Numerical Simulation of Dynamic Systems XXVI

Prof. Dr. François E. Cellier
Department of Computer Science
ETH Zurich

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However, we noticed that we can interpret the simulation results generated by the QSS solver as a *perturbation* around the analytical solution.

We shall make use of this idea to approach the analysis of numerical stability and accuracy of these solvers.

Representation of the QSS Solution as a Perturbation

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We also note that the perturbation is bounded:

$$|\Delta x_i(t)| = |q_i(t) - x_i(t)| \leq \max(\Delta Q_i, \varepsilon_i)$$

with ΔQ_i being the quantum of the i^{th} state variable, and ε_i being its associated hysteresis width.

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- We shall demonstrate the *convergence* of the QSS approximation for non-linear systems. This means that, as the *quantum* and the *hysteresis width* are being reduced to *zero*, the trajectories of the QSS solution become in the limit identical to those of the original continuous system.

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- Contrary to the classical solvers, where we usually limited the *stability analysis* to linear systems only, we shall now discuss the numerical stability of the QSS approximation for arbitrary *non-linear systems*. It shall be shown that, wherever the original system exhibits an *asymptotically stable equilibrium point*, there is a region around that equilibrium point such that, for any initial condition within that region, we can find a quantum of finite size that guarantees that the trajectories of the QSS approximation *end up bounded* in a region around that equilibrium point.

Convergence, Accuracy, and Stability in QSS II

Although these results are interesting from a theoretical point of view, as they allow us to offer assurances that the *QSS approximation of an asymptotically stable continuous system remains always numerically stable*, we don't know yet, how good the approximation actually is, as we cannot offer in general a quantitative value for the upper bound of the size of the region around the equilibrium point, in which the QSS simulation terminates.

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Unfortunately, even if the continuous system exhibits a *constant steady-state*, the QSS approximation will usually *oscillate around that steady state* as shown to the right for the scalar system $\dot{x}_a(t) = -x_a(t) + 9.5$ with $\Delta Q = 1$ and $\varepsilon = 1$:

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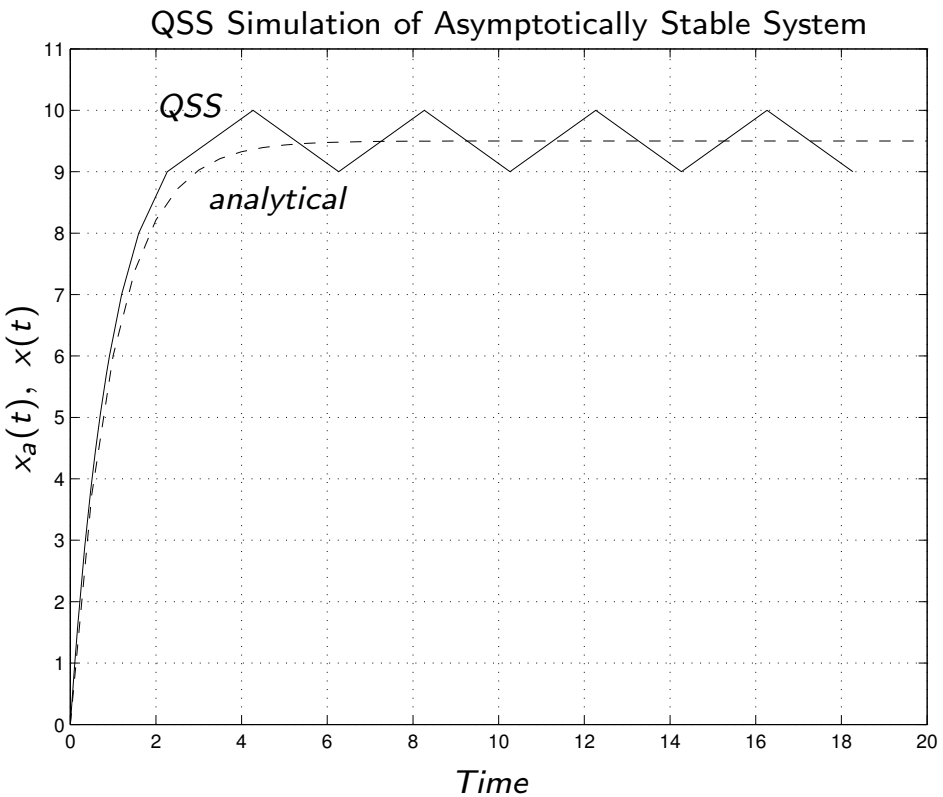

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This is a very important result that has significant practical implications.

QSS Simulations of Linear Time-invariant Systems

Let us consider a *linear time-invariant system* and its QSS approximation:

$$\dot{\mathbf{x}}_\mathbf{a}(t) = \mathbf{A} \cdot \mathbf{x}_\mathbf{a}(t) + \mathbf{B} \cdot \mathbf{u}(t) \quad ; \quad \dot{\mathbf{x}}(t) = \mathbf{A} \cdot \mathbf{q}(t) + \mathbf{B} \cdot \mathbf{u}(t)$$

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If the $\mathbf{A}$-matrix is Hurwitz, i.e., the system is analytically stable, and if $\mathbf{x}(t_0) = \mathbf{x}_\mathbf{a}(t_0)$, it can be shown that:

$$|\mathbf{x}(t) - \mathbf{x}_\mathbf{a}(t)| \leq |\mathbf{V}| \cdot |\mathbb{R}\mathrm{e}\{\mathbf{\Lambda}\}^{-1} \cdot \mathbf{\Lambda}| \cdot |\mathbf{V}^{-1}| \cdot \mathbf{\Delta Q} \quad \forall t \geq t_0$$

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- The QSS method offers a *computable global error bound*.
- The numerical solution remains *practically stable* for any value of the *quantum*.

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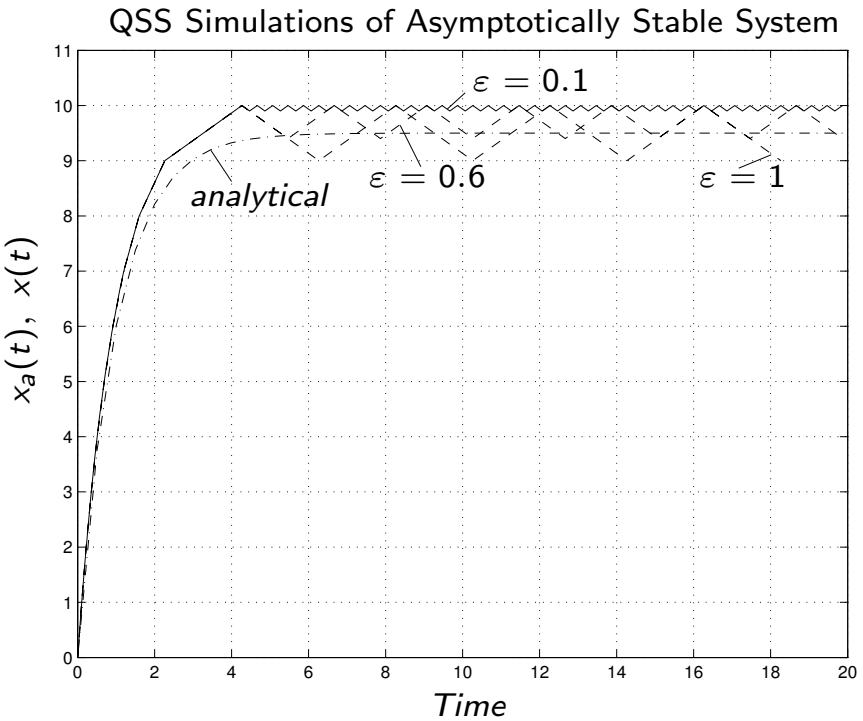

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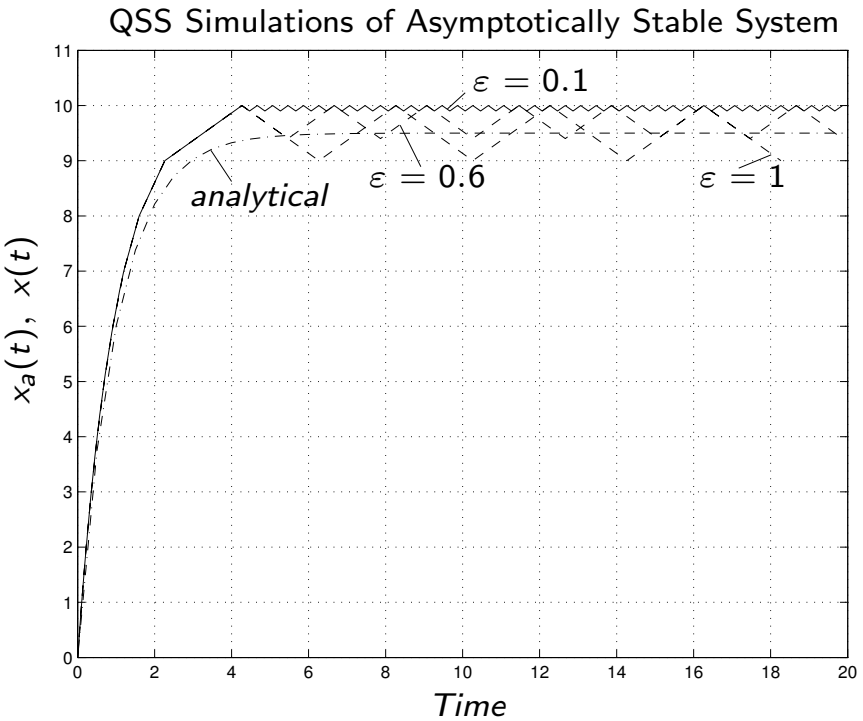

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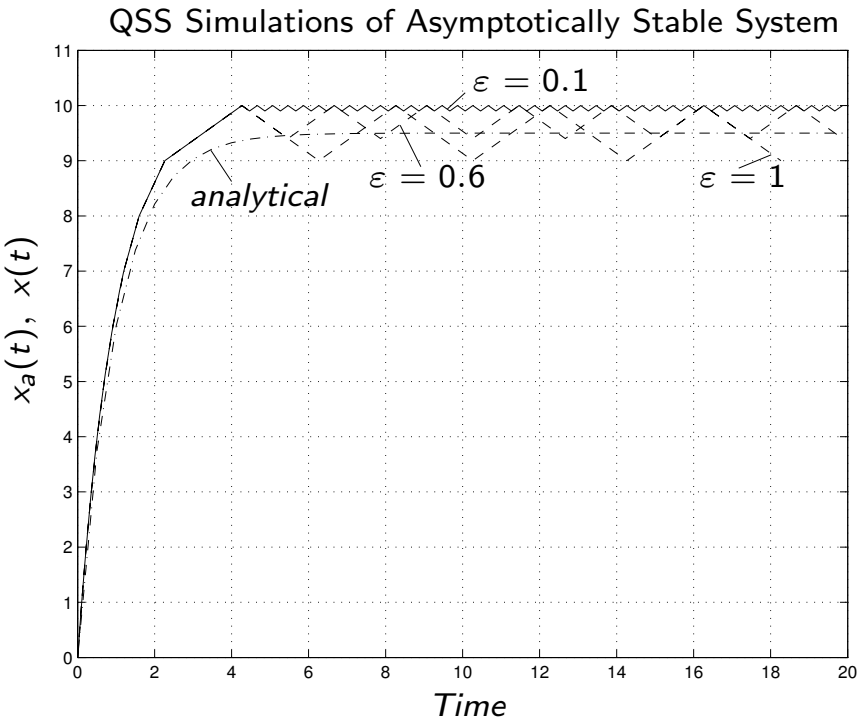

