Jordan-canonical Form:

Given the transfer function:

$$G(s) = \frac{3s^2 + 9s + 10}{s^3 + 6s^2 + 11s + 6}$$

We have already seen that we can find at once two different state-space representations for this system:

(a) Controller-canonical form:

$$\dot{\underline{x}} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -6 & -11 & -6 \end{bmatrix} \underline{x} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u$$

$$y = \begin{bmatrix} 10 & 9 & 3 \end{bmatrix} \underline{x}$$

(b) Observer-canonical form:

$$\dot{\underline{x}} = \begin{bmatrix} 0 & 0 & -6 \\ 1 & 0 & -11 \\ 0 & 1 & -6 \end{bmatrix} \underline{x} + \begin{bmatrix} 10 \\ 9 \\ 3 \end{bmatrix} u$$

$$y = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} \underline{x}$$

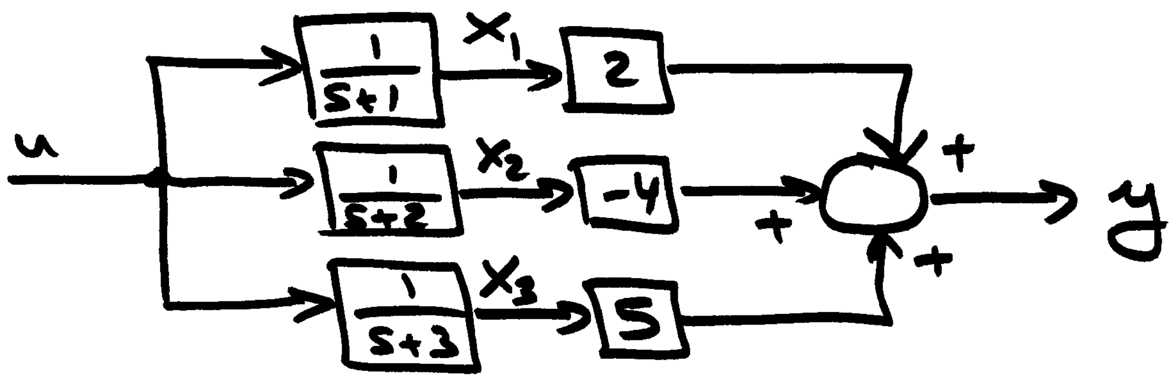
We shall now see a third canonical representation. It is based on the partial fraction expansion of the transfer function.

$$G(s) = \frac{3s^2 + 9s + 10}{s^3 + 6s^2 + 11s + 6}$$

$$= \frac{3s^2 + 9s + 10}{(s+1)\cdot(s+2)\cdot(s+3)}$$

$$= \frac{2}{s+1} + \frac{-4}{s+2} + \frac{5}{s+3}$$

Block diagram:



$$u \longrightarrow \boxed{\frac{1}{s+1}} \longrightarrow x_1$$

$$x_1(s) = \frac{1}{s+1} \cdot u(s)$$

$$\Rightarrow s x_1(s) + x_1(s) = u(s)$$

$$\dot{x}_1(t) + x_1(t) = u(t)$$

assuming zero initial conditions

$$\Rightarrow \boxed{\dot{x}_1 = -x_1 + u}$$

Similarly:

$$\dot{x}_2 = -2x_2 + u$$
$$\dot{x}_3 = -3x_3 + u$$
$$y = 2x_1 - 4x_2 + 5x_3$$

In matrix-vector form:

$$\dot{\underline{x}} = \begin{bmatrix} -1 & \phi & \phi \\ \phi & -2 & \phi \\ \phi & \phi & -3 \end{bmatrix} \underline{x} + \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} u$$

$$y = \begin{bmatrix} 2 & -4 & 5 \end{bmatrix} \underline{x}$$

This is called the Jordan-canonical form. It has the advantage of decoupling the equations as much as possible. In the given example, 3 1st-order ODEs need to be solved rather than one 3^{rd}-order equation.

