Numerical Simulation of Dynamic Systems XXV

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Introduction

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> **Given that the state variable x_i currently assumes the value $x_i(t_k)$, we would like to determine the shortest time distance h, such that $x_i(t_k + h) = x_i(t_k) \pm \Delta Q_i$.**

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Until now, we always *discretized the time axis*, while *keeping the state variables continuous*. In the sequel, we shall *discretize (quantize) the state variables*, while *keeping the time axis continuous*.

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- The integration method will use a *variable step size*, and the step size in use will depend on the gradient of that state variable.
- The step size h will be different for each state variable $\mathbf{x}$.
- We shall no longer be able to represent the discretized system by a set of *difference equations*, and we shall *lose the linearity* of the discretized system when approximating a linear continuous system.

$$\dot{\mathbf{x}} = \mathbf{A} \cdot \mathbf{x} \quad \not\Rightarrow \quad \mathbf{x_{k+1}} = \mathbf{F} \cdot \mathbf{x_k}$$

Space Discretization: A Simple Example

Let us start by considering the following first-order system:

$$\dot{x}_a(t) = -x_a(t) + 10 \cdot \varepsilon(t - 1.76)$$

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The latter model can be simulated very easily.

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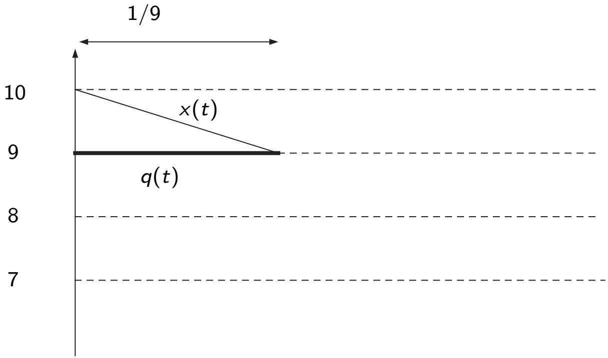

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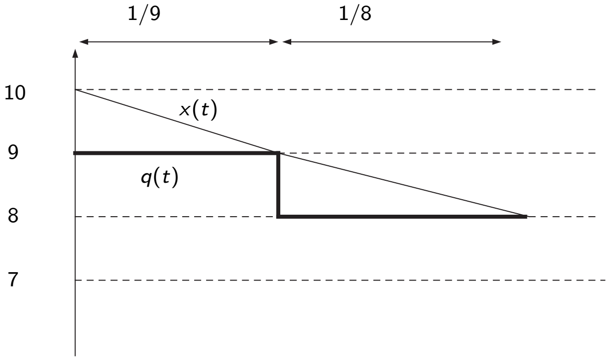

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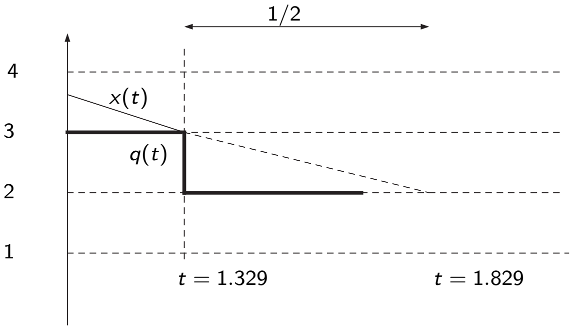

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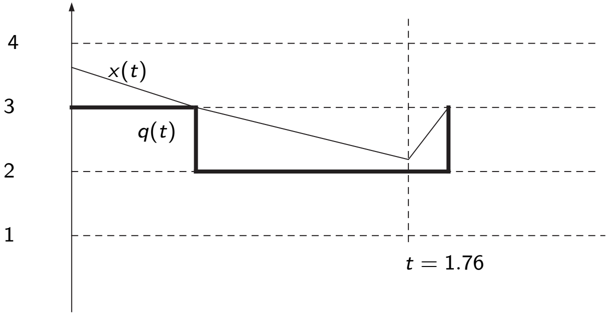

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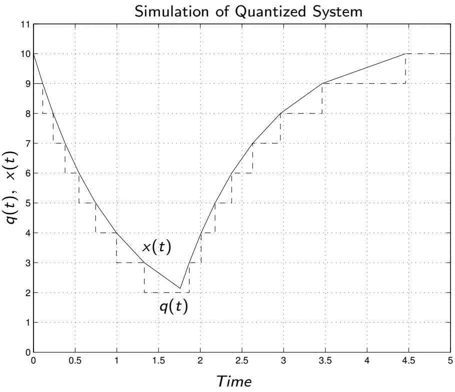

