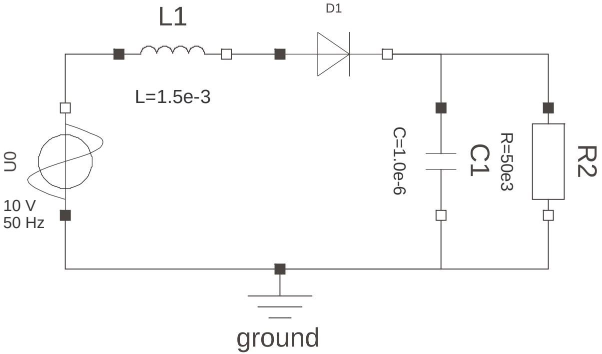
Numerical Simulation of Dynamic Systems: Hw12 - Solution

Prof. Dr. François E. Cellier
Department of Computer Science
ETH Zurich

May 28, 2013

[H9.14] Leaky Diode

Given the electrical circuit:



[H9.14] Leaky Diode II

- We wish to simulate the circuit using the rkf45rt routine developed in homework problem [H9.1].

[H9.14] Leaky Diode II

- We wish to simulate the circuit using the rkf45rt routine developed in homework problem [H9.1].
- Since the diode is in series with an inductor, the causality on the switch equation is fixed, and consequently, we are dealing here with a *variable structure system*.

[H9.14] Leaky Diode II

- We wish to simulate the circuit using the rkf45rt routine developed in homework problem [H9.1].
- Since the diode is in series with an inductor, the causality on the switch equation is fixed, and consequently, we are dealing here with a *variable structure system*.
- We shall be using the *leaky diode* approach to avoid the division by zero.

[H9.14] Leaky Diode II

- We wish to simulate the circuit using the rkf45rt routine developed in homework problem [H9.1].
- Since the diode is in series with an inductor, the causality on the switch equation is fixed, and consequently, we are dealing here with a *variable structure system*.
- We shall be using the *leaky diode* approach to avoid the division by zero.
- Choose $R_{on} = 10^{-4}\Omega$ and $G_{off} = 10^{-4} mho$, and simulate the circuit across 0.05 seconds of simulated time.

[H9.14] Leaky Diode II

- We wish to simulate the circuit using the rkf45rt routine developed in homework problem [H9.1].
- Since the diode is in series with an inductor, the causality on the switch equation is fixed, and consequently, we are dealing here with a *variable structure system*.
- We shall be using the *leaky diode* approach to avoid the division by zero.
- Choose $R_{on} = 10^{-4} \Omega$ and $G_{off} = 10^{-4} mho$, and simulate the circuit across 0.05 seconds of simulated time.
- Repeat the simulation, but this time around, choose $R_{on} = 10^{-5} \Omega$ and $G_{off} = 10^{-5} mho$.

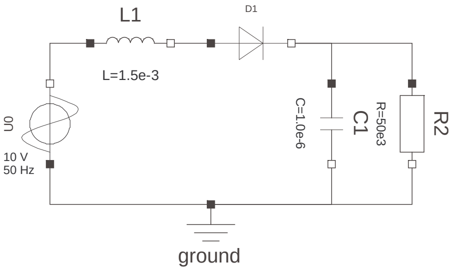
[H9.14] Leaky Diode II

- We wish to simulate the circuit using the rkf45rt routine developed in homework problem [H9.1].
- Since the diode is in series with an inductor, the causality on the switch equation is fixed, and consequently, we are dealing here with a *variable structure system*.
- We shall be using the *leaky diode* approach to avoid the division by zero.
- Choose $R_{on} = 10^{-4}\Omega$ and $G_{off} = 10^{-4} mho$, and simulate the circuit across 0.05 seconds of simulated time.
- Repeat the simulation, but this time around, choose $R_{on} = 10^{-5}\Omega$ and $G_{off} = 10^{-5} mho$.
- Split the screen into two subgraphs, and plot in the top subgraph the voltages across the capacitor together from the two simulation runs, and on the bottom subgraph the step sizes used by the two simulation runs.

