Numerical Simulation of Dynamic Systems: Hw4 - Problem

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[H3.15] Backinterpolation With Step-Size Control

We want to repeat Hw.[H3.14] once more, this time using a step-size controlled algorithm. The step-size control to be used is the following. On the *explicit semi-step*, compute now both correctors, and find ε_{rel} according to the formula:

$$\varepsilon_{\text{rel}} = \frac{\|\mathbf{x_1} - \mathbf{x_2}\|_\infty}{\max(\|\mathbf{x_1}\|_2, \|\mathbf{x_2}\|_2, \delta)}$$

If $\varepsilon_{\text{rel}} \leq 10^{-4}$, use the Gustafsson algorithm to compute the step size to be used in the next step:

$$h_{\text{new}} = \left(\frac{0.8 \cdot 10^{-4}}{\varepsilon_{\text{rel}_{\text{now}}}}\right)^{0.06} \cdot \left(\frac{\varepsilon_{\text{rel}_{\text{last}}}}{\varepsilon_{\text{rel}_{\text{now}}}}\right)^{0.08} \cdot h_{\text{old}}$$

except during the first step, when we use:

$$h_{\text{new}} = \left(\frac{0.8 \cdot 10^{-4}}{\varepsilon_{\text{rel}_{\text{now}}}}\right)^{0.2} \cdot h_{\text{old}}$$

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However, if $\varepsilon_{\text{rel}} > 10^{-4}$, we reject the step at once, i.e., we never even proceed to the implicit semi-step, and compute a new step size in accordance with the same equation as during the first step.

If a step was repeated, the step size for the immediately following next step is also computed according to that equation.

Apply this step-size control algorithm to the same problem as before, and determine the largest global relative error by comparing the solution with the analytical solution of this linear time-invariant system.

[H3.19] Order Star

Find the *damping order star* for $\mathbf{BI4}/\mathbf{5}_{0.45}$, and plot it together with the pole and zero locations. Compare with the damping order star of **BI4** that was shown in class.

Find the *frequency order star* for $\mathbf{BI4}/\mathbf{5}_{0.45}$, and plot it together with the pole and zero locations. Compare with the frequency order star **BI4** that was shown in class.

Finally, compute and plot the *order star accuracy domain* of this method.

For this problem, it may be easier to use MATLAB's contour plot, than your own domain tracking routine.