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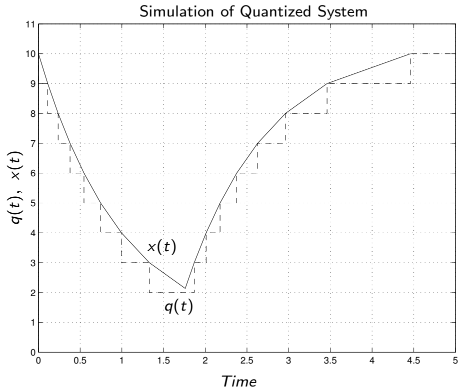

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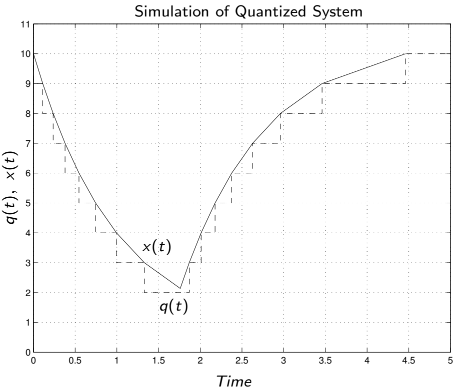

- We were able to complete the simulation in *17 very simple steps*, thereby obtaining the *exact solution* of the quantized system.
- The solution of the quantized system is similar to that of the original continuous system.

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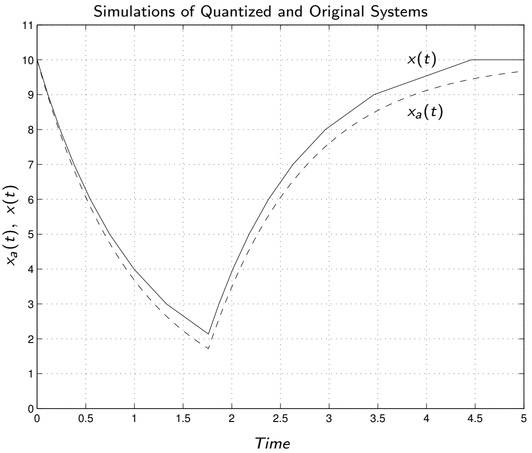

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- The model can be encoded using the *DEVS formalism*.
- DEVS stands for *Discrete EVent System specification*. The formalism was first introduced in the 1970s by *Bernard Zeigler*.
- All systems, the input/output behavior of which can be described by *sequences of discrete events*, can be represented using the DEVS formalism.

The Definition of DEVS

Atomic DEVS Models

A *DEVS model* processes a *sequence of input events* and, in reaction to those events and its own *initial discrete state*, generates a *sequence of output events*.

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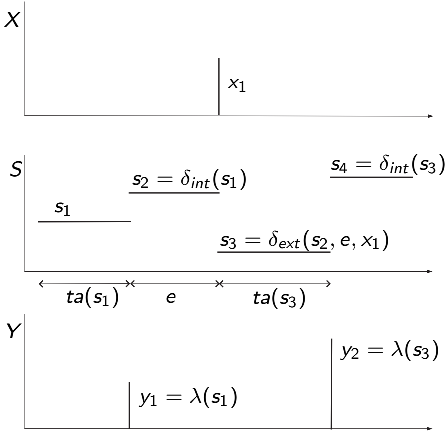
- X is the set of input values.
- Y is the set of output values.
- S is the set of state values.
- $\delta_{int}()$, $\delta_{ext}()$, $\lambda()$, and $ta()$ are functions defining the dynamics of the system.

The Definition of DEVS II

The Behavior of an Atomic DEVS Model



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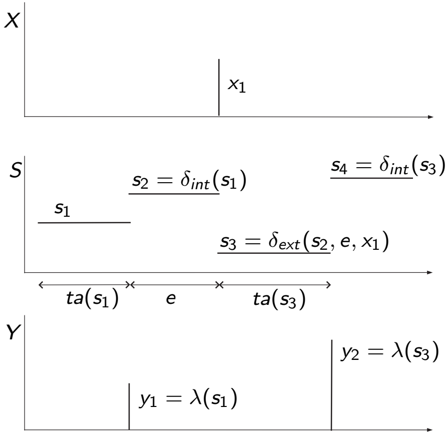