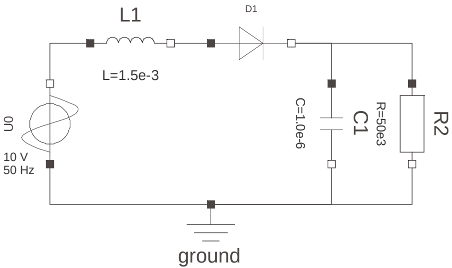
[H9.14] Leaky Diode II

- We wish to simulate the circuit using the rkf45rt routine developed in homework problem [H9.1].
- Since the diode is in series with an inductor, the causality on the switch equation is fixed, and consequently, we are dealing here with a *variable structure system*.
- We shall be using the *leaky diode* approach to avoid the division by zero.
- Choose $R_{on} = 10^{-4}\Omega$ and $G_{off} = 10^{-4} mho$, and simulate the circuit across 0.05 seconds of simulated time.
- Repeat the simulation, but this time around, choose $R_{on} = 10^{-5}\Omega$ and $G_{off} = 10^{-5} mho$.
- Split the screen into two subgraphs, and plot in the top subgraph the voltages across the capacitor together from the two simulation runs, and on the bottom subgraph the step sizes used by the two simulation runs.
- What do you conclude?

[H9.14] Leaky Diode III

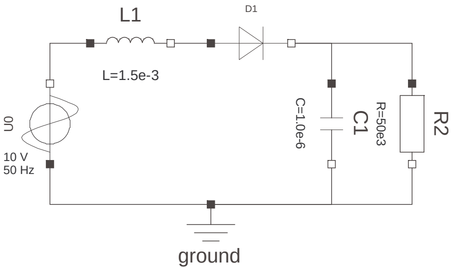


[H9.14] Leaky Diode III



$$
\begin{aligned}
1:&\quad U_0 = V_0 \cdot \sin\left(\frac{2\pi t}{t_p}\right)\\
2:&\quad u_L = L_1 \cdot \frac{di_0}{dt}\\
3:&\quad i_C = C_1 \cdot \frac{du_R}{dt}\\
4:&\quad u_R = R_2 \cdot i_R\\
5:&\quad U_0 = u_L + u_D + u_R\\
6:&\quad i_0 = i_C + i_R\\
7:&\quad u_D = [m_0 + (1 - m_0) \cdot R_{on}] \cdot s\\
8:&\quad i_0 = [m_0 \cdot G_{off} + (1 - m_0)] \cdot s
\end{aligned}
$$

[H9.14] Leaky Diode III



$$
\begin{aligned}
1:&\quad U_0 = V_0 \cdot \sin(\tfrac{2\pi t}{t_p}) \\
2:&\quad u_L = L_1 \cdot \frac{di_0}{dt} \\
3:&\quad i_C = C_1 \cdot \frac{du_R}{dt} \\
4:&\quad u_R = R_2 \cdot i_R \\
5:&\quad U_0 = u_L + u_D + u_R \\
6:&\quad i_0 = i_C + i_R \\
7:&\quad u_D = [m_0 + (1 - m_0) \cdot R_{on}] \cdot s \\
8:&\quad i_0 = [m_0 \cdot G_{off} + (1 - m_0)] \cdot s
\end{aligned}
$$

m_0 is a discrete state variable. It is *true*, when the diode is *blocking*.

[H9.14] Leaky Diode IV

1: $U_0 = V_0 \cdot \sin(\frac{2\pi t}{t_p})$
2: $u_L = L_1 \cdot \frac{di_0}{dt}$
3: $i_C = C_1 \cdot \frac{du_R}{dt}$
4: $u_R = R_2 \cdot i_R$

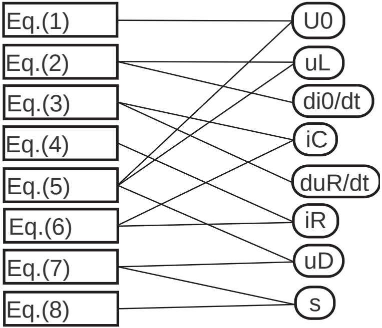
5: $U_0 = u_L + u_D + u_R$

6: $i_0 = i_C + i_R$

7: $u_D = [m_0 + (1 - m_0) \cdot R_{on}] \cdot s$
8: $i_0 = [m_0 \cdot G_{off} + (1 - m_0)] \cdot s$

