Numerical Simulation of Dynamic Systems: Hw9 - Problem

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[H7.1] Electrical Circuit, Horizontal and Vertical Sorting

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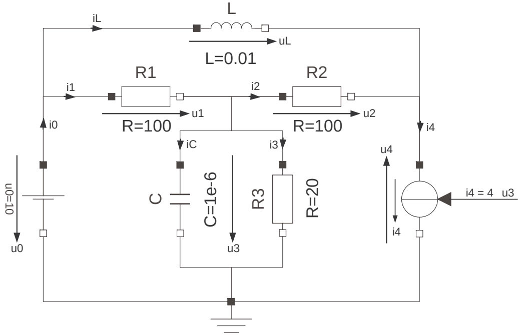

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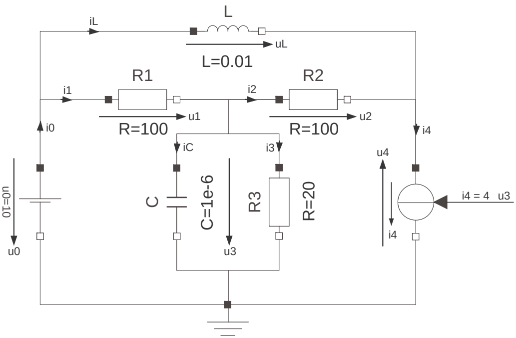

- The circuit contains a constant voltage source, u_0, and a dependent current source, i_4, that depends on the voltage across the capacitor, C, and the resistor, R_3.

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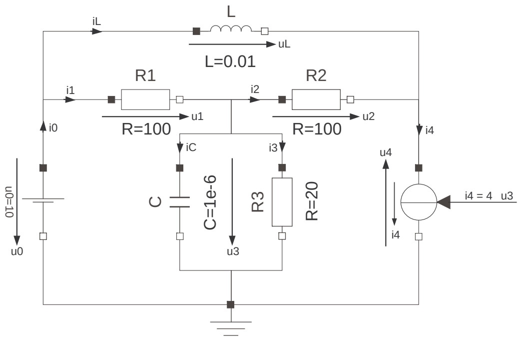

- The circuit contains a constant voltage source, u_0, and a dependent current source, i_4, that depends on the voltage across the capacitor, C, and the resistor, R_3.
- Write down the element equations for the seven circuit elements. Since the voltage u_3 is common to two circuit elements, these equations contain 13 rather than 14 unknowns. Add the voltage equations for the three meshes and the current equations for three of the four nodes.

[H7.1] Electrical Circuit, Horizontal and Vertical Sorting II

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- Simulate the ODE system across 50 μsec using RKF4/5 with Gustaffsson step-size control and with zero initial conditions on both the capacitor and the inductor.

- Plot the voltage u_3 and the current i_C, and the step size h on three separate subplots as functions of time.

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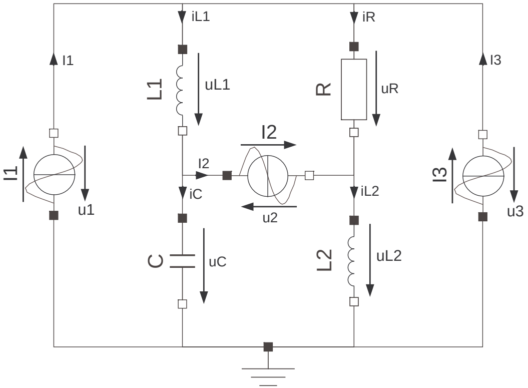

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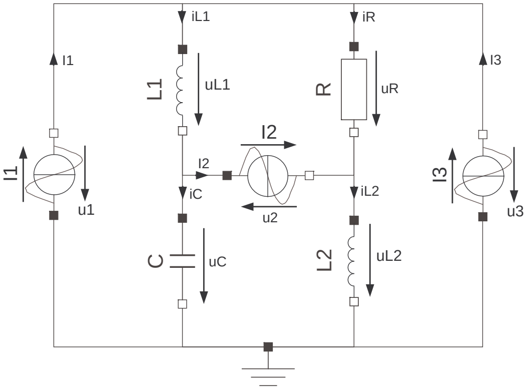