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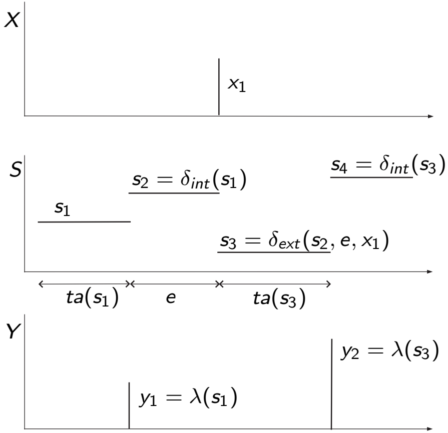

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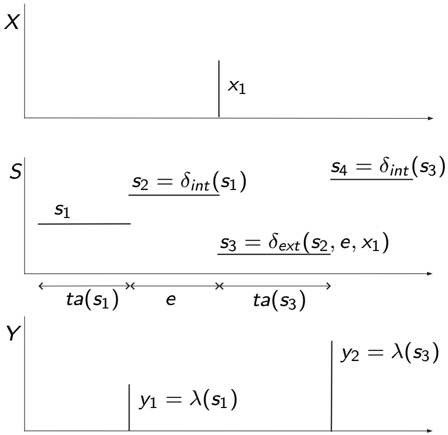

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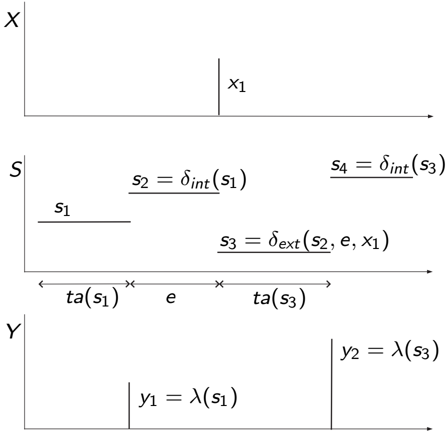

- $\delta_{int}(s)$ is the *internal transition function*.
- $\delta_{ext}(s, e, x)$ is the *external transition function*.
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- $ta(s)$ is the *time advance function*.

The Definition of DEVS III

The Specification of an Atomic DEVS Model

- Each possible state s ($s \in S$) has an associated *time advance* calculated by the *time advance function* $ta(s)$ ($ta(s) : S \rightarrow \Re_0^+$). The time advance is a non-negative real number, determining how long the system remains in a given state in absence of input events.

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- If the state adopts the value s_1 at time t_1, after $ta(s_1)$ units of time (i.e., at time $t_1 + ta(s_1)$), the system performs an *internal transition*, taking it to a new state s_2. The new state is calculated as $s_2 = \delta_{int}(s_1)$. Function δ_{int} ($\delta_{int} : S \rightarrow S$) is called the *internal transition function*.

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- When the state changes its value from s_1 to s_2, an *output event* is produced with the value $y_1 = \lambda(s_1)$. Function λ ($\lambda : S \rightarrow Y$) is called the *output function*. In this way, the functions ta, δ_{int} and λ define the *autonomous behavior of a DEVS model*.

The Definition of DEVS IV

The Specification of an Atomic DEVS Model

- When an input event arrives, the state changes instantaneously. The new state value depends not only on the value of the input event, but also on the previous state value and the elapsed time since the last transition. If the system assumes the state value s_2 at time t_2, and subsequently, an input event arrives at time $t_2 + e < ta(s_2)$ with value x_1, the new state is calculated as $s_3 = \delta_{ext}(s_2, e, x_1)$. In this case, we say that the system performs an *external transition*. Function δ_{ext} ($\delta_{ext} : S \times \Re_0^+ \times X \to S$) is called the *external transition function*. No output event is produced during an external transition.

The Definition of DEVS V

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Let us consider the following simple example: A system receives positive numbers in an asynchronous way. After it received a number x, it generates an output event with the number $x/2$ after $3 \cdot x$ time units.

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A DEVS model that correctly represents this behavior is the following:

$$M_F = (X, Y, S, \delta_{int}, \delta_{ext}, \lambda, ta), \text{ where}$$
$$X = Y = S = \Re^+$$
$$\delta_{int}(s) = \infty$$
$$\delta_{ext}(s, e, x) = x$$
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Observe that the state can assume a time advance equal to ∞. When this occurs, we say that the system is in a *passive state*, since it will no longer change its state, unless and until it receives an input event.