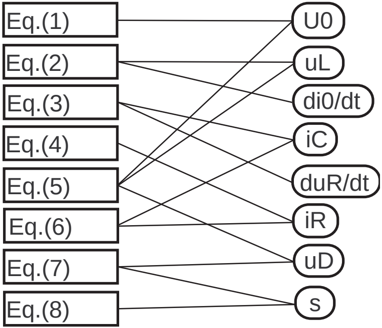
[H9.14] Leaky Diode IV

$$
\begin{array}{llll}
1: & U_0 & = & V_0 \cdot \sin(\frac{2\pi t}{t_p}) \\
2: & u_L & = & L_1 \cdot \frac{di_0}{dt} \\
3: & i_C & = & C_1 \cdot \frac{du_R}{dt} \\
4: & u_R & = & R_2 \cdot i_R \\
5: & U_0 & = & u_L + u_D + u_R \\
6: & i_0 & = & i_C + i_R \\
7: & u_D & = & [m_0 + (1 - m_0) \cdot R_{on}] \cdot s \\
8: & i_0 & = & [m_0 \cdot G_{off} + (1 - m_0)] \cdot s
\end{array}
$$



[H9.14] Leaky Diode IV

$$
\begin{aligned}
1:&\quad U_0 &=&\quad V_0 \cdot \sin\left(\frac{2\pi t}{t_p}\right)\\
2:&\quad u_L &=&\quad L_1 \cdot \frac{di_0}{dt}\\
3:&\quad i_C &=&\quad C_1 \cdot \frac{du_R}{dt}\\
4:&\quad u_R &=&\quad R_2 \cdot i_R\\
5:&\quad U_0 &=&\quad u_L + u_D + u_R\\
6:&\quad i_0 &=&\quad i_C + i_R\\
7:&\quad u_D &=&\quad [m_0 + (1 - m_0) \cdot R_{on}] \cdot s\\
8:&\quad i_0 &=&\quad [m_0 \cdot G_{off} + (1 - m_0)] \cdot s
\end{aligned}
$$



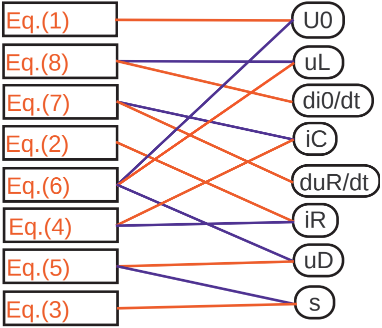
As expected, variable s does not show up inside an algebraic loop. It must be computed from the last equation.

[H9.14] Leaky Diode V

We causalize as much as we can:

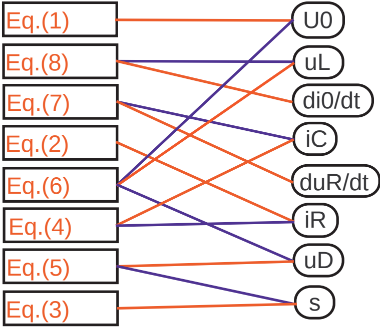
[H9.14] Leaky Diode V

We causalize as much as we can:



[H9.14] Leaky Diode V

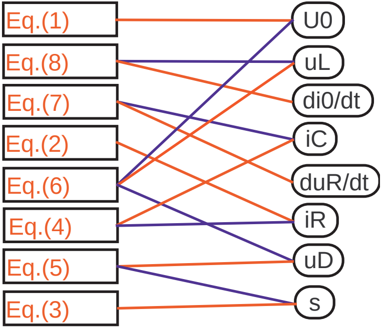
We causalize as much as we can:



$$
\begin{aligned}
1:&\quad U_0 &=&\quad V_0 \cdot \sin\left(\frac{2\pi t}{t_p}\right)\\
2:&\quad i_R &=&\quad \frac{1}{R_2} \cdot u_R\\
3:&\quad s &=&\quad \frac{1}{m_0 \cdot G_{off} + (1 - m_0)} \cdot i_0\\
4:&\quad i_C &=&\quad i_0 - i_R\\
5:&\quad u_D &=&\quad [m_0 + (1 - m_0) \cdot R_{on}] \cdot s\\
6:&\quad u_L &=&\quad U_0 - u_D - u_R\\
7:&\quad \frac{du_R}{dt} &=&\quad \frac{1}{C_1} \cdot i_C\\
8:&\quad \frac{di_0}{dt} &=&\quad \frac{1}{L_1} \cdot u_L
\end{aligned}
$$