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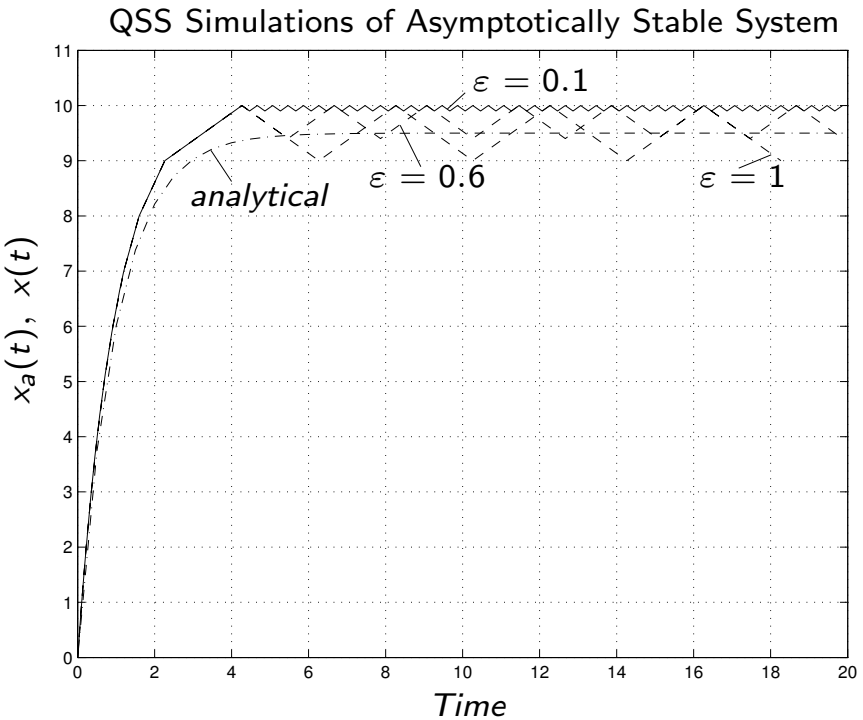

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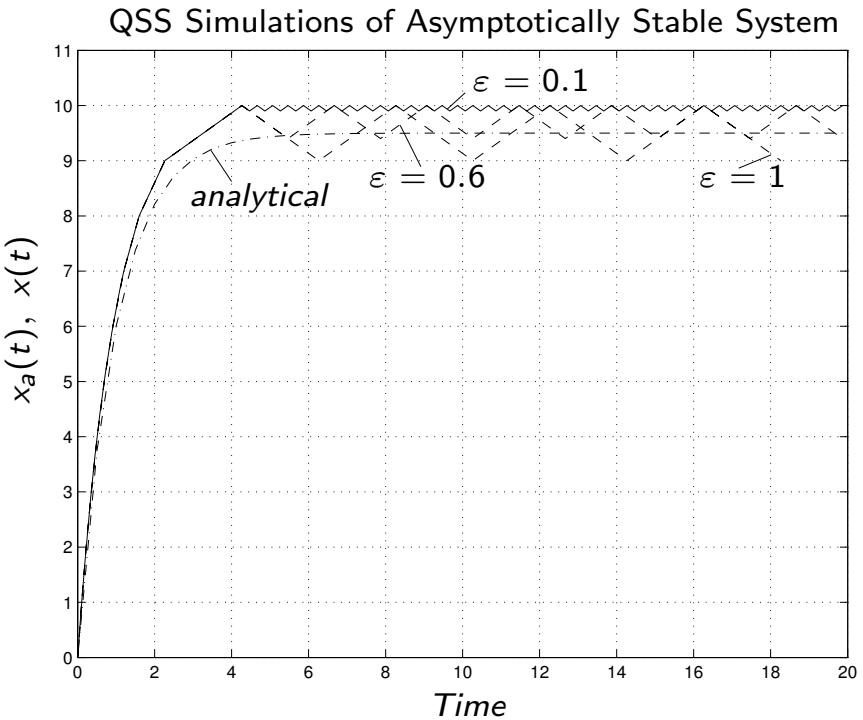

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- A *practical rule* is to choose the quantum proportional to the *amplitude* of every state variable.

Input Signals in the QSS Method

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However, when we approximate an input signal by a piecewise constant trajectory, we introduce a *new type of error*. In this case, the expression for the *global error bound* needs to be augmented with an additional term:

$$|\mathbf{x}(t) - \mathbf{x_a}(t)| \leq |\mathbf{V}| \cdot |\mathbb{R}\mathrm{e}\{\mathbf{\Lambda}\}^{-1} \cdot \mathbf{\Lambda}| \cdot |\mathbf{V}^{-1}| \cdot \mathbf{\Delta Q} + |\mathbf{V}| \cdot |\mathbb{R}\mathrm{e}\{\mathbf{\Lambda}\}^{-1} \cdot \mathbf{V}^{-1} \cdot \mathbf{B}| \cdot \mathbf{\Delta u}$$

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This means that the number of integration steps grows *inversely proportional to the tolerated error*. For this reason, if we tighten our *accuracy requirements* hundredfold, we need to reduce the *quantum* by a factor of 100, which in turn increases the *number of integration steps* by a factor of 100.

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We can see this easily when looking at the approximation of an input ramp signal. The signal gets represented by a stair case. The more accurate we need to represent the ramp, the smaller we need to make the individual steps, and the more steps we shall need.

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We can see this easily when looking at the approximation of an input ramp signal. The signal gets represented by a stair case. The more accurate we need to represent the ramp, the smaller we need to make the individual steps, and the more steps we shall need.

This is unacceptable for more stringent accuracy requirements. Most engineering applications call for relative error tolerances of 10^{-4} or better. For such accuracy requirements, QSS turns out to be highly inefficient. We consequently need *QSS methods of higher orders of approximation accuracy*.

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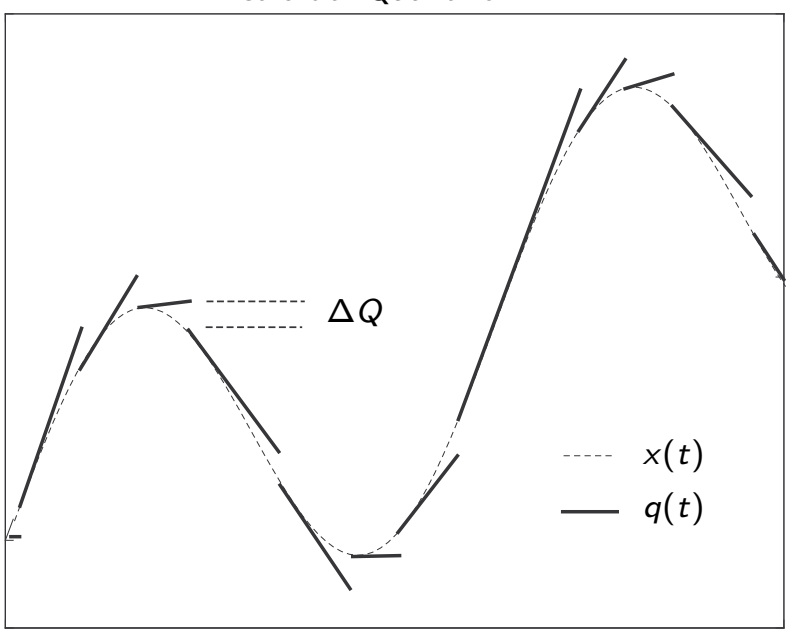

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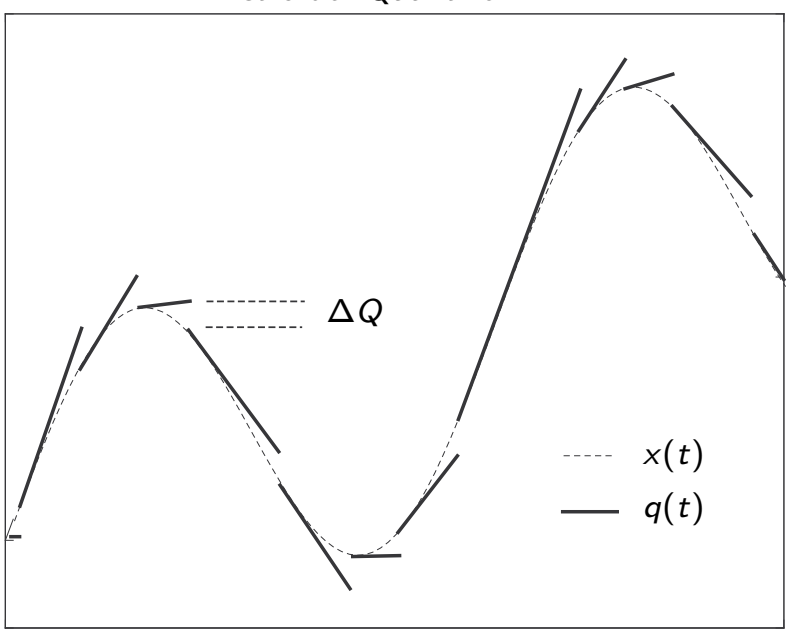

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$$q(t) = \begin{cases} x(t), & \text{if } t = t_0 \vee |q(t^-) - x(t^-)| = \Delta Q \\ q(t_j) + m_j \cdot (t - t_j), & \text{otherwise} \end{cases}$$

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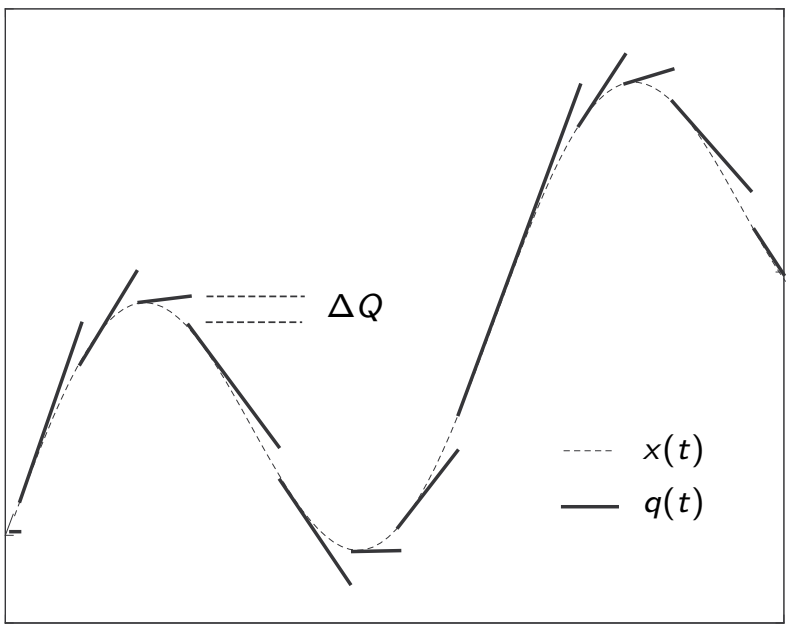

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The sequence $t_0, \ldots, t_j, \ldots$ is defined as:

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and the gradients are $m_0 = 0$, $m_j = \dot{x}(t_j^-)$ for $j = 1, \ldots, k, \ldots$.

