudo-Boolean variables in the form of functions. Thus, toggling a Boolean variable can be written as:

$$ms = \text{not}(ms);$$

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m_0 is a Boolean function of m_1, m_s, and its own past value $\mathrm{pre}(m_0)$. Because of the dependence of m_0 on its own past, also m_0 is a discrete state variable.

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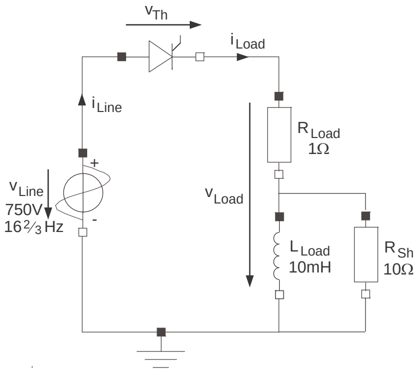
m_0 needs to be updated at the end of every discrete event.

[H9.7] Thyristor VIII



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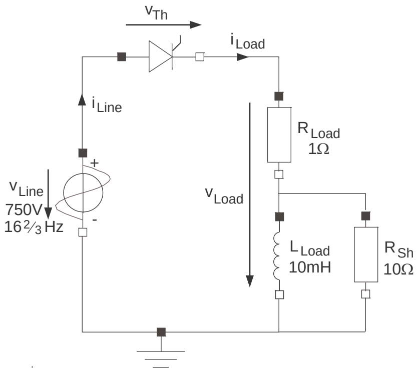

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2:	v_{RLoad}	$=$	$R_{Load} \cdot i_{Load}$
3:	v_{RSh}	$=$	$L_{Load} \cdot \frac{di_L}{dt}$
4:	v_{RSh}	$=$	$R_{Sh} \cdot i_{RSh}$
5:	v_{Load}	$=$	$v_{RLoad} + v_{RSh}$
6:	v_{Line}	$=$	$v_{Th} + v_{Load}$
7:	i_{Load}	$=$	$i_L + i_{RSh}$
8:	v_{Th}	$=$	$m_0 \cdot s$
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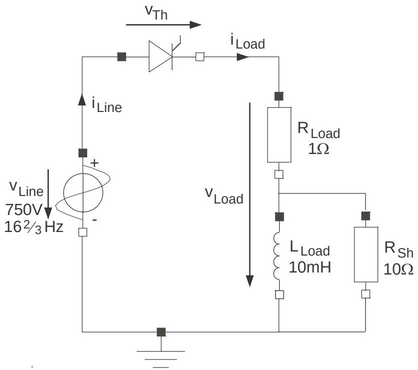

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5: $v_{Load} = v_{RLoad} + v_{RSh}$
6: $v_{Line} = v_{Th} + v_{Load}$

7: $i_{Load} = i_L + i_{RSh}$

8: $v_{Th} = m_0 \cdot s$
9: $i_{Load} = (1 - m_0) \cdot s$

m_0 is a discrete state variable. It is *true*, when the thyristor is *off*.

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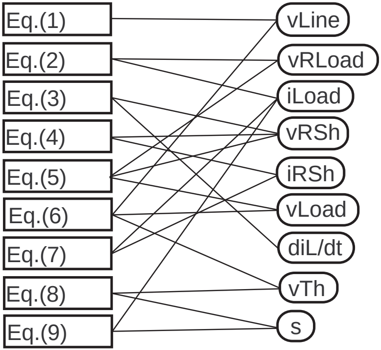
7: $i_{Load} = i_L + i_{RSh}$

8: $v_{Th} = m_0 \cdot s$
9: $i_{Load} = (1 - m_0) \cdot s$



[H9.7] Thyristor X

We causalize as much as we can:

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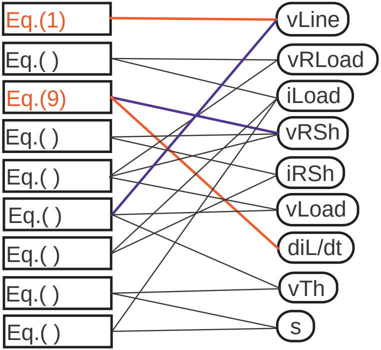