[H9.14] Leaky Diode V

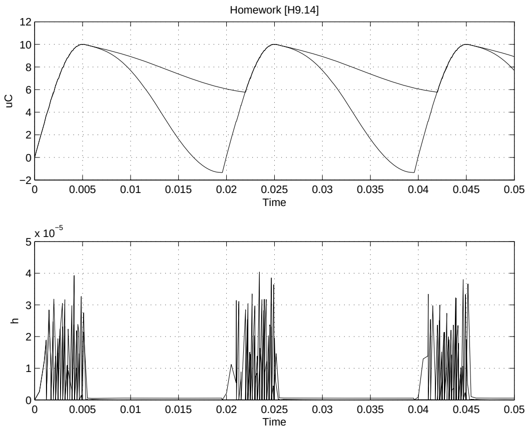
We causalize as much as we can:



$$
\begin{aligned}
1:&\quad U_0 = V_0 \cdot \sin\left(\frac{2\pi t}{t_p}\right)\\
2:&\quad i_R = \frac{1}{R_2} \cdot u_R\\
3:&\quad s = \frac{1}{m_0 \cdot G_{off} + (1 - m_0)} \cdot i_0\\
4:&\quad i_C = i_0 - i_R\\
5:&\quad u_D = [m_0 + (1 - m_0) \cdot R_{on}] \cdot s\\
6:&\quad u_L = U_0 - u_D - u_R\\
7:&\quad \frac{du_R}{dt} = \frac{1}{C_1} \cdot i_C\\
8:&\quad \frac{di_0}{dt} = \frac{1}{L_1} \cdot u_L
\end{aligned}
$$

We were able to causalize all equations at once. The system feels like an *index-0 system*. However with an ideal diode, we would be confronted with a *conditional index change*.

[H9.14] Leaky Diode VI



[H9.14] Leaky Diode VII

- We notice that the simulation took forever. When the diode is blocking, an ideal diode would have led to a division by zero, whereas the leaky diode leads to an incredibly stiff model. **We should have used a stiff system solver!**

[H9.14] Leaky Diode VII

- We notice that the simulation took forever. When the diode is blocking, an ideal diode would have led to a division by zero, whereas the leaky diode leads to an incredibly stiff model. **We should have used a stiff system solver!**

- The simulation results look vastly different. Depending on the value of a fudge parameter that has no direct physical meaning, we got simulation trajectories that look significantly different.

[H9.14] Leaky Diode VII

- We notice that the simulation took forever. When the diode is blocking, an ideal diode would have led to a division by zero, whereas the leaky diode leads to an incredibly stiff model. **We should have used a stiff system solver!**

- The simulation results look vastly different. Depending on the value of a fudge parameter that has no direct physical meaning, we got simulation trajectories that look significantly different.

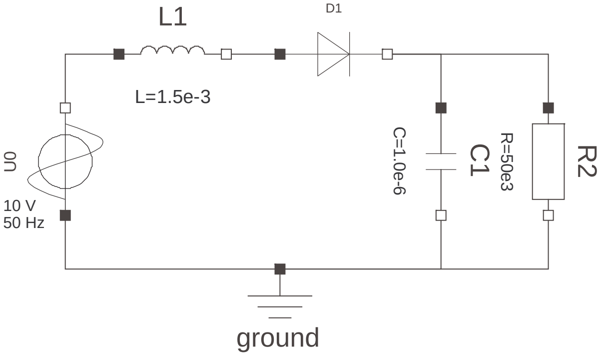
- **This simulation is garbage!** It should never happen that the simulation trajectories are highly sensitive to a non-physical fudge parameter.

[H9.14] Leaky Diode VII

- We notice that the simulation took forever. When the diode is blocking, an ideal diode would have led to a division by zero, whereas the leaky diode leads to an incredibly stiff model. **We should have used a stiff system solver!**
- The simulation results look vastly different. Depending on the value of a fudge parameter that has no direct physical meaning, we got simulation trajectories that look significantly different.
- **This simulation is garbage!** It should never happen that the simulation trajectories are highly sensitive to a non-physical fudge parameter.
- **The circuit is garbage!** A real diode always has residual resistance values. If we build this circuit, its behavior will depend on how well we solder the diode into the circuit. Thus, we cannot expect to get any level of reproducibility out of this circuit. Two different specimen will behave very differently.

[H9.15] Mixed-mode Integration

Given the electrical circuit:



[H9.15] Mixed-mode Integration II

- We wish to simulate the circuit using the rkf45rt routine developed in homework problem [H9.1].

[H9.15] Mixed-mode Integration II

- We wish to simulate the circuit using the rkf45rt routine developed in homework problem [H9.1].
- Since the diode is in series with an inductor, the causality on the switch equation is fixed, and consequently, we are dealing here with a *variable structure system*.