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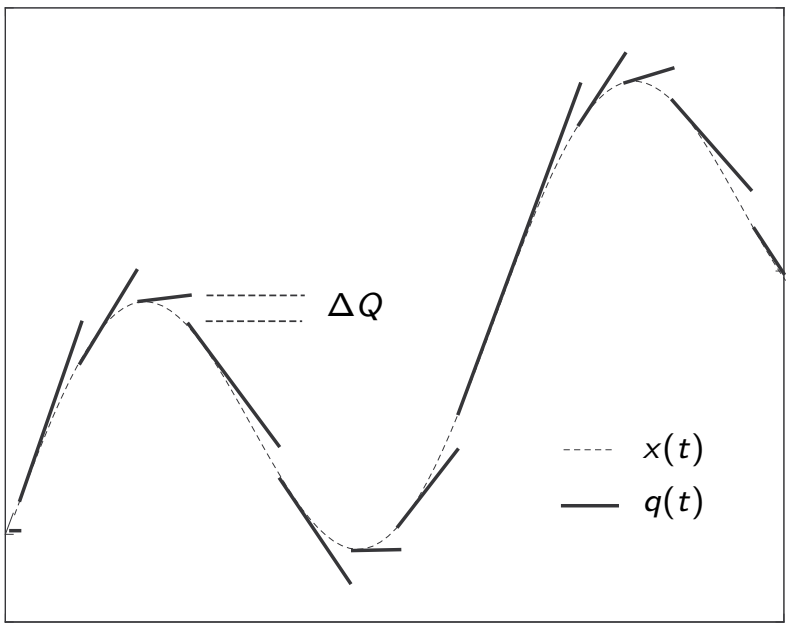

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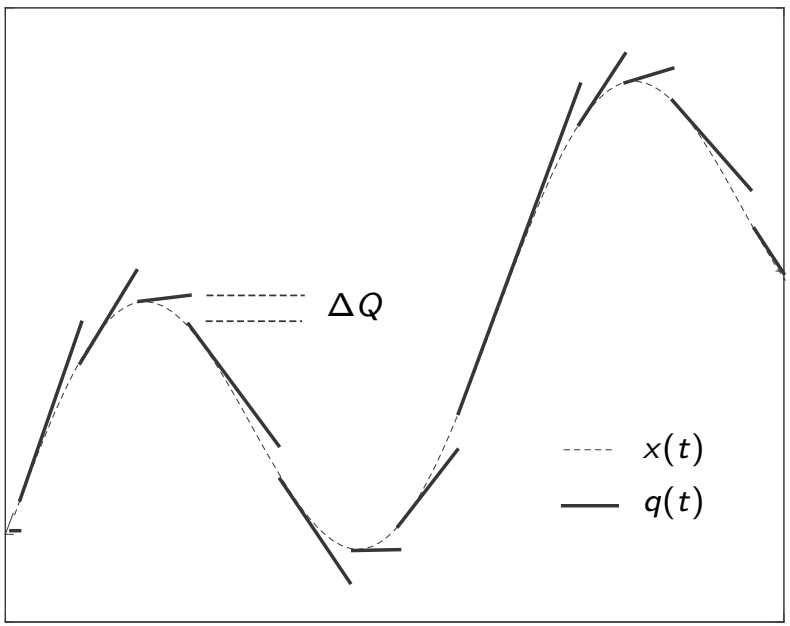

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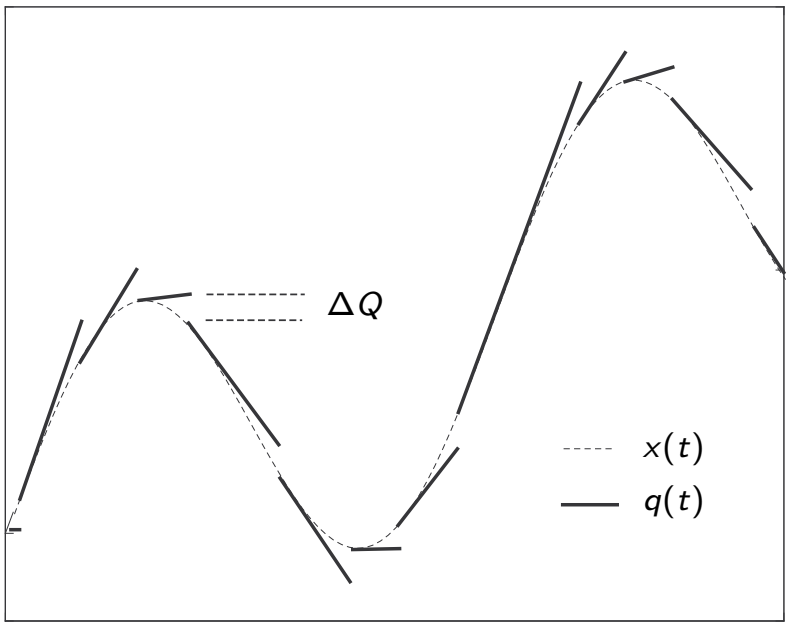

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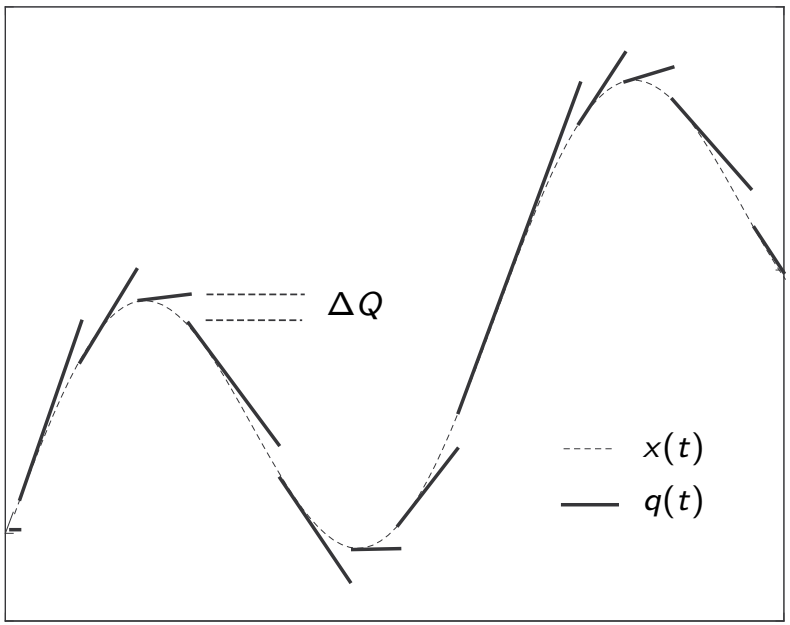

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- This avoids the possibility of occurrence of *illegitimate models*.

Second-order accurate QSS Method (QSS2)

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Given the non-linear continuous system:

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When using QSS2, a ramp input signal does not cause any events at all, as it can be represented accurately by a linear trajectory.

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The simulation results do, in the non-linear case, not coincide exactly with the definition of the QSS2 methods, but rather represent an approximation only.

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The basic idea for obtaining an equivalent DEVS model of a QSS2 approximation is the same as in QSS, i.e., we divide the system to be modeled into *static functions* and *quantized integrators*. The following differences are to be noted:

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Except for these differences, the resulting DEVS model will look exactly the same for **QSS2** as for **QSS**.

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The QSS2 approximation of a *linear time-invariant system*:

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In the case of linear time-invariant systems, QSS and QSS2 share the *identical stability and accuracy properties*.

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- More precisely, the cost grows *inversely proportional* to the *square root* of the tolerated error in QSS2.
- For example, if we wish to improve the accuracy of a simulation by a factor of $10,000$, the cost increases by a factor of $10,000$ when using QSS, whereas it only increases by a factor of 100 when using QSS2.

An Illustrative Example

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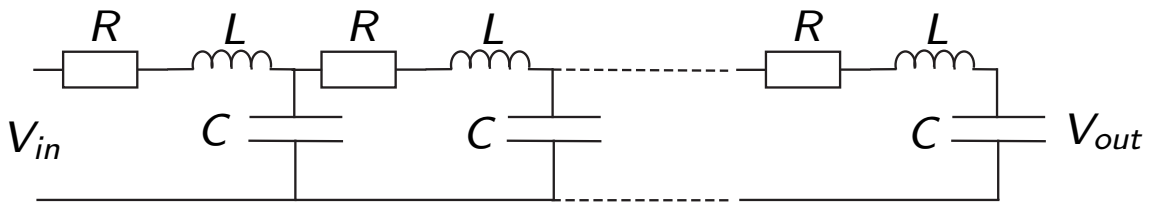

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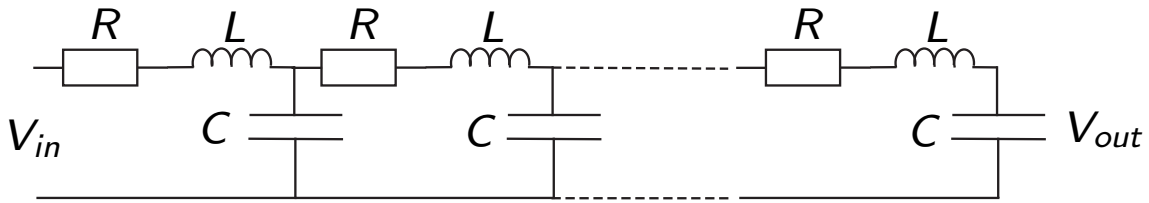

Let us consider 5 segments. There results a *linear time-invariant system* of order 10 with a *band-structured sparse system matrix*:

$$\mathbf{A} = \begin{pmatrix} -R/L & -1/L & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1/C & 0 & -1/C & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1/L & -R/L & -1/L & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1/C & 0 & -1/C & 0 & 0 & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1/C & 0 \end{pmatrix}$$

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Using **PowerDEVS**, we constructed the *block diagram* and simulated the circuit using QSS2 with a *trapezoidal input* (the typical form of signals in integrated digital circuits). We used a quantum of $4mV$ for the potentials and $10\mu A$ for the currents.

