Digital Control Systems

Second Edition

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Saunders College Publishing
A Harcourt Brace Jovanovich College Publisher

Ft. Worth Philadelphia San Diego New York Orlando Austin
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Table of Laplace Transforms, z-Transforms, and Modified z-Transforms

Laplace Transform $F(s)$	Time Function $f(t), t > 0$	z-Transform $F(z)$	Modified z-Transform $F(z, m)$
1	$\delta(t)$	1	0
e^{-kTs}	$\delta(t - kT)$	z^{-k}	z^{-k-1+m}
$\frac{1}{s}$	$u_s(t)$	$\frac{z}{z-1}$	$\frac{1}{z-1}$
$\frac{1}{s^2}$	t	$\frac{Tz}{(z-1)^2}$	$\frac{mT}{z-1} + \frac{T}{(z-1)^2}$
$\frac{2}{s^3}$	t^2	$\frac{T^2 z(z+1)}{(z-1)^3}$	$T^2 \frac{m^2 z^2 + (2m - 2m^2 + 1)z + (m-1)^2}{(z-1)^3}$
$\frac{(k-1)!}{s^k}$	t^{k-1}	$\lim_{a \to 0} (-1)^{k-1} \frac{\partial^{k-1}}{\partial a^{k-1}} \left[\frac{z}{z - e^{-aT}} \right]$	$\lim_{a \to 0} (-1)^{k-1} \frac{\partial^{k-1}}{\partial a^{k-1}} \left(\frac{e^{-amT}}{z - e^{-aT}} \right)$
$\frac{1}{s+a}$	e^{-at}	$\frac{z}{z - e^{-aT}}$	$\frac{e^{-amT}}{z - e^{-aT}}$
$\frac{1}{(s+a)^2}$	te^{-at}	$\frac{Tze^{-aT}}{(z - e^{-aT})^2}$	$\frac{Te^{-amT}[e^{-aT} + m(z - e^{-aT})]}{(z - e^{-aT})^2}$
$\frac{(k-1)!}{(s+a)^k}$	$t^k e^{-at}$	$(-1)^k \frac{\partial^k}{\partial a^k} \left[\frac{z}{z - e^{-aT}} \right]$	$(-1)^k \frac{\partial^k}{\partial a^k} \left[\frac{e^{-amT}}{z - e^{-aT}} \right]$
$\frac{a}{s(s+a)}$	$1 - e^{-at}$	$\frac{z(1 - e^{-aT})}{(z-1)(z - e^{-aT})}$	$\frac{(1 - e^{-amT})z + (e^{-amT} - e^{-aT})}{(z-1)(z - e^{-aT})}$

Laplace Transform $F(s)$	Time Function $f(t),\ t > 0$	z-Transform $F(z)$	Modified z-Transform $F(z, m)$
$\dfrac{1}{(s+a)(s+b)}$	$\dfrac{1}{(b-a)}(e^{-at} - e^{-bt})$	$\dfrac{1}{(b-a)}\left[\dfrac{z}{z-e^{-aT}} - \dfrac{z}{z-e^{-bT}}\right]$	$\dfrac{1}{(b-a)}\left[\dfrac{e^{-amT}}{z-e^{-aT}} - \dfrac{e^{-bmT}}{z-e^{-bT}}\right]$
$\dfrac{a}{s^2(s+a)}$	$t - \dfrac{1}{a}(1 - e^{-at})$	$\dfrac{Tz}{(z-1)^2} - \dfrac{(1-e^{-aT})z}{a(z-1)(z-e^{-aT})}$	$\dfrac{T}{(z-1)^2} + \dfrac{amT-1}{a(z-1)} + \dfrac{e^{-amT}}{a(z-e^{-aT})}$
$\dfrac{1}{(s+a)^2}$	te^{-at}	$\dfrac{Tze^{-aT}}{(z-e^{-aT})^2}$	$\dfrac{Te^{-amT}[e^{-aT} + m(z-e^{-aT})]}{(z-e^{-aT})^2}$
$\dfrac{a}{s^3(s+a)}$	$\dfrac{1}{2}\left(t^2 - \dfrac{2}{a}t + \dfrac{2}{a^2} - \dfrac{2}{a^2}e^{-at}\right)$	$\dfrac{T^2 z}{(z-1)^3} + \dfrac{(aT-2)Tz}{2a(z-1)^2} + \dfrac{z}{a^2(z-1)} - \dfrac{z}{a^2(z-e^{-aT})}$	$\dfrac{T^2}{(z-1)^3} + \dfrac{T^2(m+\frac{1}{2})a - T}{a(z-1)^2} + \dfrac{(amT)^2/2 - amT + 1}{a^2(z-1)} - \dfrac{e^{-amT}}{a^2(z-e^{-aT})}$
$\dfrac{a^2}{s(s+a)^2}$	$u_s(t) - (1+at)e^{-at}$	$\dfrac{z}{z-1} - \dfrac{z}{z-e^{-aT}} - \dfrac{aTe^{-aT}z}{(z-e^{-aT})^2}$	$\dfrac{1}{z-1} - \left[\dfrac{1+amT}{z-e^{-aT}} + \dfrac{aTe^{-aT}}{(z-e^{-aT})^2}\right]e^{-amT}$
$\dfrac{a^2}{s^2(s+a)^2}$	$t - \dfrac{2}{a} + \left(t + \dfrac{2}{a}\right)e^{-at}$	$\dfrac{1}{a}\left[\dfrac{(aT+2)z - 2z^2}{(z-1)^2} + \dfrac{2z}{z-e^{-aT}} + \dfrac{aTe^{-aT}z}{(z-e^{-aT})^2}\right]$	$\dfrac{1}{a}\left\{\dfrac{aT}{(z-1)^2} + \dfrac{amT-2}{z-1} + \left[\dfrac{aTe^{-aT}}{(z-e^{-aT})^2} - \dfrac{amT-2}{z-e^{-aT}}\right]e^{-amT}\right\}$
$\dfrac{\omega}{s^2+\omega^2}$	$\sin \omega t$	$\dfrac{z \sin \omega T}{z^2 - 2z\cos \omega T + 1}$	$\dfrac{\sin m\omega T + \sin(1-m)\omega T}{z^2 - 2z\cos \omega T + 1}$