[H9.15] Mixed-mode Integration II

- We wish to simulate the circuit using the rkf45rt routine developed in homework problem [H9.1].
- Since the diode is in series with an inductor, the causality on the switch equation is fixed, and consequently, we are dealing here with a *variable structure system*.
- We shall be using the *mixed-mode integration* approach to avoid the division by zero.

[H9.15] Mixed-mode Integration II

- We wish to simulate the circuit using the rkf45rt routine developed in homework problem [H9.1].
- Since the diode is in series with an inductor, the causality on the switch equation is fixed, and consequently, we are dealing here with a *variable structure system*.
- We shall be using the *mixed-mode integration* approach to avoid the division by zero.
- Inline the inductor using backward Euler. Step-size control will now be limited to controlling the single *continuous state variable* defined by the voltage across the capacitor. The current through the inductor is now a *discrete state variable*. The step size of the backward Euler integrator will simply be in sync with that of the RKF4/5 algorithm.

[H9.15] Mixed-mode Integration II

- We wish to simulate the circuit using the rkf45rt routine developed in homework problem [H9.1].
- Since the diode is in series with an inductor, the causality on the switch equation is fixed, and consequently, we are dealing here with a *variable structure system*.
- We shall be using the *mixed-mode integration* approach to avoid the division by zero.
- Inline the inductor using backward Euler. Step-size control will now be limited to controlling the single *continuous state variable* defined by the voltage across the capacitor. The current through the inductor is now a *discrete state variable*. The step size of the backward Euler integrator will simply be in sync with that of the RKF4/5 algorithm.
- Simulate the circuit across 0.05 seconds of simulated time.

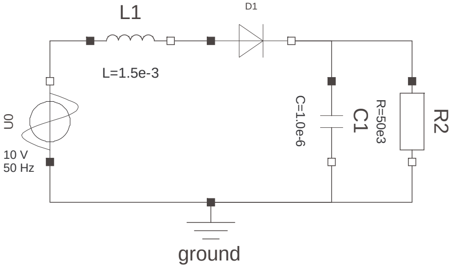
[H9.15] Mixed-mode Integration III

- Split the screen into two subgraphs, and plot in the top subgraph the voltages across the capacitor together from the current simulation run and from the more precise run of homework problem [H9.14], and on the bottom subgraph the step sizes used by the current simulation run and the more precise run of homework problem [H9.14].

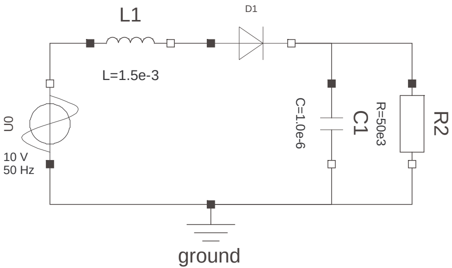
[H9.15] Mixed-mode Integration III

- Split the screen into two subgraphs, and plot in the top subgraph the voltages across the capacitor together from the current simulation run and from the more precise run of homework problem [H9.14], and on the bottom subgraph the step sizes used by the current simulation run and the more precise run of homework problem [H9.14].
- What do you conclude?

[H9.15] Mixed-mode Integration IV

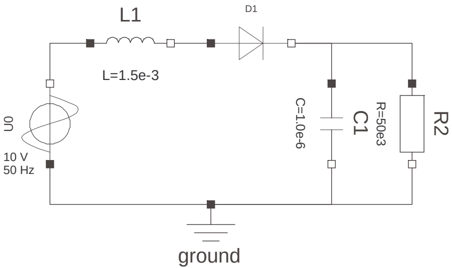


[H9.15] Mixed-mode Integration IV



$$
\begin{aligned}
1:&\quad U_0 &=&\quad V_0 \cdot \sin\left(\frac{2\pi t}{t_p}\right)\\
2:&\quad u_L &=&\quad L_1 \cdot di_0\\
3:&\quad i_0 &=&\quad \mathrm{pre}(i_0) + h \cdot di_0\\
4:&\quad i_C &=&\quad C_1 \cdot \frac{du_R}{dt}\\
5:&\quad u_R &=&\quad R_2 \cdot i_R\\
6:&\quad U_0 &=&\quad u_L + u_D + u_R\\
7:&\quad i_0 &=&\quad i_C + i_R\\
8:&\quad u_D &=&\quad m_0 \cdot s\\
9:&\quad i_0 &=&\quad (1 - m_0) \cdot s
\end{aligned}
$$