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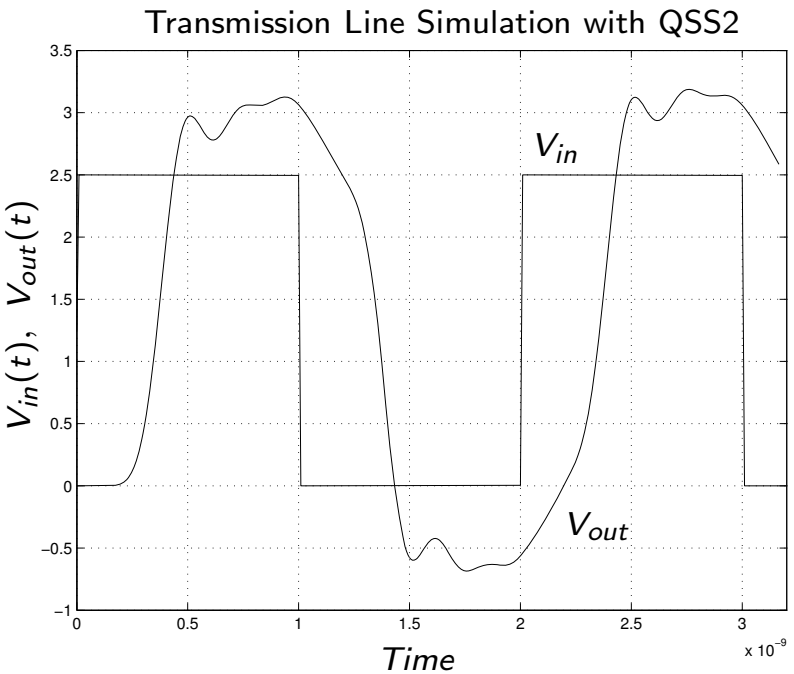

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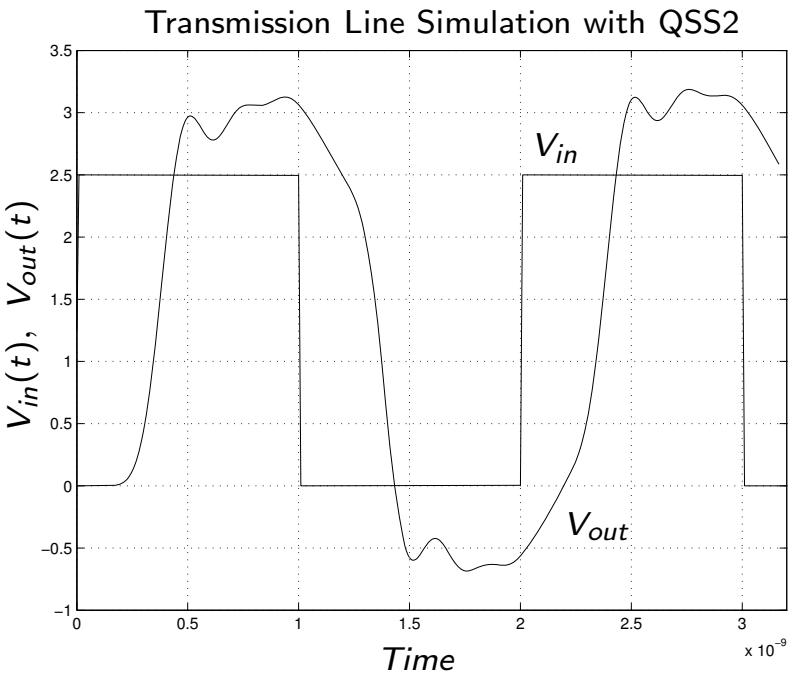

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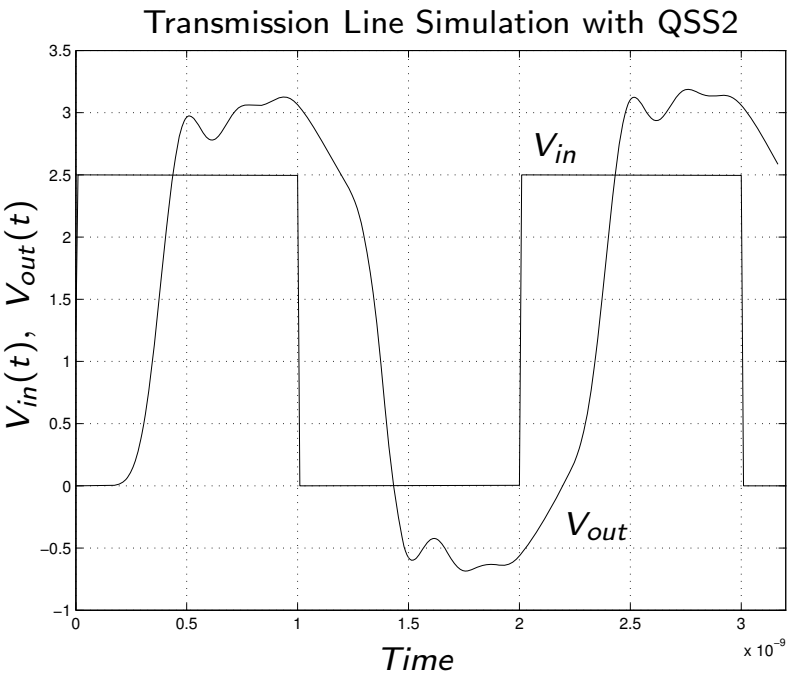

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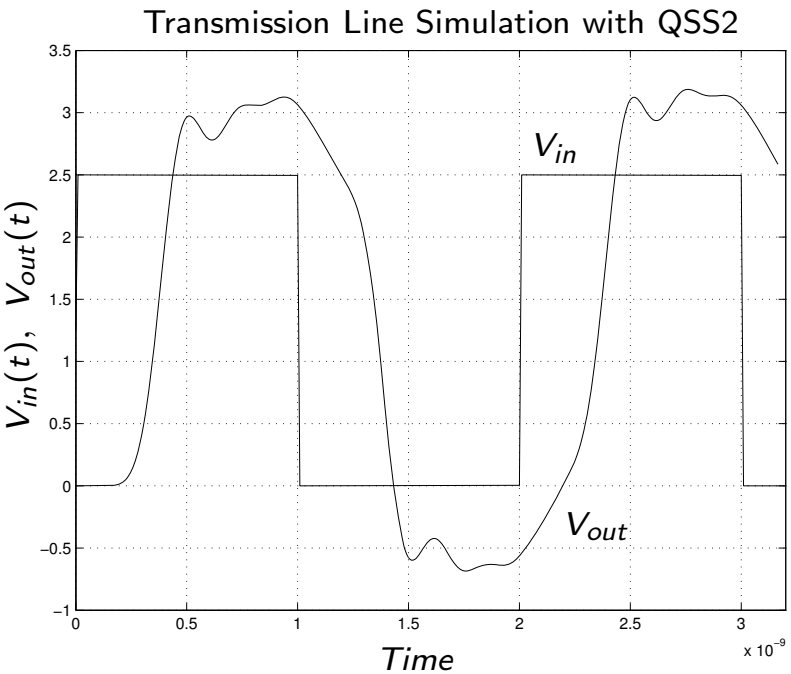

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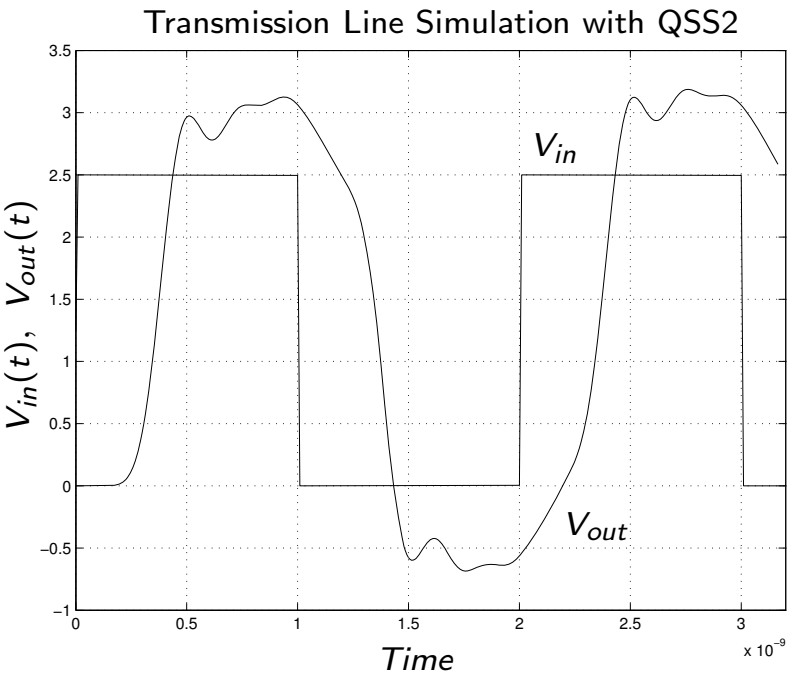

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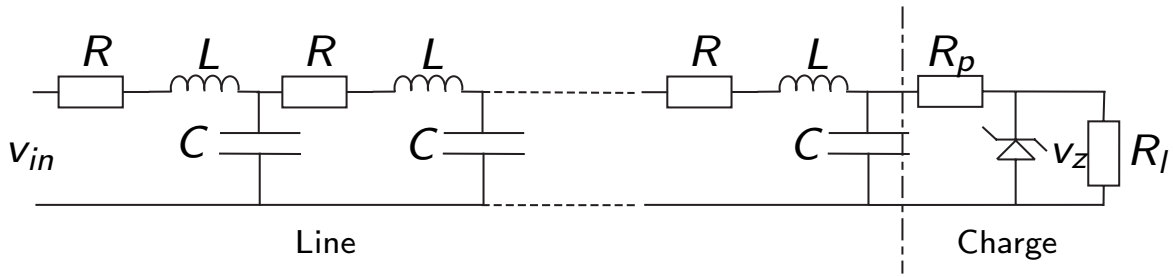
- Reducing the quantum 100 times, we estimate that the *number of steps* will be augmented approximately 10 times. The new error bound for V_{out} is then $2.5mV$.

DAE Simulation with QSS Methods

The figure below shows the same circuit as the previous example with added *surge voltage protection* formed by a resistor and a *Zener diode*.



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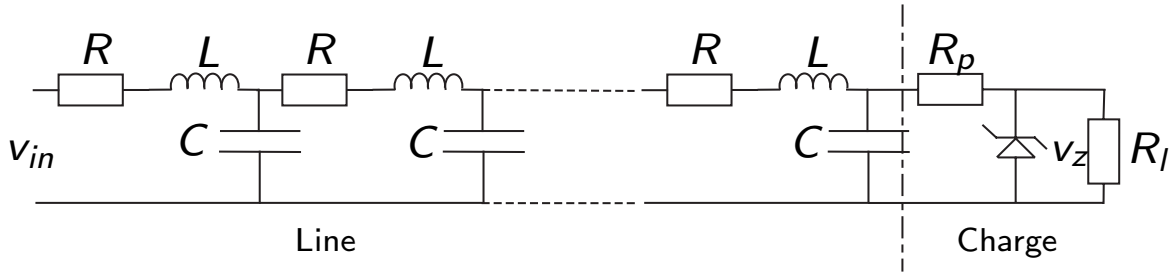

The new system of equations is a *DAE*:

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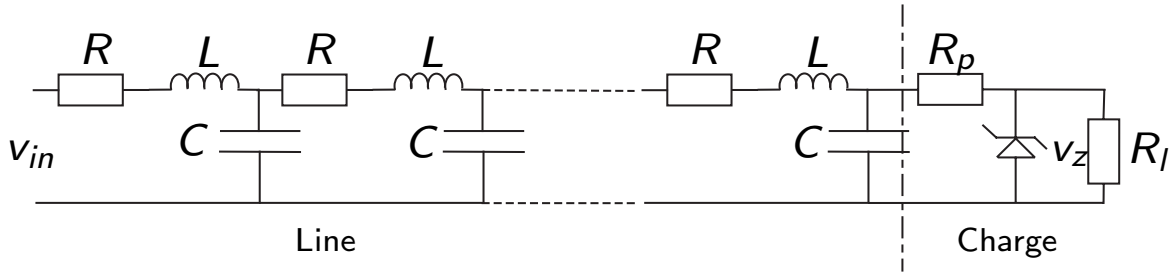