The Definition of DEVS VI

The Specification of an Atomic DEVS Model

Let us analyze what happens with the model M_1 when it receives an input event trajectory. Consider for instance that input events occur at times $t = 1$, $t = 3$, and $t = 10$ with the values 2, 1, and 5, respectively. Suppose that initially we have $t = 0$, $s = \infty$ and $e = 0$.

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$s = \infty$
$e = 0$
$ta(s) = ta(\infty) = \infty$

time $t = 1^{-}$:
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$e = 1$

time $t = 1$:
$s = \delta_{ext}(s, e, x) = \delta_{ext}(\infty, 1, 2) = 2$

time $t = 1^{+}$:
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time $t = 3$:
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time $t = 3^+$:
- $s = 1$
- $e = 0$
- $ta(s) = ta(1) = 3$

time $t = 6$:
- output event with value $\lambda(s) = \lambda(1) = 0.5$
- $s = \delta_{int}(s) = \delta_{int}(1) = \infty$

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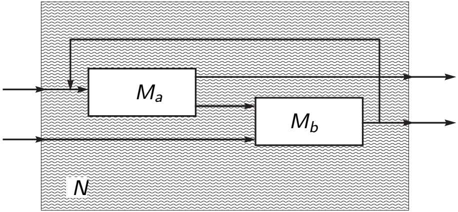

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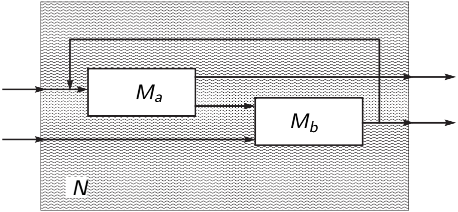

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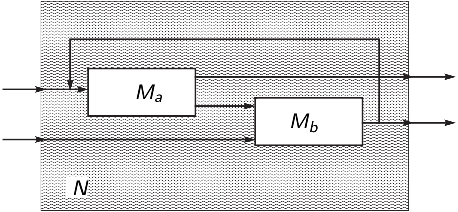

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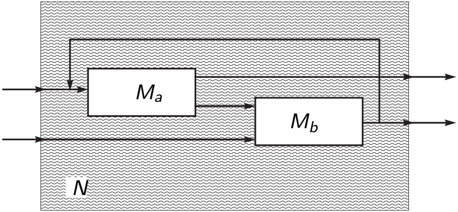

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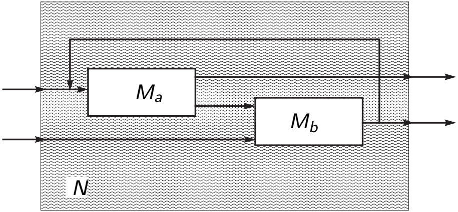

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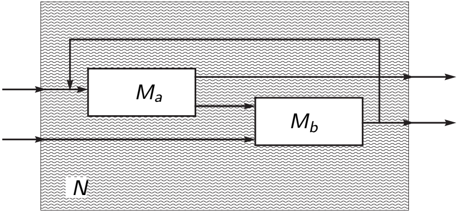
etc.

Coupled DEVS Models

Atomic DEVS models can be coupled to form more complex models. The most simple manner for defining the coupling between DEVS models is through the use of *input and output ports*.



We notice the following couplings:

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etc.

The resulting coupled model N can be used as if it were a new atomic model.

Example: DEVS Model of a Static Function

Let us consider a system that calculates a *static function* $f(u_0, u_1)$, where u_0 and u_1 are real-valued piecewise constant trajectories generated by other subsystems. We can represent piecewise constant trajectories by *sequences of events*, if we relate each event to a change in the trajectory value.

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- When an input event arrives, it is assigned the value $\sigma = 0$. In this way, an immediate output event is being scheduled.

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3. Propagate the output event produced by d^* to all atomic models connected to it through its output ports while executing the corresponding external transition functions. Then return to step 1 above.