[H9.15] Mixed-mode Integration IV



$$
\begin{aligned}
1: &\quad U_0 = V_0 \cdot \sin\left(\frac{2\pi t}{t_p}\right)\\
2: &\quad u_L = L_1 \cdot di_0\\
3: &\quad i_0 = \mathrm{pre}(i_0) + h \cdot di_0\\
4: &\quad i_C = C_1 \cdot \frac{du_R}{dt}\\
5: &\quad u_R = R_2 \cdot i_R\\
6: &\quad U_0 = u_L + u_D + u_R\\
7: &\quad i_0 = i_C + i_R\\
8: &\quad u_D = m_0 \cdot s\\
9: &\quad i_0 = (1 - m_0) \cdot s
\end{aligned}
$$

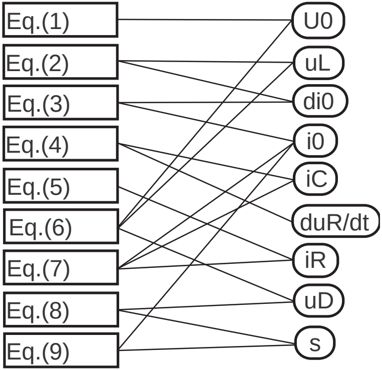
m_0 is a discrete state variable. It is *true*, when the diode is *blocking*.

[H9.15] Mixed-mode Integration V

$$
\begin{aligned}
1:&\quad U_0 = V_0 \cdot \sin(\frac{2\pi t}{t_p}) \\
2:&\quad u_L = L_1 \cdot di_0 \\
3:&\quad i_0 = \text{pre}(i_0) + h \cdot di_0 \\
4:&\quad i_C = C_1 \cdot \frac{du_R}{dt} \\
5:&\quad u_R = R_2 \cdot i_R \\
\\
6:&\quad U_0 = u_L + u_D + u_R \\
\\
7:&\quad i_0 = i_C + i_R \\
\\
8:&\quad u_D = m_0 \cdot s \\
9:&\quad i_0 = (1 - m_0) \cdot s
\end{aligned}
$$

[H9.15] Mixed-mode Integration V

$$
\begin{aligned}
1:&\quad U_0 = V_0 \cdot \sin\left(\frac{2\pi t}{t_p}\right)\\
2:&\quad u_L = L_1 \cdot di_0\\
3:&\quad i_0 = \mathrm{pre}(i_0) + h \cdot di_0\\
4:&\quad i_C = C_1 \cdot \frac{du_R}{dt}\\
5:&\quad u_R = R_2 \cdot i_R\\
6:&\quad U_0 = u_L + u_D + u_R\\
7:&\quad i_0 = i_C + i_R\\
8:&\quad u_D = m_0 \cdot s\\
9:&\quad i_0 = (1 - m_0) \cdot s
\end{aligned}
$$

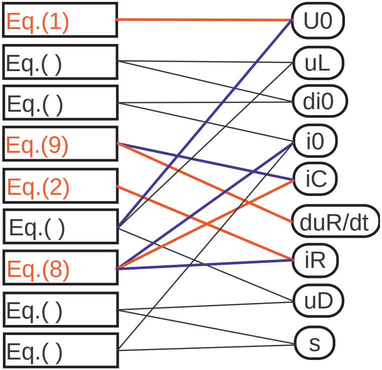


[H9.15] Mixed-mode Integration VI

We causalize as much as we can:

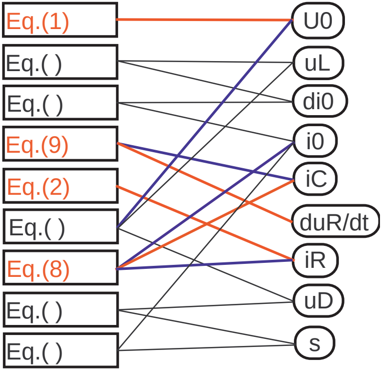
[H9.15] Mixed-mode Integration VI

We causalize as much as we can:



[H9.15] Mixed-mode Integration VI

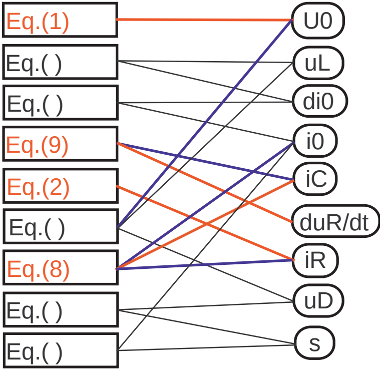
We causalize as much as we can:



$$
\begin{aligned}
&1: && U_0 &&= V_0 \cdot \sin\left(\frac{2\pi t}{t_p}\right) \\
&?: && u_L &&= L_1 \cdot di_0 \\
&?: && i_0 &&= \mathrm{pre}(i_0) + h \cdot di_0 \\
&9: && i_C &&= C_1 \cdot \frac{du_R}{dt} \\
&2: && u_R &&= R_2 \cdot i_R \\
&?: && U_0 &&= u_L + u_D + u_R \\
&8: && i_0 &&= i_C + i_R \\
&?: && u_D &&= m_0 \cdot s \\
&?: && i_0 &&= (1 - m_0) \cdot s
\end{aligned}
$$