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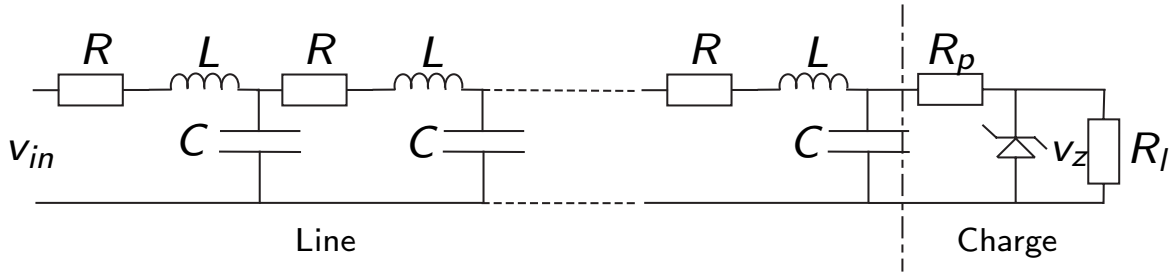

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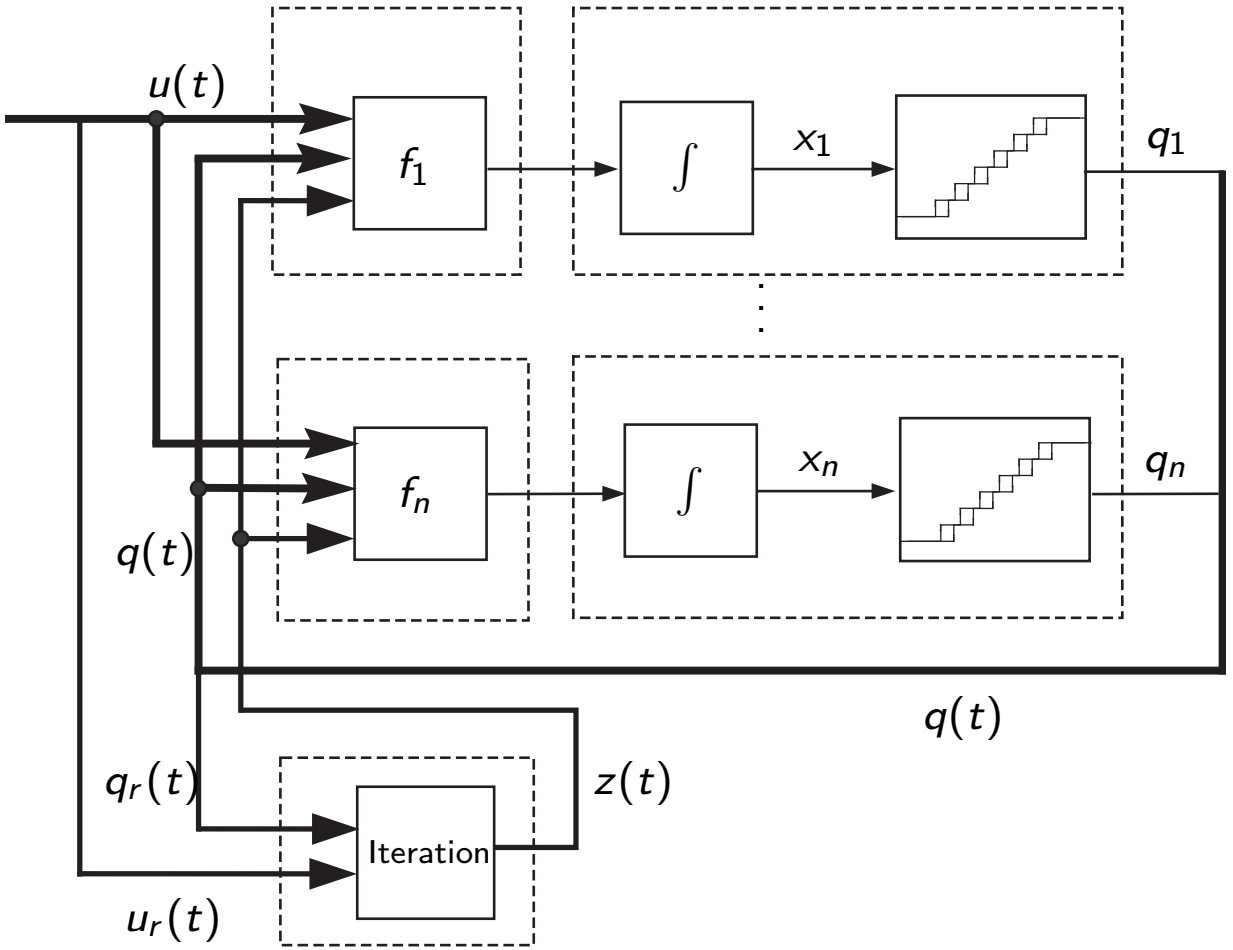

$$\begin{aligned} \dot{\mathbf{x}} &= \mathbf{f}(\mathbf{q}, \mathbf{u}, \mathbf{z}) \\ 0 &= \mathbf{g}(\mathbf{q_r}, \mathbf{u_r}, \mathbf{z}) \end{aligned}$$

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For these reasons, the *main advantage* of the QSS methods lies in *simulations of systems with discontinuities*.

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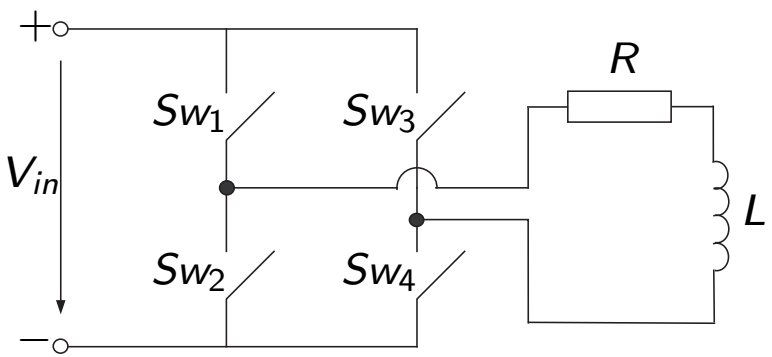

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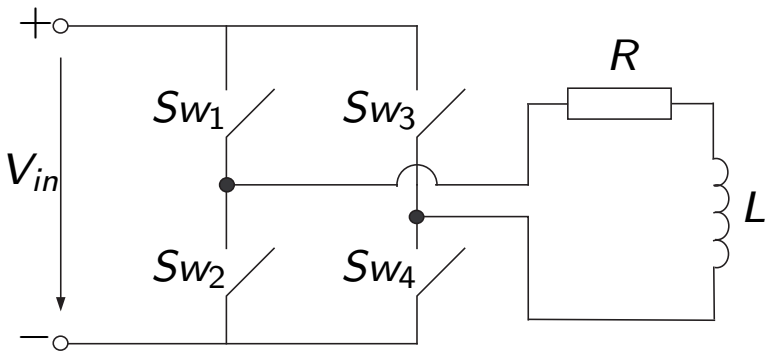

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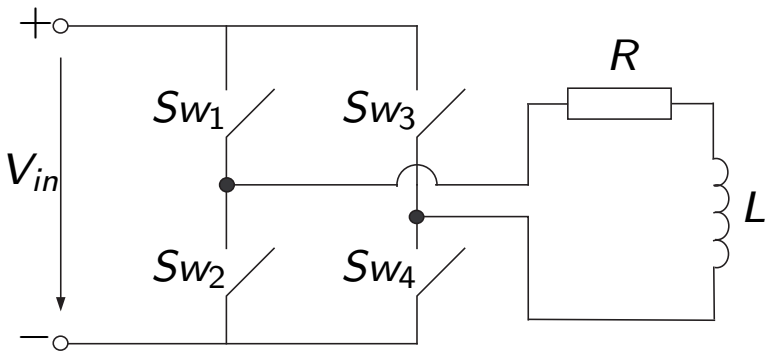

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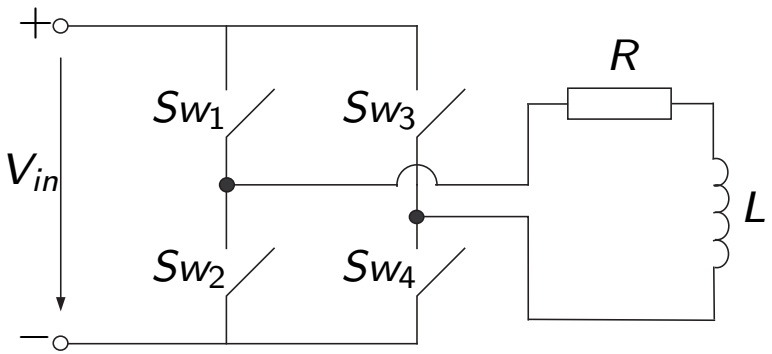

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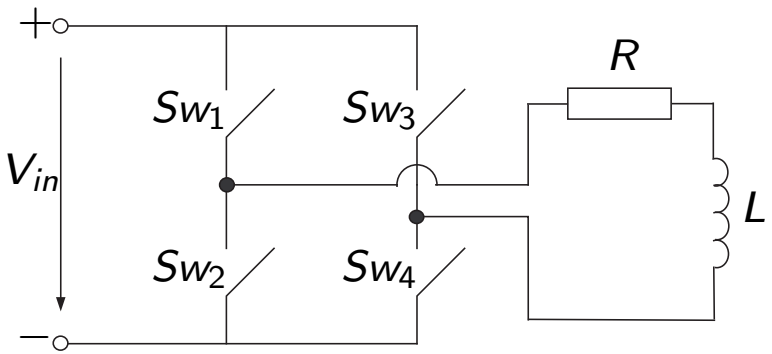

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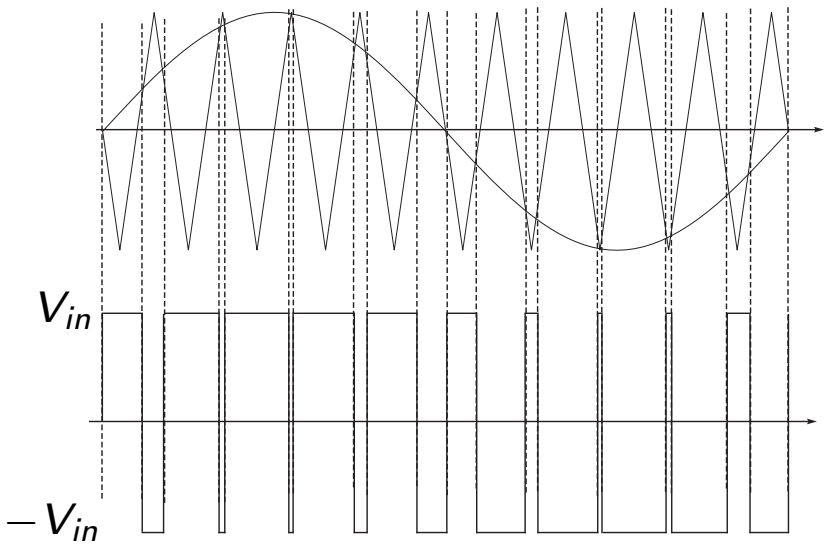

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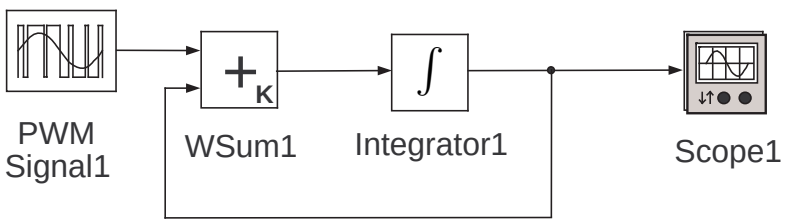