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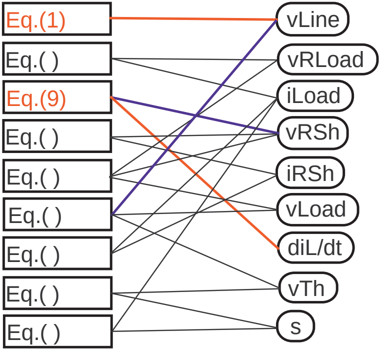

$$
\begin{aligned}
1:&\quad v_{Line} &=&\quad V_0 \cdot \sin\left(\frac{2\pi t}{t_p}\right)\\
?:&\quad v_{RLoad} &=&\quad R_{Load} \cdot i_{Load}\\
9:&\quad v_{RSh} &=&\quad L_{Load} \cdot \frac{di_L}{dt}\\
?:&\quad v_{RSh} &=&\quad R_{Sh} \cdot i_{RSh}\\
&&&\\
?:&\quad v_{Load} &=&\quad v_{RLoad} + v_{RSh}\\
?:&\quad v_{Line} &=&\quad v_{Th} + v_{Load}\\
&&&\\
?:&\quad i_{Load} &=&\quad i_L + i_{RSh}\\
&&&\\
?:&\quad v_{Th} &=&\quad m_0 \cdot s\\
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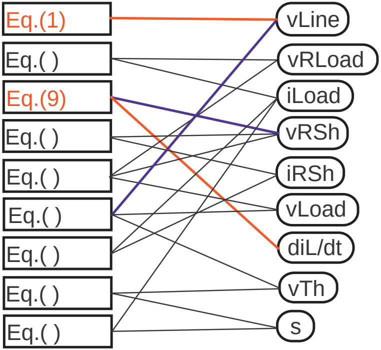

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?:&\quad v_{Load} &=&\quad v_{RLoad} + v_{RSh}\\
?:&\quad v_{Line} &=&\quad v_{Th} + v_{Load}\\
?:&\quad i_{Load} &=&\quad i_L + i_{RSh}\\
?:&\quad v_{Th} &=&\quad m_0 \cdot s\\
?:&\quad i_{Load} &=&\quad (1 - m_0) \cdot s
\end{aligned}
$$

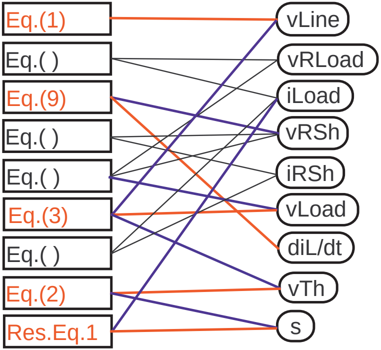
We end up with an algebraic loop in seven equations and seven unknowns. The switch equation (variable s) is part of the loop.

[H9.7] Thyristor XI

We choose s as our first tearing variable:

[H9.7] Thyristor XI

We choose *s* as our first tearing variable:



[H9.7] Thyristor XI

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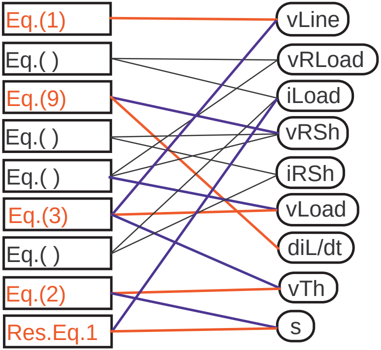
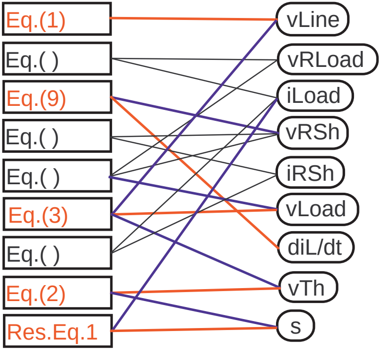


1:	v_{Line}	$=$	$V_0 \cdot \sin(\frac{2\pi t}{t_p})$
?:	v_{RLoad}	$=$	$R_{Load} \cdot i_{Load}$
9:	v_{RSh}	$=$	$L_{Load} \cdot \frac{di_L}{dt}$
?:	v_{RSh}	$=$	$R_{Sh} \cdot i_{RSh}$
?:	v_{Load}	$=$	$v_{RLoad} + v_{RSh}$
3:	v_{Line}	$=$	$v_{Th} + v_{Load}$
?:	i_{Load}	$=$	$i_L + i_{RSh}$
2:	v_{Th}	$=$	$m_0 \cdot s$
res.eq.1:	i_{Load}	$=$	$(1 - m_0) \cdot s$

[H9.7] Thyristor XI

We choose s as our first tearing variable:



1:	v_{Line}	$=$	$V_0 \cdot \sin(\frac{2\pi t}{t_p})$
?:	v_{RLoad}	$=$	$R_{Load} \cdot i_{Load}$
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?:	v_{RSh}	$=$	$R_{Sh} \cdot i_{RSh}$
?:	v_{Load}	$=$	$v_{RLoad} + v_{RSh}$
3:	v_{Line}	$=$	$v_{Th} + v_{Load}$
?:	i_{Load}	$=$	$i_L + i_{RSh}$
2:	v_{Th}	$=$	$m_0 \cdot s$
res.eq.1:	i_{Load}	$=$	$(1 - m_0) \cdot s$

We end up with a second algebraic loop in four equations and four unknowns.