[H9.15] Mixed-mode Integration VI

We causalize as much as we can:



$$
\begin{array}{llcl}
1: & U_0 & = & V_0 \cdot \sin(\frac{2\pi t}{t_p}) \\
?: & u_L & = & L_1 \cdot di_0 \\
?: & i_0 & = & \mathrm{pre}(i_0) + h \cdot di_0 \\
9: & i_C & = & C_1 \cdot \frac{du_R}{dt} \\
2: & u_R & = & R_2 \cdot i_R \\
 & & & \\
?: & U_0 & = & u_L + u_D + u_R \\
 & & & \\
8: & i_0 & = & i_C + i_R \\
 & & & \\
?: & u_D & = & m_0 \cdot s \\
?: & i_0 & = & (1 - m_0) \cdot s
\end{array}
$$

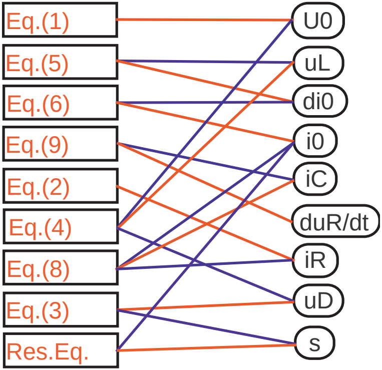
We ended up with an algebraic loop in five equations and five unknowns. Luckily, the switch equation (variable s) is inside the algebraic loop.

[H9.15] Mixed-mode Integration VII

We choose s as our tearing variable:

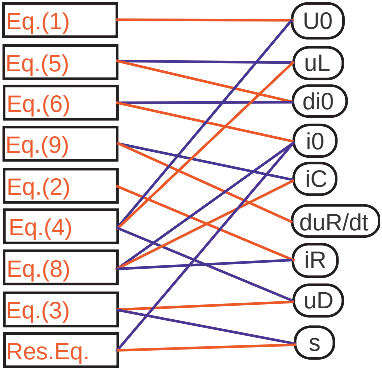
[H9.15] Mixed-mode Integration VII

We choose *s* as our tearing variable:



[H9.15] Mixed-mode Integration VII

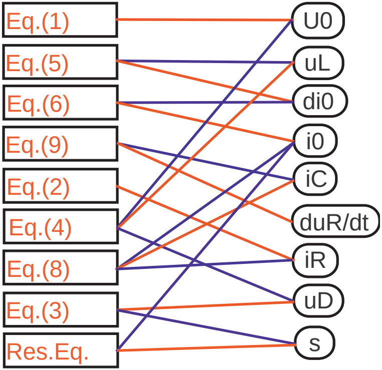
We choose s as our tearing variable:



1:	U_0	$=$	$V_0 \cdot \sin(\frac{2\pi t}{t_p})$
2:	i_R	$=$	$\frac{1}{R_2} \cdot u_R$
3:	u_D	$=$	$m_0 \cdot s$
4:	u_L	$=$	$U_0 - u_D - u_R$
5:	di_0	$=$	$\frac{1}{L_1} \cdot u_L$
6:	i_0	$=$	$\mathrm{pre}(i_0) + h \cdot di_0$
res.eq.:	s	$=$	$\frac{1}{1-m_0} \cdot i_0$
8:	i_C	$=$	$i_0 - i_R$
9:	$\frac{du_R}{dt}$	$=$	$\frac{1}{C_1} \cdot i_C$

[H9.15] Mixed-mode Integration VII

We choose s as our tearing variable:



1:	U_0	$=$	$V_0 \cdot \sin(\frac{2\pi t}{t_p})$
2:	i_R	$=$	$\frac{1}{R_2} \cdot u_R$
3:	u_D	$=$	$m_0 \cdot s$
4:	u_L	$=$	$U_0 - u_D - u_R$
5:	di_0	$=$	$\frac{1}{L_1} \cdot u_L$
6:	i_0	$=$	$\text{pre}(i_0) + h \cdot di_0$
res.eq.:	s	$=$	$\frac{1}{1-m_0} \cdot i_0$
8:	i_C	$=$	$i_0 - i_R$
9:	$\frac{du_R}{dt}$	$=$	$\frac{1}{C_1} \cdot i_C$

We were able to causalize all of the remaining equations in this way.