Laplace Transform $F(s)$	Time Function $f(t), t > 0$	z-Transform $F(z)$	Modified z-Transform $F(z, m)$
$\frac{s}{s^2+\omega^2}$	$\cos \omega t$	$\frac{z(z-\cos \omega T)}{z^2 - 2z\cos \omega T + 1}$	$\frac{\cos m\omega T - \cos(1-m)\omega T}{z^2 - 2z\cos\omega T + 1}$
$\frac{\omega}{s^2-\omega^2}$	$\sinh \omega t$	$\frac{z \sinh \omega T}{z^2 - 2z\cosh \omega T + 1}$	$\frac{\sinh m\omega T + \sinh(1-m)\omega T}{z^2 - 2z\cosh\omega T + 1}$
$\frac{s}{s^2-\omega^2}$	$\cosh \omega t$	$\frac{z(z-\cosh \omega T)}{z^2 - 2z\cosh \omega T + 1}$	$\frac{\cosh m\omega Tz - \cosh(1-m)\omega T}{z^2 - 2z\cosh\omega T + 1}$
$\frac{\omega}{(s+a)^2+\omega^2}$	$e^{-at}\sin \omega t$	$\frac{ze^{-aT}\sin \omega T}{z^2 - 2ze^{-aT}\cos \omega T + e^{-2aT}}$	$\frac{e^{-amT}[z\sin m\omega T + e^{-aT}\sin(1-m)\omega T]}{z^2 - 2ze^{-aT}\cos\omega T + e^{-2aT}}$
$\frac{a^2+\omega^2}{s[(s+a)^2+\omega^2]}$	$1 - e^{-at}\sec\phi\cos(\omega t + \phi)$	$\frac{z}{z-1} - \frac{z^2 - ze^{-aT}\sec\phi\cos(\omega T - \phi)}{z^2 - 2ze^{-aT}\cos\omega T + e^{-2aT}}$	$\frac{1}{z-1} - \frac{e^{-maT}\sec\phi(Az - B)}{z^2 - 2ze^{-aT}\cos\omega T + e^{-2aT}}$
	$\phi = \tan^{-1}(-a/\omega)$		$A = \cos(m\omega T + \phi)$
			$B = e^{-aT}\cos[(1-m)\omega T - \phi]$
$\frac{s+a}{(s+a)^2+\omega^2}$	$e^{-at}\cos \omega t$	$\frac{z^2 - ze^{-aT}\cos\omega T}{z^2 - 2ze^{-aT}\cos\omega T + e^{-2aT}}$	$\frac{e^{-maT}[z\cos m\omega T + e^{-aT}\sin(1-m)\omega T]}{z^2 - 2ze^{-aT}\cos\omega T + e^{-2aT}}$