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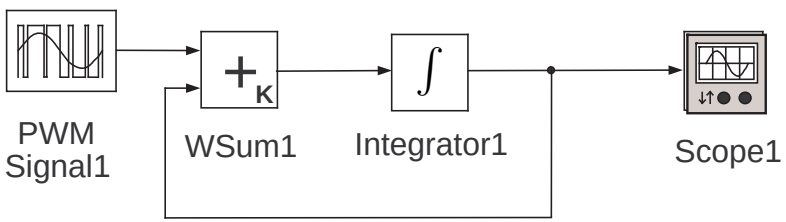

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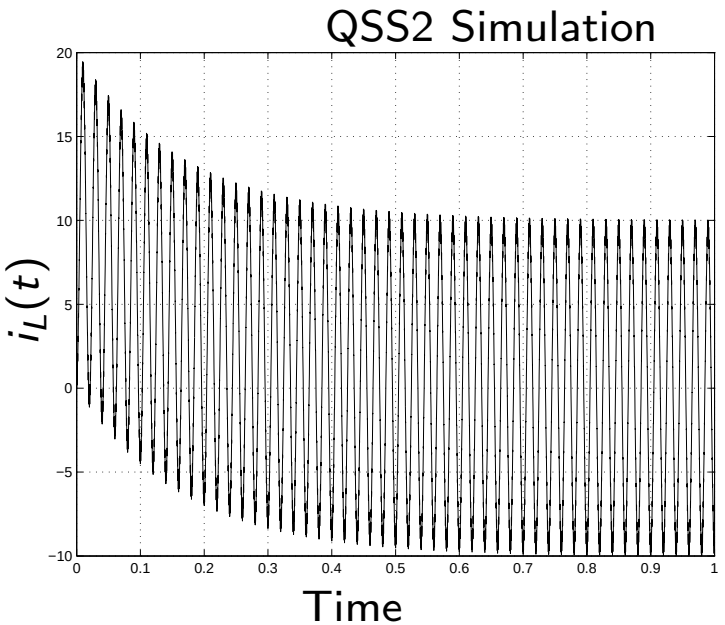

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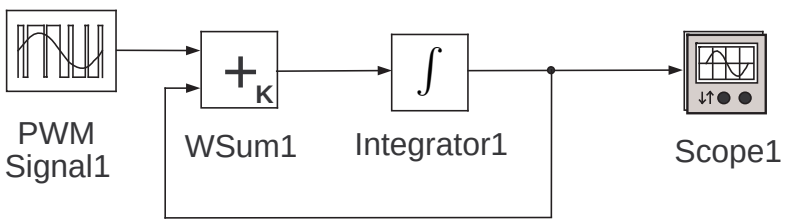

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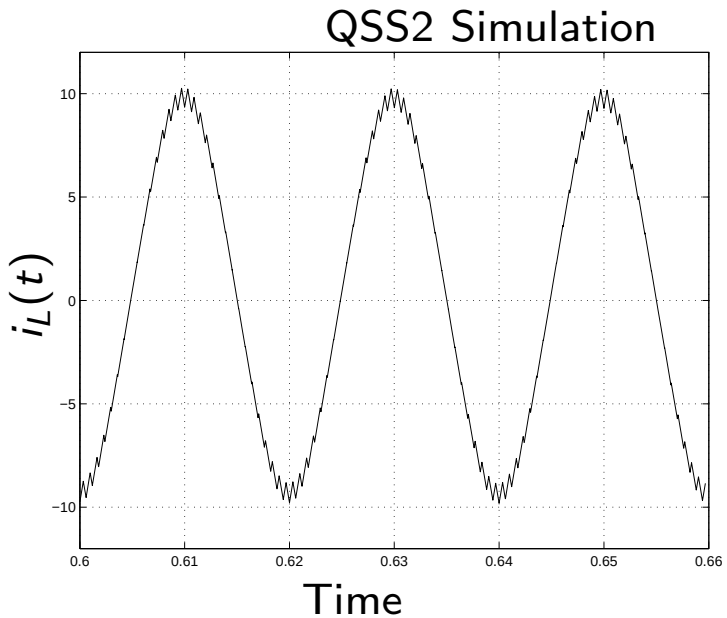

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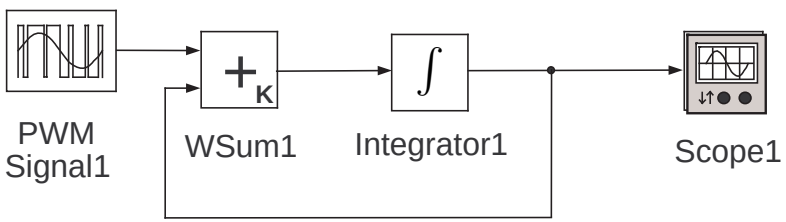

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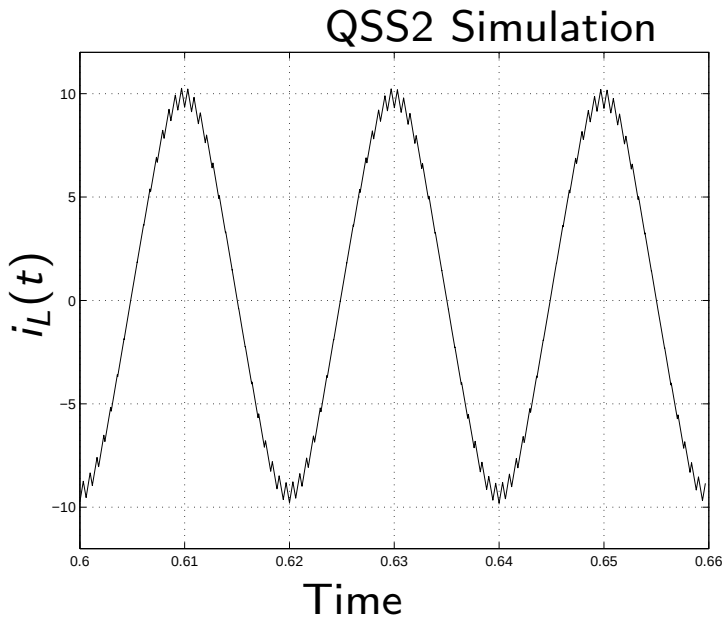

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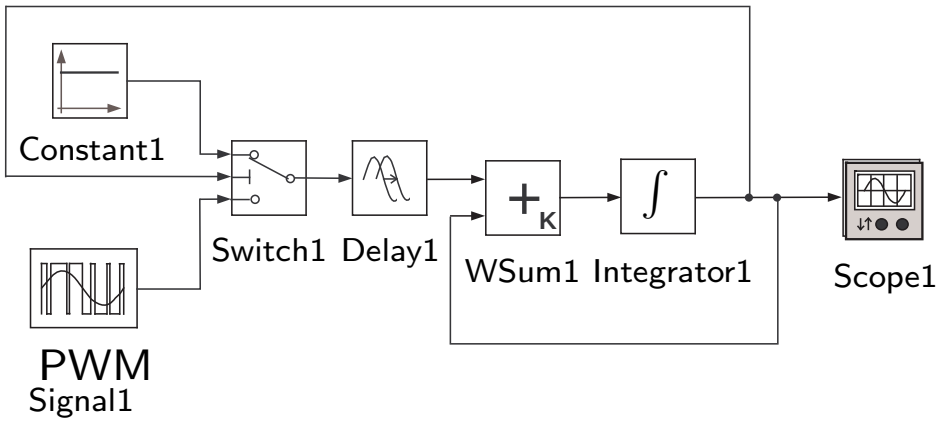

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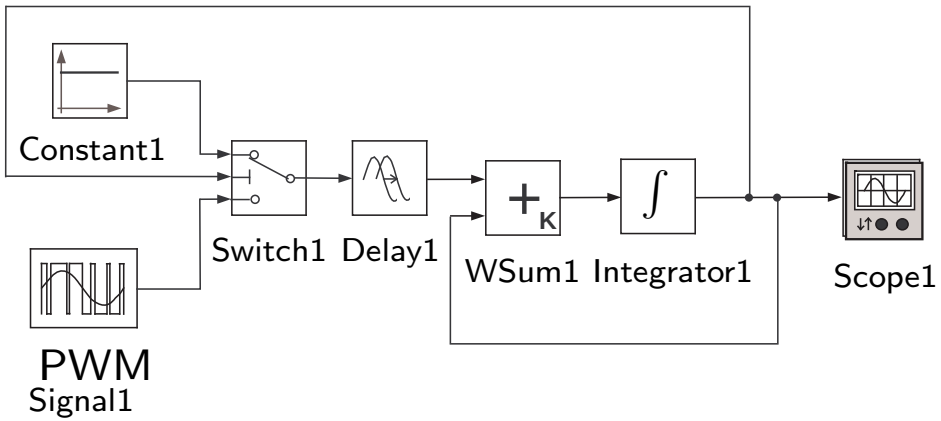

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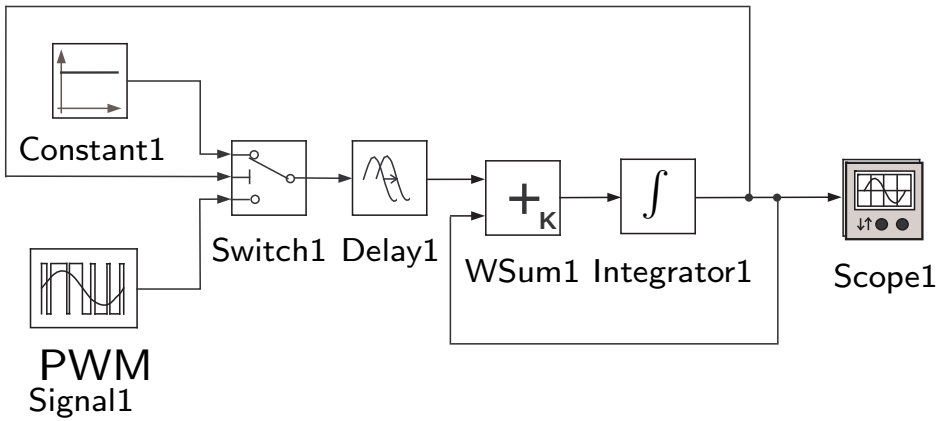

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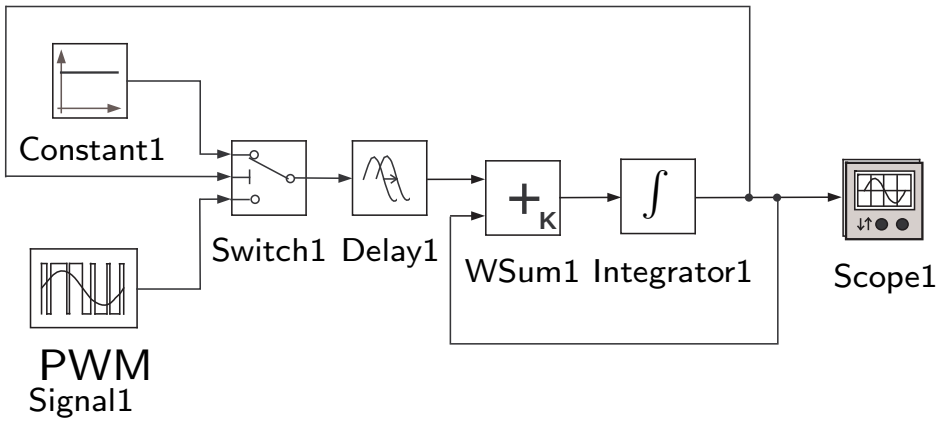