[H9.15] Mixed-mode Integration VIII

Substitution gives us the completely causal set of simulation equations:

$$
\begin{array}{llcl}
1: & U_0 & = & V_0 \cdot \sin(\frac{2\pi t}{t_p}) \\
2: & i_R & = & \frac{1}{R_2} \cdot u_R \\
3: & s & = & \frac{L_1 \cdot \mathrm{pre}(i_0) + h \cdot (U_0 - u_R)}{L_1 \cdot (1 - m_0) + h \cdot m_0} \\
4: & u_D & = & m_0 \cdot s \\
5: & u_L & = & U_0 - u_D - u_R \\
6: & di_0 & = & \frac{1}{L_1} \cdot u_L \\
7: & i_0 & = & \mathrm{pre}(i_0) + h \cdot di_0 \\
8: & i_C & = & i_0 - i_R \\
9: & \frac{du_R}{dt} & = & \frac{1}{C_1} \cdot i_C
\end{array}
$$

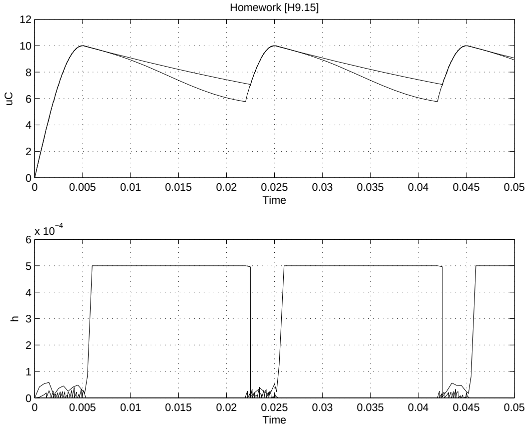
[H9.15] Mixed-mode Integration VIII

Substitution gives us the completely causal set of simulation equations:

$$
\begin{array}{llcl}
1: & U_0 & = & V_0 \cdot \sin(\frac{2\pi t}{t_p}) \\
2: & i_R & = & \frac{1}{R_2} \cdot u_R \\
3: & s & = & \frac{L_1 \cdot \mathrm{pre}(i_0) + h \cdot (U_0 - u_R)}{L_1 \cdot (1 - m_0) + h \cdot m_0} \\
4: & u_D & = & m_0 \cdot s \\
5: & u_L & = & U_0 - u_D - u_R \\
6: & di_0 & = & \frac{1}{L_1} \cdot u_L \\
7: & i_0 & = & \mathrm{pre}(i_0) + h \cdot di_0 \\
8: & i_C & = & i_0 - i_R \\
9: & \frac{du_R}{dt} & = & \frac{1}{C_1} \cdot i_C
\end{array}
$$

If the diode operates in its blocking mode ($m_0 = 1$), we unfortunately end up with h in the denominator of the equation computing s.

[H9.15] Mixed-mode Integration IX



[H9.15] Mixed-mode Integration X

- The new simulation executes very fast. There is no stiffness problem at all.

[H9.15] Mixed-mode Integration X

- The new simulation executes very fast. There is no stiffness problem at all.
- During the time, when the diode is blocking, the step size increases to the maximum value allowed, i.e., it increases to the communication interval.

[H9.15] Mixed-mode Integration X

- The new simulation executes very fast. There is no stiffness problem at all.
- During the time, when the diode is blocking, the step size increases to the maximum value allowed, i.e., it increases to the communication interval.
- The simulation results look once again different. I would have needed to use values of $R_{on} = 10^{-10}\Omega$ and $G_{off} = 10^{-10} mho$, in order to get simulation results out of the former model that look similar to those of the latter model. However, this would have been out of the question without using a stiff system solver. My notebook would have died of old age, before that simulation would have ended.

References

1. Cellier, F.E. and M. Krebs (2007), "Analysis and Simulation of Variable Structure Systems Using Bond Graphs and Inline Integration," *Proc. ICBGM07, 8th SCS Intl. Conf. on Bond Graph Modeling and Simulation*, San Diego, California, pp. 29-34.