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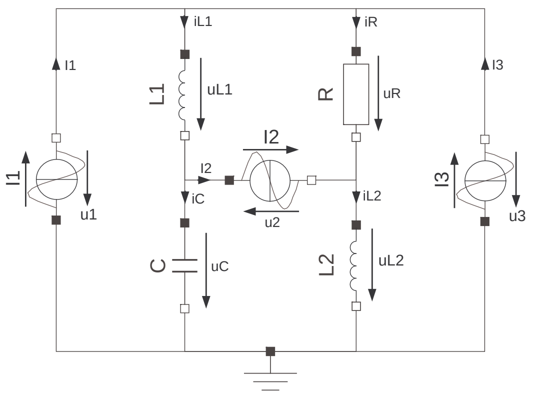

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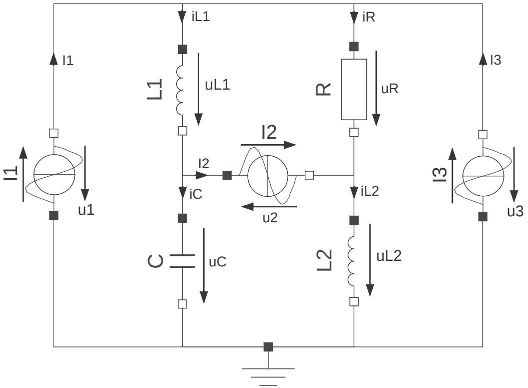

- Write down the complete set of equations describing this circuit. Draw the structure digraph and begin causalizing the equations. Determine a constraint equation.
- Apply the Pantelides algorithm to reduce the perturbation index to 1. Then apply the tearing algorithm with substitution to bring the perturbation index down to 0.
- Write down the structure incidence matrices of the index-1 DAE and the index-0 ODE systems, and show that they are in BLT form, and in LT form, respectively.

[H7.8] Chemical Reactions, Pantelides Algorithm

The following set of DAEs:

$$
\begin{aligned}
\frac{dC}{dt} &= K_1(C_0 - C) - R \\
\frac{dT}{dt} &= K_1(T_0 - T) + K_2 R - K_3(T - T_C) \\
0 &= R - K_3 \exp\left(\frac{-K_4}{T}\right) C \\
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describes a chemical isomerization reaction.

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describes a chemical isomerization reaction.

C is the reactant concentration, T is the reactant temperature, and R is the reactant rate per unit volume. C_0 is the feed reactant concentration, and T_0 is the feed reactant temperature. u is the desired concentration, and T_C is the control temperature that we need to produce u.

[H7.8] Chemical Reactions, Pantelides Algorithm II

- We want to turn the problem around (inverse model control) and determine the necessary control temperature T_C as a function of the desired concentration u. Thus, u will be an input to our model, and T_C is the output.

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- Draw the structure digraph. You shall notice at once that one of the equations has no connections to it. Thus, it is a constraint equation that needs to be differentiated, while an integrator associated with the constraint equation needs to be thrown out.

- We now have five equations in five unknowns. Draw the enhanced structure digraph, and start causalizing the equations. You shall notice that a second constraint equation appears. Hence the original DAE system had been an index-3 DAE system. Differentiate that constraint equation as well, and throw out the second integrator. In the process, new pseudo-derivatives are introduced that call for additional differentiations.

[H7.8] Chemical Reactions, Pantelides Algorithm III

- This time around, you end up with eight equations in eight unknowns. Draw the once more enhanced structure digraph, and causalize the equations. This is an example, in which (by accident) the Pantelides algorithm reduces the perturbation index in one step from 2 to 0, i.e., the final set of equations does not contain an algebraic loop.

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- Draw a block diagram that shows how the output T_C can be computed from the three inputs u, $\frac{du}{dt}$, and $\frac{d^2u}{dt^2}$.