[H9.7] Thyristor XII

We choose a second residual equation, and now, we can causalize the remaining equations:

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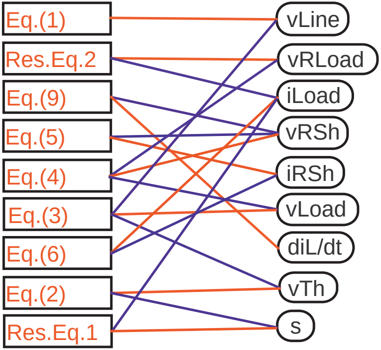

[H9.7] Thyristor XII

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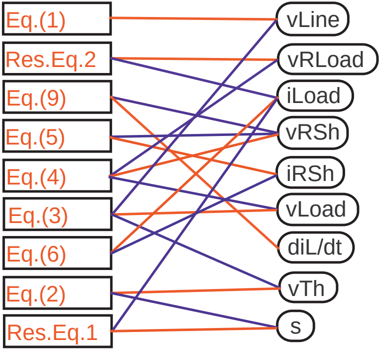

1:	v_{Line}	$=$	$V_0 \cdot \sin(\frac{2\pi t}{t_p})$
2:	v_{Th}	$=$	$m_0 \cdot s$
3:	v_{Load}	$=$	$v_{Line} - v_{Th}$
4:	v_{RSh}	$=$	$v_{Load} - v_{RLoad}$
5:	i_{RSh}	$=$	$\frac{1}{R_{Sh}} \cdot v_{RSh}$
6:	i_{Load}	$=$	$i_L + i_{RSh}$
res.eq.2:	v_{RLoad}	$=$	$R_{Load} \cdot i_{Load}$
res.eq.1:	s	$=$	$\frac{1}{1-m_0} \cdot i_{Load}$
9:	$\frac{di_L}{dt}$	$=$	$\frac{1}{L_{Load}} \cdot v_{RSh}$

[H9.7] Thyristor XIII

Substitution gives us two linear equations in the two unknown tearing variables, s and v_{RLoad}:

$$[R_{Sh} \cdot (1 - m_0) + m_0] \cdot s + v_{RLoad} = R_{Sh} \cdot i_L + v_{Line}$$
$$(m_0 \cdot R_{Load}) \cdot s + (R_{Load} + R_{Sh}) \cdot v_{RLoad} = (R_{Load} \cdot R_{Sh}) \cdot i_L + R_{Load} \cdot v_{Line}$$

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$$(m_0 \cdot R_{Load}) \cdot s + (R_{Load} + R_{Sh}) \cdot v_{RLoad} = (R_{Load} \cdot R_{Sh}) \cdot i_L + R_{Load} \cdot v_{Line}$$

or:

$$\begin{pmatrix} R_{Sh} \cdot (1 - m_0) + m_0 & 1 \\ m_0 \cdot R_{Load} & R_{Load} + R_{Sh} \end{pmatrix} \cdot \begin{pmatrix} s \\ v_{RLoad} \end{pmatrix} = \begin{pmatrix} R_{Sh} & 1 \\ R_{Load} \cdot R_{Sh} & R_{Load} \end{pmatrix} \cdot \begin{pmatrix} i_L \\ v_{Line} \end{pmatrix}$$

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$$(m_0 \cdot R_{Load}) \cdot s + (R_{Load} + R_{Sh}) \cdot v_{RLoad} = (R_{Load} \cdot R_{Sh}) \cdot i_L + R_{Load} \cdot v_{Line}$$

or:

$$\begin{pmatrix} R_{Sh} \cdot (1 - m_0) + m_0 & 1 \\ m_0 \cdot R_{Load} & R_{Load} + R_{Sh} \end{pmatrix} \cdot \begin{pmatrix} s \\ v_{RLoad} \end{pmatrix} = \begin{pmatrix} R_{Sh} & 1 \\ R_{Load} \cdot R_{Sh} & R_{Load} \end{pmatrix} \cdot \begin{pmatrix} i_L \\ v_{Line} \end{pmatrix}$$

We are now ready to code.

