Numerical Simulation of Dynamic Systems: Hw5 - Problem

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[H4.2] Nyström-Milne Predictor-Corrector Techniques

Follow the reasoning of the Adams-Bashforth-Moulton predictor-corrector techniques, and develop similar pairs of algorithms using a Nyström predictor stage followed by a Milne corrector stage.

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Plot the stability domains for NyMi3 and NyMi4. What do you conclude?

[H4.4] Milne Integration

Usually, the term "Milne integration algorithm," when used in the literature, denotes a specific predictor-corrector technique, namely:

predictor:

$$\dot{\mathbf{x}}_{\mathbf{k}} = \mathbf{f}(\mathbf{x}_{\mathbf{k}}, t_k)$$
$$\mathbf{x}^{\mathbf{P}}_{\mathbf{k+1}} = \mathbf{x}_{\mathbf{k-3}} + \frac{h}{3}(8\dot{\mathbf{x}}_{\mathbf{k}} - 4\dot{\mathbf{x}}_{\mathbf{k-1}} + 8\dot{\mathbf{x}}_{\mathbf{k-2}})$$

corrector:

$$\dot{\mathbf{x}}^{\mathbf{P}}_{\mathbf{k+1}} = \mathbf{f}(\mathbf{x}^{\mathbf{P}}_{\mathbf{k+1}}, t_{k+1})$$
$$\mathbf{x}^{\mathbf{C}}_{\mathbf{k+1}} = \mathbf{x}_{\mathbf{k-1}} + \frac{h}{3}(\dot{\mathbf{x}}^{\mathbf{P}}_{\mathbf{k+1}} + 4\dot{\mathbf{x}}_{\mathbf{k}} + \dot{\mathbf{x}}_{\mathbf{k-1}})$$

The corrector is clearly *Simpson's rule*. However, the predictor is something new that we haven't seen yet.

[H4.4] Milne Integration II

Derive the order of approximation accuracy of the predictor. To this end, use the Newton-Gregory backward polynomial in order to derive a set of formulae with a distance of four steps apart between their two state values.

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Derive the order of approximation accuracy of the predictor. To this end, use the Newton-Gregory backward polynomial in order to derive a set of formulae with a distance of four steps apart between their two state values.

Plot the stability domain of the predictor-corrector method, and compare it with that of NyMi4. What do you conclude? Why did *William E. Milne* propose to use this particular predictor?

[H4.10] The Nordsieck Form

In the class presentations, I showed the transformation matrix that converts the state history vector into an equivalent *Nordsieck vector*. Since, at the time of conversion, we also have the current state derivative information available, it is more common to drop the oldest state information in the state history vector, and replace it by the current state derivative information. Consequently, we are looking for a transformation matrix **T** of the form:

$$\begin{pmatrix} x_k \\ h \cdot \dot{x}_k \\ \frac{h^2}{2} \cdot \ddot{x}_k \\ \frac{h^3}{6} \cdot x_k^{(\mathrm{iii})} \end{pmatrix} = \mathbf{T} \cdot \begin{pmatrix} x_k \\ h \cdot \dot{x}_k \\ x_{k-1} \\ x_{k-2} \end{pmatrix}$$

The matrix **T** can easily be found by manipulating the individual equations of the transformation matrix shown in class.

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Find corresponding **T**-matrices of dimensions 3×3 and 5×5.