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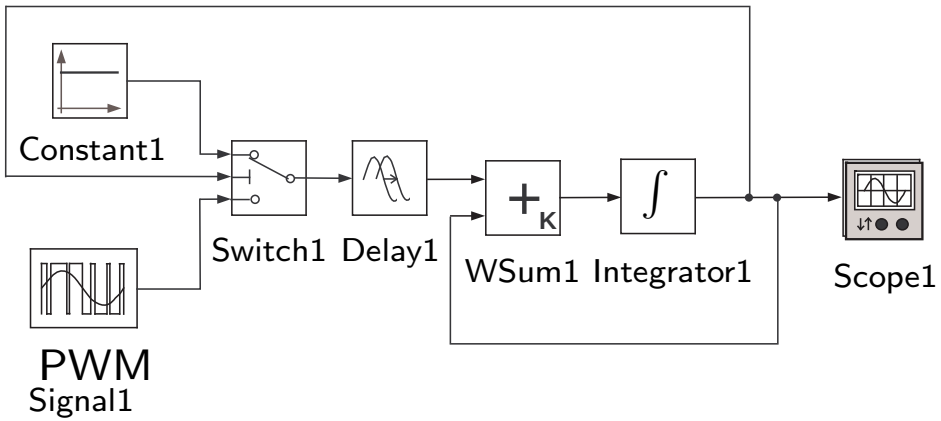

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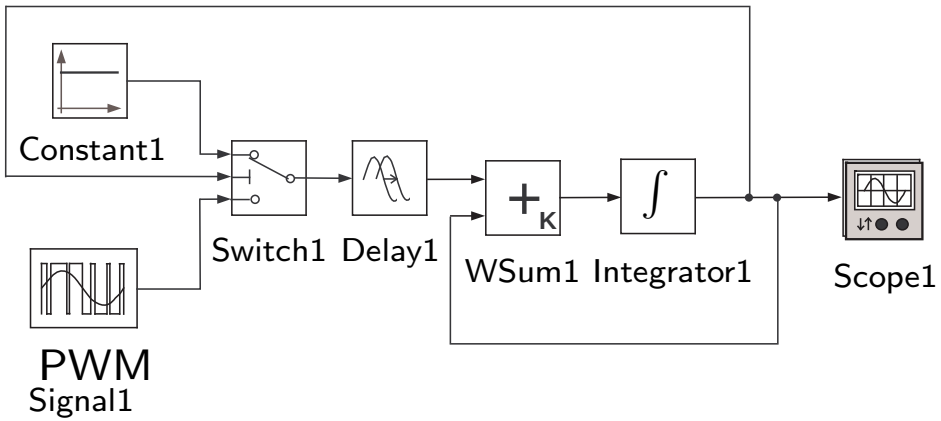

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The calculation of the *commutation time* is trivial, because the signal is either piecewise linear (QSS2) or piecewise parabolic (QSS3).

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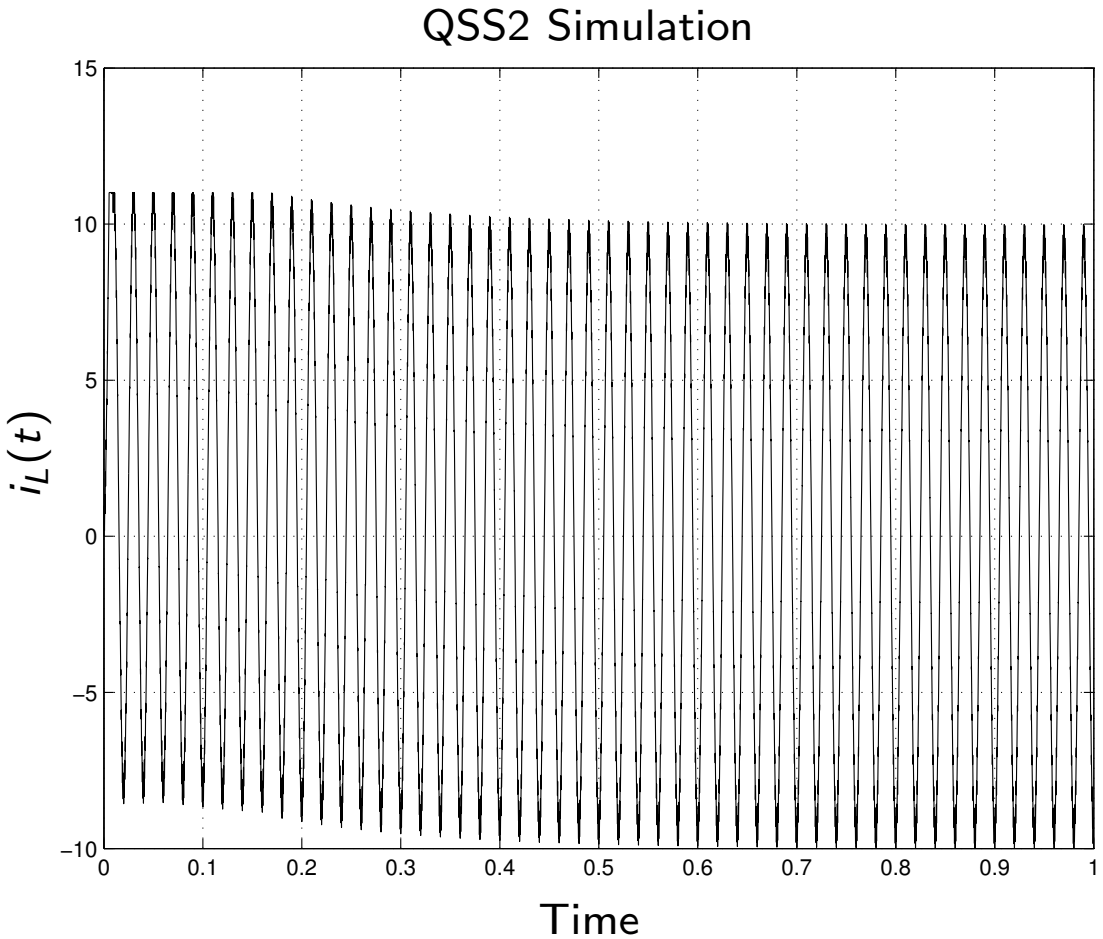

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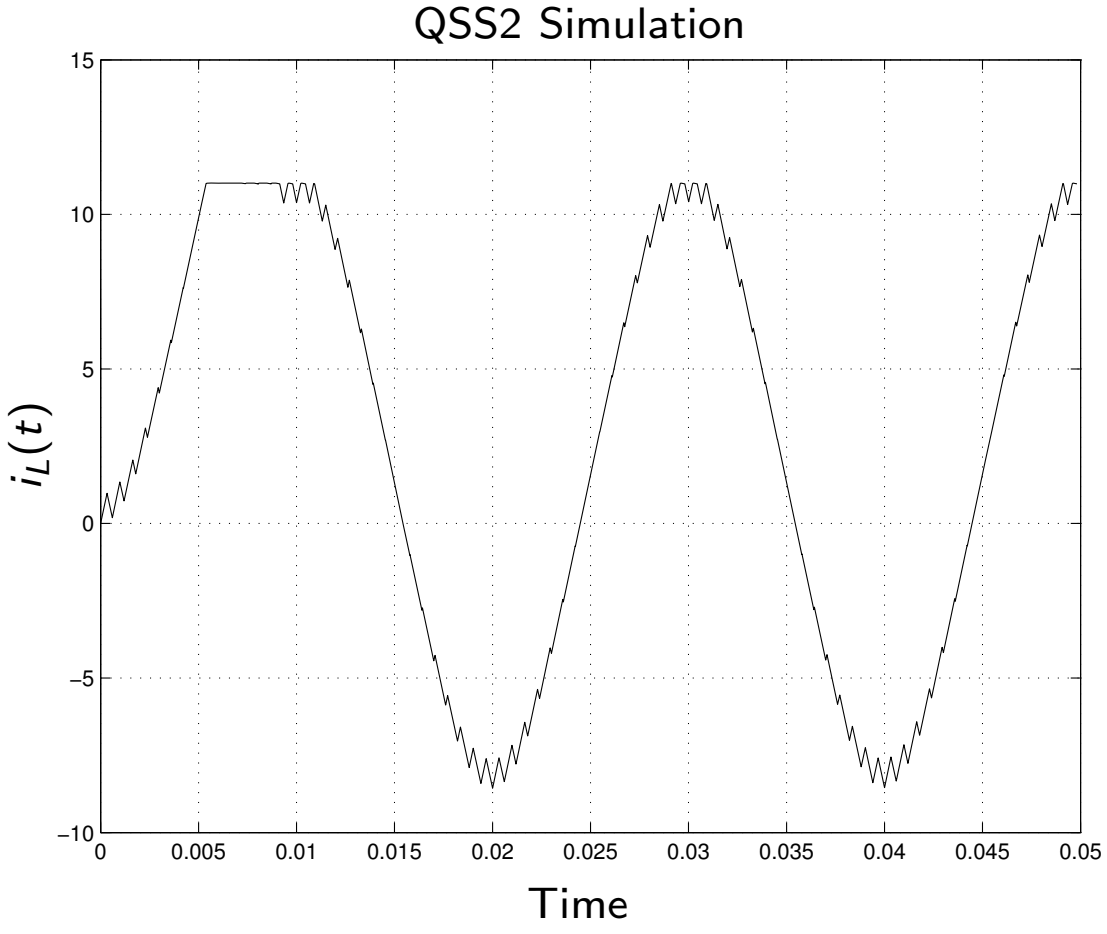

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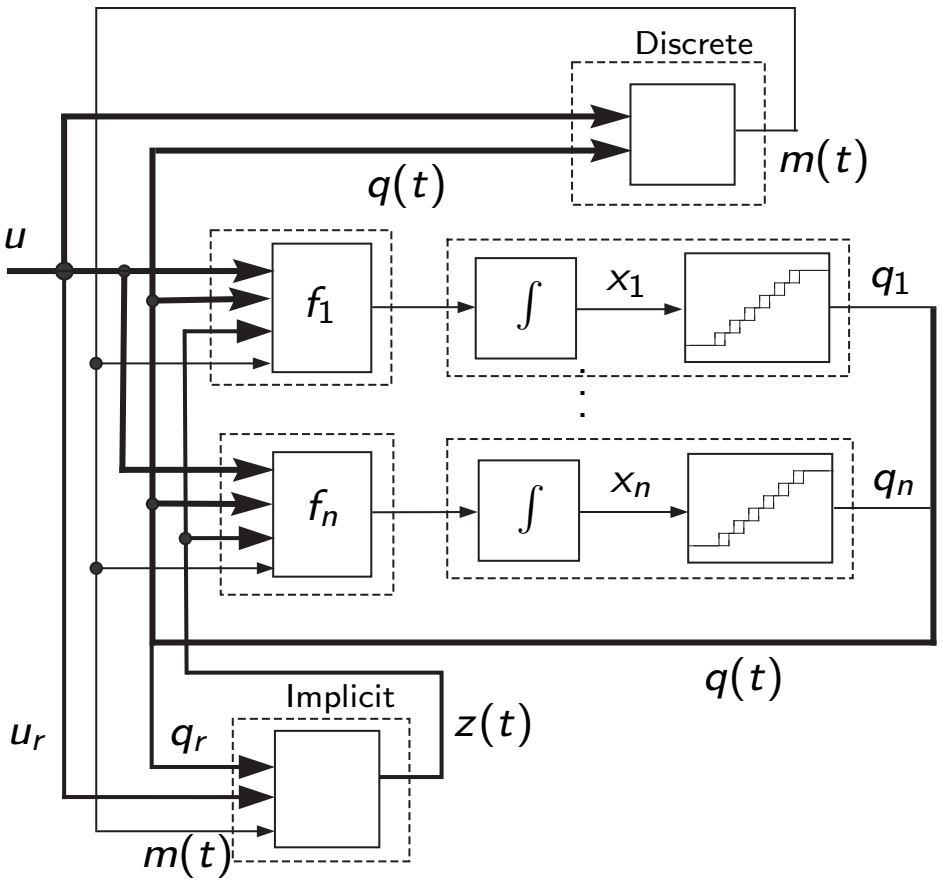