[H9.7] Thyristor XIV

```
function [xcdot] = cst_eq(xc, xd, t)
    %
    % State - space model of [H9.7]
    %
    RLoad = 1;
    RSh = 10;
    LLoad = 0.01;
    V0 = 750;
    p = 16 + 2/3;
    tp = 1/p;
    %
    iL = xc(1);
    m0 = xd(1);
    m1 = xd(2);
    ms = xd(3);
    %
    vLine = V0*sin(2 * pi * t/tp);
    inpt = [iL; vLine];
    a11 = RSh * (1 - m0) + m0;
    a12 = 1;
    a21 = m0 * RLoad;
    a22 = RLoad + RSh;
    A = [a11, a12; a21, a22];
    b11 = RSh;
    b12 = 1;
    b21 = RLoad * RSh;
    b22 = RLoad;
    B = [b11, b12; b21, b22];
    tear = A\B * inpt;
    s = tear(1);
    vRLoad = tear(2);
    vTh = m0 * s;
    vLoad = vLine - vTh;
    vRSh = vLoad - vRLoad;
    iRSh = vRSh/RSh;
    iLoad = iL + iRSh;
    diL = vRSh/LLoad;
    %
    xcdot = diL;
    %
return
```

[H9.7] Thyristor XV

```
function [y] = out_eq(xc, xd, t)
    %
    % Output model of [H9.7]
    %
    RLoad = 1;
    RSh = 10;
    LLoad = 0.01;
    V0 = 750;
    p = 16 + 2/3;
    tp = 1/p;
    %
    iL = xc(1);
    m0 = xd(1);
    m1 = xd(2);
    ms = xd(3);
    %
    vLine = V0*sin(2 * pi * t/tp);
    inpt = [iL; vLine];
    a11 = RSh * (1 - m0) + m0;
    a12 = 1;
    a21 = m0 * RLoad;
    a22 = RLoad + RSh;
    A = [a11, a12; a21, a22];
    b11 = RSh;
    b12 = 1;
    b21 = RLoad * RSh;
    b22 = RLoad;
    B = [b11, b12; b21, b22];
    tear = A\B * inpt;
    s = tear(1);
    vRLoad = tear(2);
    vTh = m0 * s;
    vLoad = vLine - vTh;
    vRSh = vLoad - vRLoad;
    iRSh = vRSh/RSh;
    iLoad = iL + iRSh;
    diL = vRSh/LLoad;
    %
    y = zeros(2, 1);
    y(1) = vLoad;
    y(2) = iLoad;
    %
return
```

[H9.7] Thyristor XVI

```
function [f] = zcf(xc, xd, t)
    %
    % Zero - crossing function of [H9.7]
    %
    RLoad = 1;
    RSh = 10;
    LLoad = 0.01;
    V0 = 750;
    p = 16 + 2/3;
    tp = 1/p;
    %
    iL = xc(1);
    m0 = xd(1);
    m1 = xd(2);
    ms = xd(3);
    %
    vLine = V0*sin(2 * pi * t/tp);
    inpt = [iL; vLine];
    a11 = RSh * (1 - m0) + m0;
    a12 = 1;
    a21 = m0 * RLoad;
    a22 = RLoad + RSh;
    A = [a11, a12; a21, a22];
    b11 = RSh;
    b12 = 1;
    b21 = RLoad * RSh;
    b22 = RLoad;
    B = [b11, b12; b21, b22];
    tear = A\B * inpt;
    s = tear(1);
    vRLoad = tear(2);
    vTh = m0 * s;
    vLoad = vLine - vTh;
    vRSh = vLoad - vRLoad;
    iRSh = vRSh/RSh;
    iLoad = iL + iRSh;
    diL = vRSh/LLoad;
    %
    f = s;
    %
return
```

[H9.7] Thyristor XVII

We still need to discuss the *thyristor logic*. Let us check how the **Modelica Standard Library (MSL)** tackles the problem:

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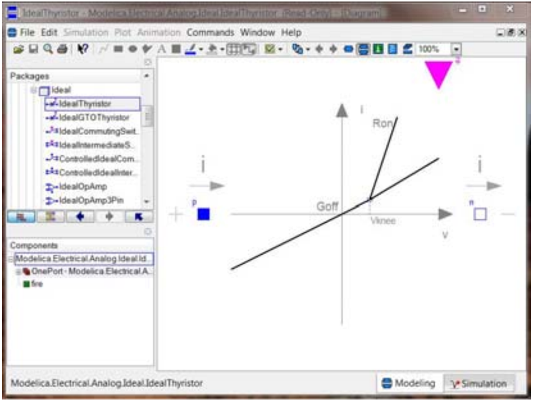