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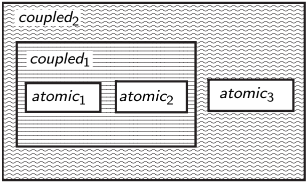

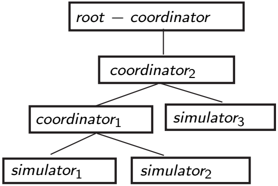

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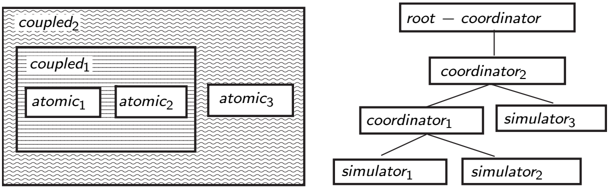

There exist several software tools that support directly the simulation of DEVS models. The one that we shall be using is called **PowerDEVS**. It was developed by *Ernesto Kofman* at the Universidad Nacional de Rosario (Argentina). It is the DEVS modeling and simulation environment that is most suitable for our purposes.

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a *dynamic system*:

$$\dot{x}(t) = d_x(t)$$
$$q(t) = \text{floor}[x(t)]$$

and a *static function*:

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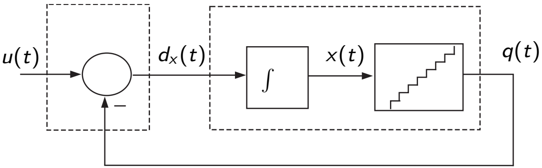

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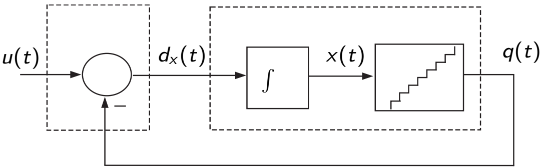

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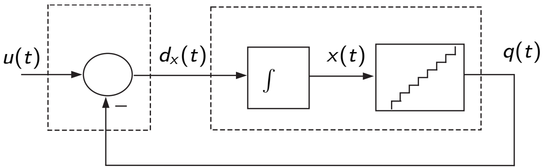

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\tilde{\sigma} = \begin{cases} \frac{q+1-x}{x_v} & \text{if } x_v > 0 \\ \frac{q-x}{x_v} & \text{if } x_v < 0 \\ \infty & \text{otherwise} \end{cases}
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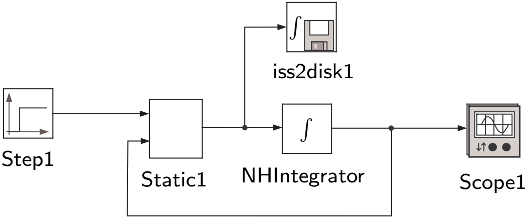

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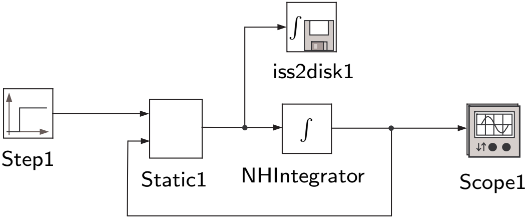

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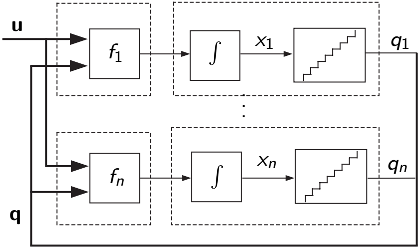

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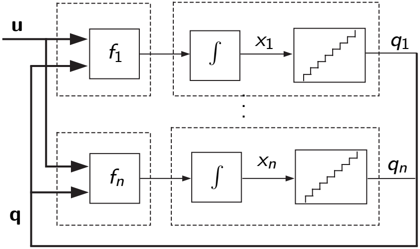

We can model a generic time-invariant quantized system using DEVS models of the static function and quantized integrator types.

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Luckily, this problem can be solved easily by adding *hysteresis*.

Quantization Functions with Hysteresis

If we add *hysteresis* to the relationship between $x(t)$ and $q(t)$, the oscillations in $q(t)$ can only be produced by *large oscillations* in $x(t)$ that cannot occur instantaneously, as long as the magnitude of the state derivatives remains bounded.