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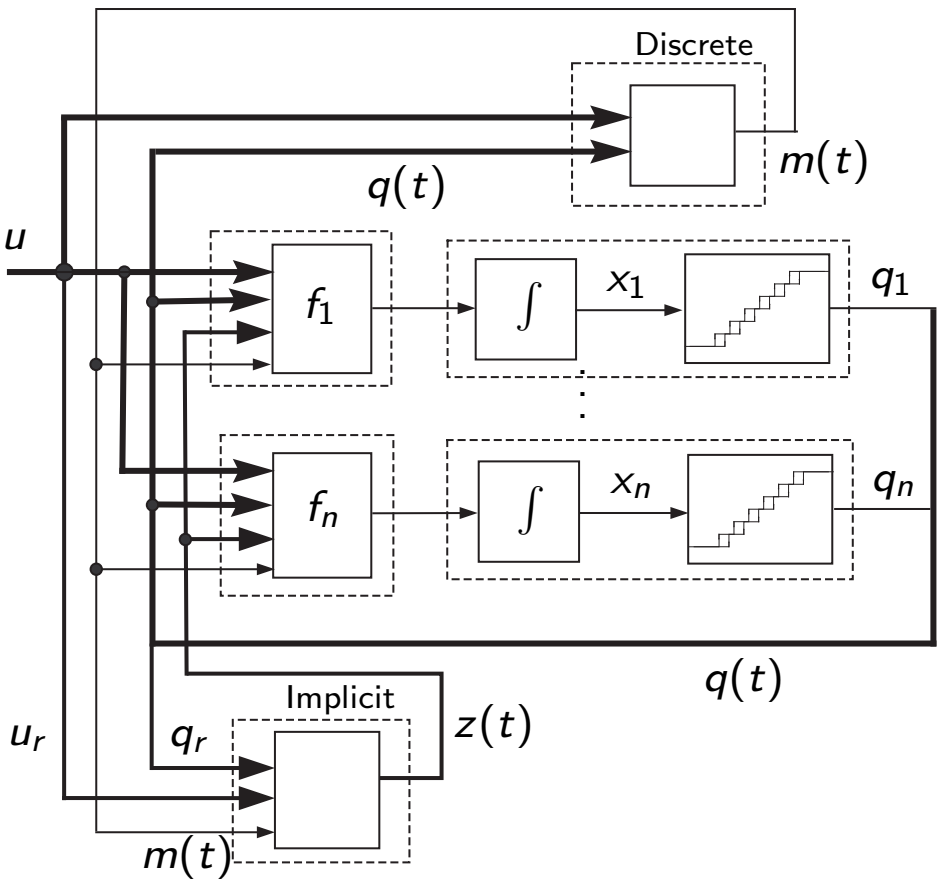

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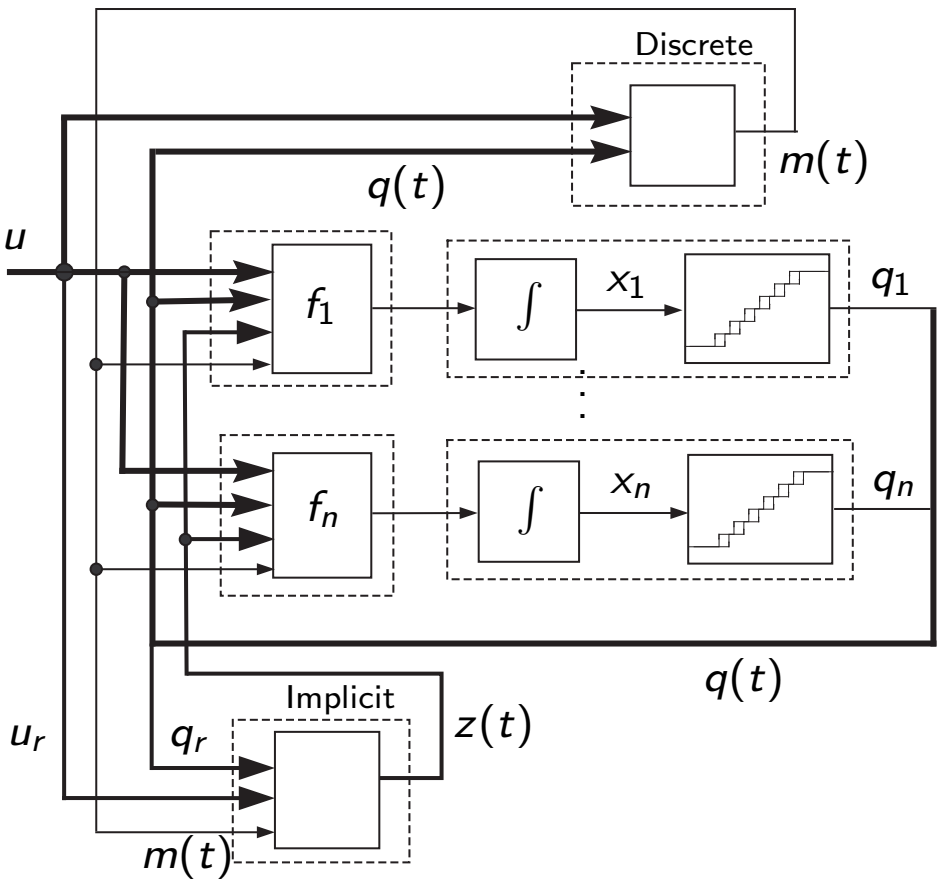

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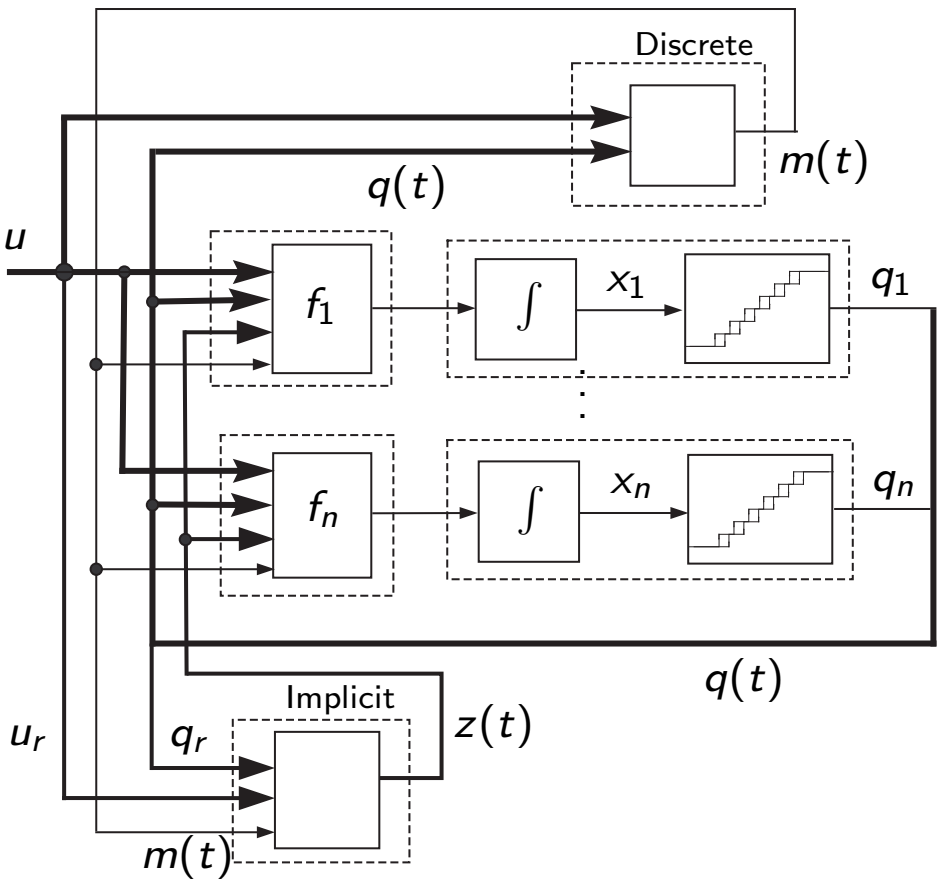

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- To support the computation of the signal $m(t)$, **PowerDEVS** offers a set of different blocks including *switches*, *quantizers*, *comparators*, etc.

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References

1. Cellier, F.E., E. Kofman, G. Migoni, and M. Bortolotto (2008), "Quantized State System Simulation," *Proc. GCMS'08, Grand Challenges in Modeling and Simulation*, part of *SCSC'08, Summer Computer Simulation Conference*, Edinburgh, Scotland, pp. 504-510.

2. Sanz, V., A. Urquía, S. Dormido, and F.E. Cellier (2010), "System Modeling Using the Parallel DEVS Formalism and the Modelica Language," *Simulation Modelling Practice and Theory*, **18**(7), pp.998-1018.

3. Sanz, V., F.E. Cellier, A. Urquía, and S. Dormido (2009), "Modeling of the ARGESIM 'Crane and Embedded Controller' System Using the DEVSLib Modelica Library," *Proc. ADHS'09: 3rd IFAC Conference on Analysis and Design of Hybrid Systems*, Zaragoza, Spain.

4. Beltrame, T. and F.E. Cellier (2006), "Quantized State System Simulation in Dymola/Modelica Using the DEVS Formalism," *Simulation News Europe*, **16**(3), pp.3-12.

5. Beltrame, Tamara (2006), *Design and Development of a Dymola/Modelica Library for Discrete Event-oriented Systems Using DEVS Methodology*, MS Thesis, Dept. of Computer Science, ETH Zurich, Switzerland.

References II

1. Floros, X., F.E. Cellier, and E. Kofman (2010), "Discretizing Time or States? A Comparative Study between DASSL and QSS," *Proc. 3rd International Workshop on Equation-based Object-oriented Modeling Languages and Tools*, Oslo, Norway, pp.107-115.

2. Floros, X., F. Bergero, F.E. Cellier, and E. Kofman (2011), "Automated Simulation of Modelica Models with QSS Methods – The Discontinuous Case," *Proc. 8th International Modelica Conference*, Dresden, Germany.