Numerical Simulation of Dynamic Systems: Hw4 - Problem

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[H3.15] Backinterpolation With Step-Size Control

We want to repeat Hw.[H3.14] once more, this time using a step-size controlled algorithm. The step-size control to be used is the following. On the *explicit semi-step*, compute now both correctors, and find ε_{rel} according to the formula:

$$\varepsilon_{\text{rel}} = \frac{\|\mathbf{x_1} - \mathbf{x_2}\|_\infty}{\max(\|\mathbf{x_1}\|_2, \|\mathbf{x_2}\|_2, \delta)}$$

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If $\varepsilon_{\text{rel}} \leq 10^{-4}$, use the Gustafsson algorithm to compute the step size to be used in the next step:

$$h_{\text{new}} = \left(\frac{0.8 \cdot 10^{-4}}{\varepsilon_{\text{rel}_{\text{now}}}}\right)^{0.06} \cdot \left(\frac{\varepsilon_{\text{rel}_{\text{last}}}}{\varepsilon_{\text{rel}_{\text{now}}}}\right)^{0.08} \cdot h_{\text{old}}$$

except during the first step, when we use:

$$h_{\text{new}} = \left(\frac{0.8 \cdot 10^{-4}}{\varepsilon_{\text{rel}_{\text{now}}}}\right)^{0.2} \cdot h_{\text{old}}$$

[H3.15] Backinterpolation With Step-Size Control II

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Apply this step-size control algorithm to the same problem as before, and determine the largest global relative error by comparing the solution with the analytical solution of this linear time-invariant system.

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For this problem, it may be easier to use MATLAB's contour plot, than your own domain tracking routine.