[H9.7] Thyristor XVII

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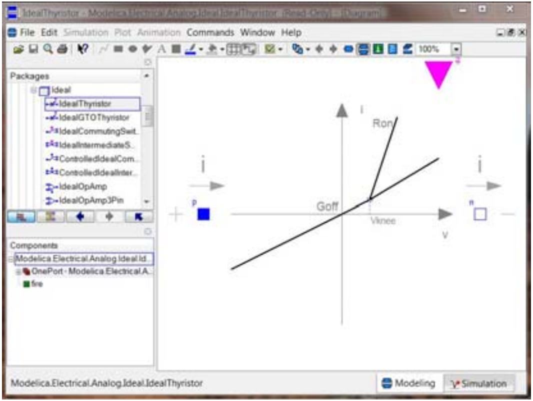
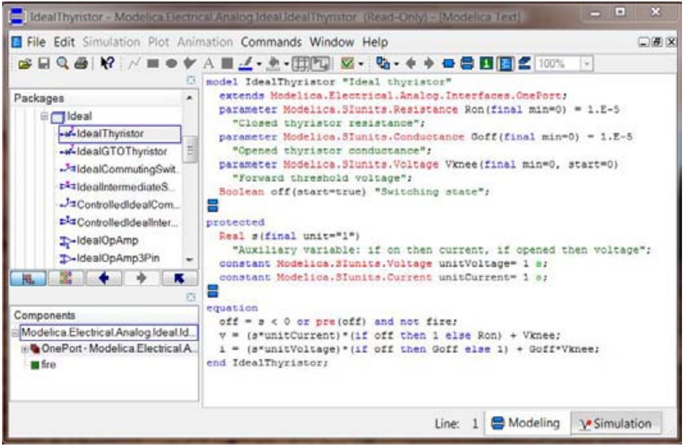


The MSL uses a *leaky diode*.

[H9.7] Thyristor XVIII



[H9.7] Thyristor XIX

Using our ideal diode:

$$
\begin{aligned}
off &= s < 0 \textbf{ or } \mathrm{pre}(off) \textbf{ and not } fire; \\
v_{Th} &= \textbf{if } off \textbf{ then } s \textbf{ else } 0; \\
i_{Load} &= \textbf{if } off \textbf{ then } 0 \textbf{ else } s;
\end{aligned}
$$

[H9.7] Thyristor XIX

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or in terms of our variables:

$$\begin{aligned} m_s &= s < 0; \\ m_0 &= m_s \textbf{ or } \text{pre}(m_0) \textbf{ and not } m_1; \\ v_{Th} &= \textbf{if } m_0 \textbf{ then } s \textbf{ else } 0; \\ i_{Load} &= \textbf{if } m_0 \textbf{ then } 0 \textbf{ else } s; \end{aligned}$$

[H9.7] Thyristor XIX

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$$

or in terms of our variables:

$$
\begin{aligned}
m_s &= s < 0; \\
m_0 &= m_s \textbf{ or } \mathrm{pre}(m_0) \textbf{ and not } m_1; \\
v_{Th} &= \textbf{if } m_0 \textbf{ then } s \textbf{ else } 0; \\
i_{Load} &= \textbf{if } m_0 \textbf{ then } 0 \textbf{ else } s;
\end{aligned}
$$

and using **Matlab's** pseudo-Boolean variables and functions:

$$
m_{0_{new}} = \mathrm{or}(m_s, \mathrm{and}(m_0, \mathrm{not}(m_1)));
$$

[H9.7] Thyristor XX

```
function [xdnew] = dst_eq(xc, xd, t, evt_nbr)
    %
    % Discrete event model of [H9.7]
    %
    p = 16 + 2/3;
    tp = 1/p;
    %
    iL = xc(1);
    m0 = xd(1);
    m1 = xd(2);
    ms = xd(3);
    %
    if evt_nbr == 1,
        %
        % The thyristor control is switched on
        % and the next event of the same type is scheduled
        %
        m1 = 1;
        push_evt(t + tp/2, 1);
    end
    %
    if evt_nbr == 2,
        %
        % The thyristor control is switched off
        % and the next event of the same type is scheduled
        %
        m1 = 0;
        push_evt(t + tp/2, 2);
    end
    if evt_nbr == -1,
        %
        % A state event has occurred
        % We need to toggle the diode switch
        %
        ms = not(ms);
    end
    %
    % We need to compute the new value of m0
    %
    m0 = or(ms,and(m0,not(m1)));
    %
    xdnew = zeros(3, 1);
    xdnew(1) = m0;
    xdnew(2) = m1;
    xdnew(3) = ms;
    %
return
```

[H9.7] Thyristor XXI