Numerical Simulation of Dynamic Systems: Hw2 - Solution

Prof. Dr. François E. Cellier
Department of Computer Science
ETH Zurich

March 12, 2013

[H2.1] Marginal Stability

Given the following linear time–invariant continuous–time system:

$$\dot{\mathbf{x}} = \begin{pmatrix} 1250 & -25113 & -60050 & -42647 & -23999 \\ 500 & -10068 & -24057 & -17092 & -9613 \\ 250 & -5060 & -12079 & -8586 & -4826 \\ -750 & 15101 & 36086 & 25637 & 14420 \\ 250 & -4963 & -11896 & -8438 & -4756 \end{pmatrix} \cdot \mathbf{x} + \begin{pmatrix} 5 \\ 2 \\ 1 \\ -3 \\ 1 \end{pmatrix} \cdot u$$

$$\mathbf{y} = \begin{pmatrix} -1 & 26 & 59 & 43 & 23 \end{pmatrix} \cdot \mathbf{x}$$

with initial conditions:

$$\mathbf{x_0} = \begin{pmatrix} 1 \\ -2 \\ 3 \\ -4 \\ 5 \end{pmatrix}$$

[H2.1] Marginal Stability II

Determine the step size, h_{marg}, for which FE will give marginally stable results.

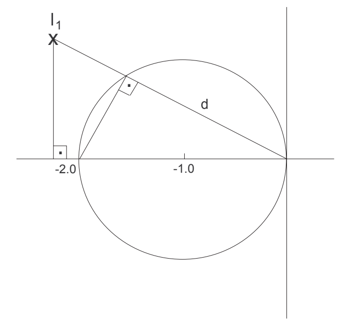
Simulate the system across 10 seconds of simulated time with step input using the FE algorithm with the following step sizes:

1. $h = 0.1 \cdot h_{\text{marg}}$,
2. $h = 0.95 \cdot h_{\text{marg}}$,
3. $h = h_{\text{marg}}$,
4. $h = 1.05 \cdot h_{\text{marg}}$, and
5. $h = 2 \cdot h_{\text{marg}}$.

Discuss the results.

[H2.1] Marginal Stability III

Determine the step size, h_{marg}, for which FE will give marginally stable results.



$$d : (-2) = \mathbb{R}\text{e}(\lambda_1) : |\lambda_1|$$

Thus:

$$h_{marg} = \min_{\forall i}\left(\frac{d_i}{|\lambda_i|}\right) = \min_{\forall i}\left(\frac{-2 \cdot \mathbb{R}\text{e}(\lambda_i)}{|\lambda_i|^2}\right)$$

$$\Rightarrow \quad l = \text{eig}(A); \quad hmarg = \min(-2 * \text{real}(l) \ ./ \ (\text{abs}(l) \ .* \ \text{abs}(l)));$$

[H2.1] Marginal Stability IV

We find:

```
l = eig(A)

l =

   -4.0000 + 3.0000i
   -4.0000 - 3.0000i
   -5.0000
   -2.0000
   -1.0000
```

All eigenvalues are in the left-half complex plane, i.e., the system is analytically stable.

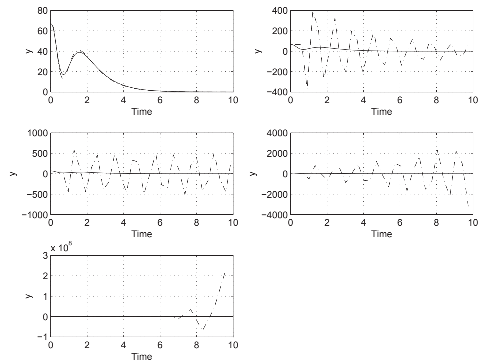
```
hmarg =

   0.3200
```

The numerical solution is marginally stable for $h_{marg} = 0.32$.

[H2.1] Marginal Stability V

Simulation results:



Even for $h = 0.032 = 0.1 \cdot h_{marg}$, the solution is not very accurate.

The FE algorithm requires very small step sizes in order to generate accurate simulation results.

[H2.5] Stability Domain

For the *predictor-corrector FE-BE method*, find the stability domains if:

1. no corrector is used,
2. one corrector is used,
3. two correctors are used,
4. three correctors are used, and
5. four correctors are used.

Plot the five stability domains on top of each other, and discuss the results.

[H2.5] Stability Domain II

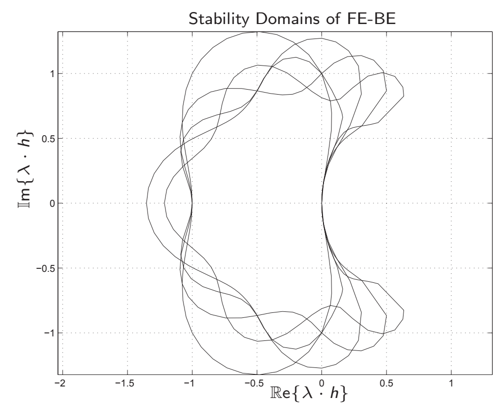


Figure: Stability domains of FE-BE algorithms