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Definition (Function of Quantization with Hysteresis)

Given an ordered sequence of increasing real-valued numbers $(\ldots, Q_{-1}, Q_0, Q_1, \ldots)$, we say that $q(t)$ is related to $x(t)$ through a quantization function with hysteresis, if:

$$q(t) = \begin{cases} Q_m & \text{if } t = t_0 & \wedge & Q_m \leq x(t_0) < Q_{m+1} \\ Q_{k+1} & \text{if } x(t) = Q_{k+1} & \wedge & q(t^-) = Q_k \\ Q_{k-1} & \text{if } x(t) = Q_k - \varepsilon_k & \wedge & q(t^-) = Q_k \\ q(t^-) & \text{otherwise} & & \end{cases}$$

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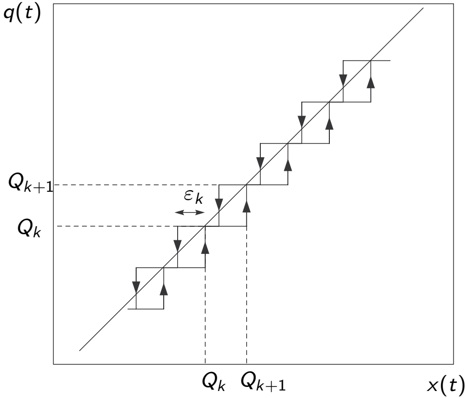
The discrete values Q_k are called *quantization levels*, and the distance $Q_{k+1} - Q_k$ is called *quantum*. The quantum is often chosen constant. ε_k is the *hysteresis width*.

Quantization Functions with Hysteresis II

The graph depicted below shows a *quantization function with hysteresis* with a *uniform quantum*.



QSS Method: Definition

Given the *time-invariant continuous system*:

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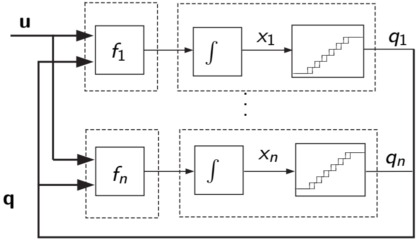

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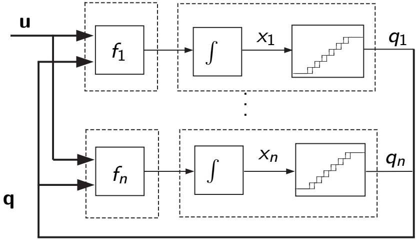

As before, the QSS can be subdivided into *static functions* and *quantized integrators*.

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- It should be mentioned, however, that the QSS algorithm is independent of DEVS. We chose DEVS for the implementation of the QSS method, because DEVS simplified our work. However, we could have programmed the QSS method also independently of DEVS using any other event description formalism.

Simulation with QSS: An Illustrative Example

Let us consider the following second-order system and its QSS approximation:

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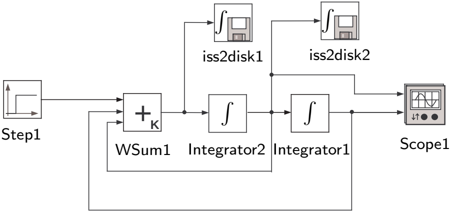

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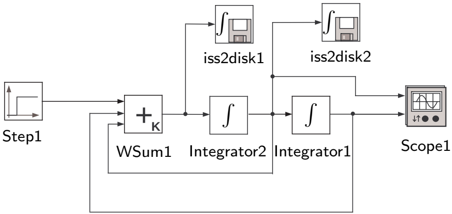

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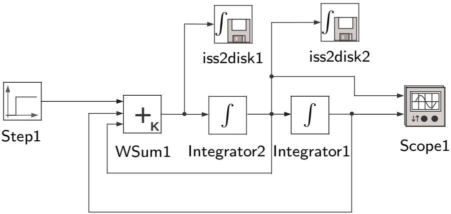

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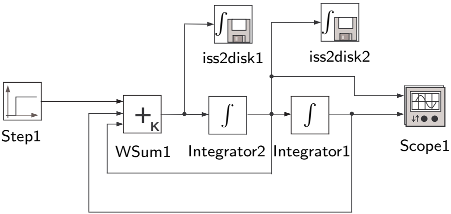

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- The *quantum* and the *hysteresis* are parameters of each integrator (here: $Q_{k+1} - Q_k = \Delta Q = \epsilon_k = 0.05$).
- The QSS method intrinsically exploits *sparsity* (events are only propagated between directly connected blocks).