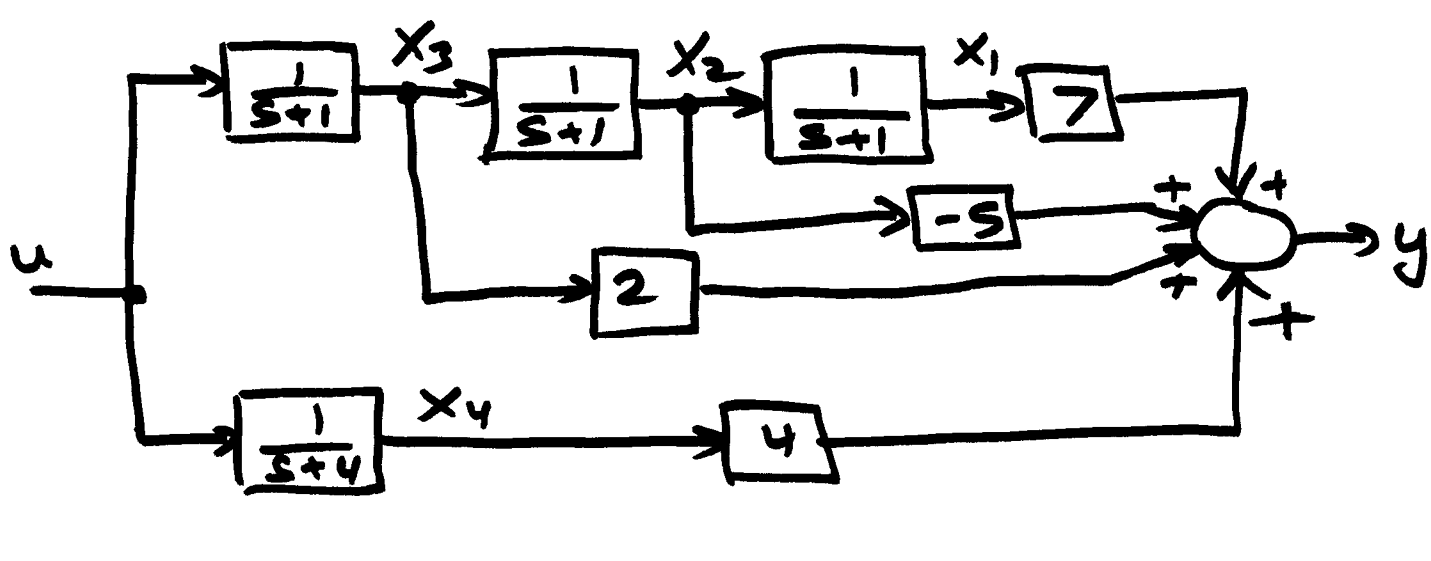
Example:

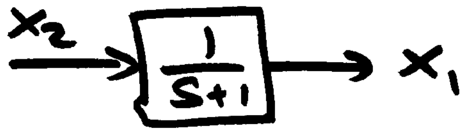
$$G(s) = \frac{6s^3 + 19s^2 + 12s + 20}{s^4 + 7s^3 + 15s^2 + 13s + 4}$$

$$= \frac{6s^3 + 19s^2 + 12s + 20}{(s+1)^3 \cdot (s+4)}$$

$$= \frac{7}{(s+1)^3} + \frac{-5}{(s+1)^2} + \frac{2}{s+1} + \frac{4}{s+4}$$

Block diagram:





In analogy with the previous example:

$$\dot{x}_1 = -x_1 + x_2$$
$$\dot{x}_2 = -x_2 + x_3$$
$$\dot{x}_3 = -x_3 + u$$
$$\dot{x}_4 = -4x_4 + u$$
$$y = 7x_1 - 5x_2 + 2x_3 + 4x_4$$

$$\Rightarrow \quad \dot{\underline{x}} = \left[\begin{array}{ccc|c} -1 & +1 & 0 & 0 \\ 0 & -1 & +1 & 0 \\ 0 & 0 & -1 & 0 \\ \hline 0 & 0 & 0 & -4 \end{array}\right] \underline{x} + \left[\begin{array}{c} 0 \\ 0 \\ 1 \\ \hline 1 \end{array}\right] u$$

$$y = \left[\begin{array}{ccc|c} 7 & -5 & 2 & 4 \end{array}\right] \underline{x}$$

The Jordan-canonical form has a system (A) matrix

That is block-diagonal. Each diagonal block (so-called Jordan block) contains one root (possibly multiple) along the diagonal, and a superdiagonal of 1's. The corresponding elements of the input ($\underline{b}$) vector are all zero except for the last element that is 1. The elements of the output ($\underline{c}'$) vector are the residua associated with the terms of the PFE.