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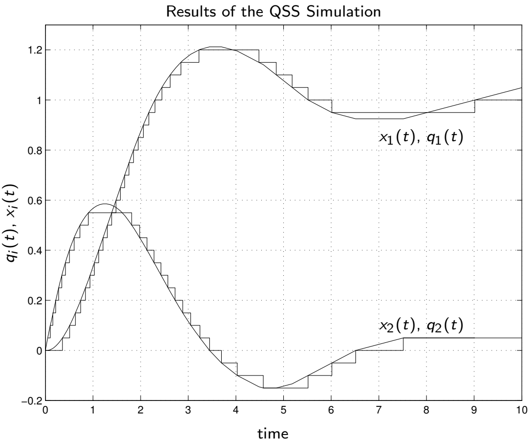

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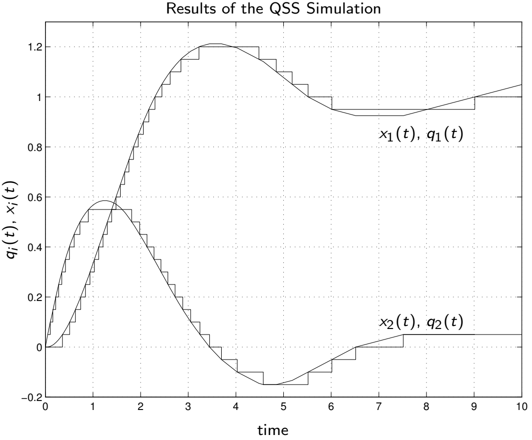

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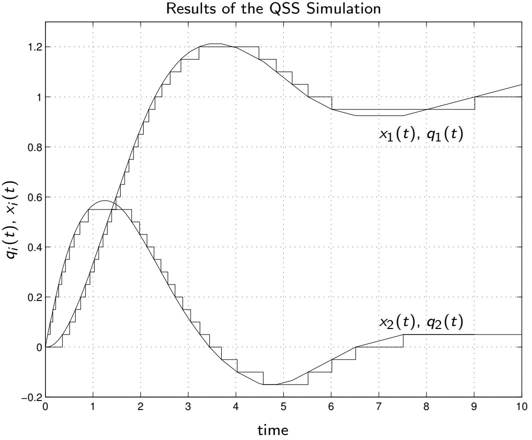

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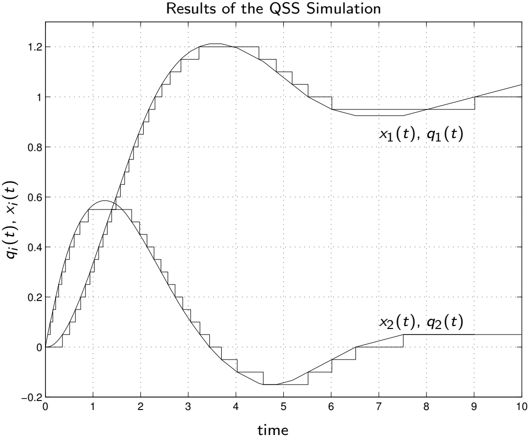

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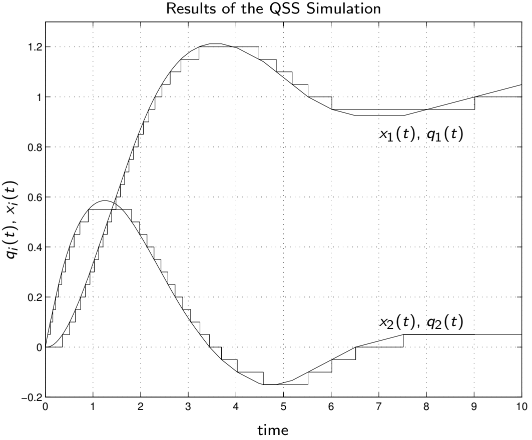
- The presence of the *hysteresis* is easy to observe where the signs of the state derivatives $\dot{x}_i(t)$ change.

Simulation with QSS: An Illustrative Example II

The *simulation results* are shown below:



- The trajectories of the *state variables* $x_i(t)$ are *piecewise linear*.
- The trajectories of the *quantized states* $q_i(t)$ are *piecewise constant*.
- The presence of the *hysteresis* is easy to observe where the signs of the state derivatives $\dot{x}_i(t)$ change.
- The obtained solution is quite close to the analytical solution.

Conclusions

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- The QSS algorithm exploits the *sparsity of the model topology*. Events are propagated only between blocks that are directly connected.
- Unfortunately, the QSS algorithm cannot be easily programmed as a **Matlab** function. Instead, we also introduced a new tool, **PowerDEVS**, that has been specifically designed for the numerical simulation of continuous systems using QSS algorithms.

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