Impulse-bond Graphs

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Keywords: impulse modeling, mechanical impacts, graphical modeling, object-oriented modeling, physical systems modeling.

Abstract

This paper presents a new supplemental tool for bondgraphic modeling, called the impulse-bond graph (IBG). These graphs are an extension of the classic bondgraphic notation and provide a proper way for describing discrete processes in physical systems. The Impulse-bond graphs are especially useful for modeling collision and transition processes in mechanical systems, but they can be applied to other domains as well. Modeling a system with the aid of IBGs is intuitive, since the IBG can be derived from the corresponding regular bond graph (BG) by applying a small number of fixed conversion rules. Hence, an experienced modeler can use his or her knowledge about classic bondgraphic models of continuous physical systems to create a corresponding IBG.

1. MOTIVATION

Although mechanical systems can be completely described by continuous processes, it is not always convenient to model everything in this fashion. Certain processes can be simulated more efficiently and more accurately by the use of discrete event modeling. The most obvious application is the modeling of elastic or semi-elastic collisions between two objects. Such collisions lead to an apparently sudden change of the velocity difference:

$$\Delta v_{post} = -\varepsilon \cdot \Delta v_{pre}$$

Of course, it is possible to model hard impacts also by purely continuous models [10], but this leads invariably to a very stiff model, and such models are difficult to numerically integrate with high precision. Such models can only be simulated with a high computational effort, and yet, the precision of the simulation results may nevertheless be unacceptably poor.

The need for modeling sudden velocity changes can also occur in seemingly soft and continuous processes. The transition of a sliding object from slippage to ideal adhesion is such an example: A constant friction force leads to the reduction of the slip velocity. When the corresponding inert body element has finally come to rest, the constraint equation (v = 0) ensures adhesion.

This is a purely continuous process, but such a system becomes unfortunately very stiff for small sliding velocities, and the velocity vector of the body is very unlikely to become zero for numerical reasons. Hence the modeler is advised to define a range of adhesion for small sliding velocities. Whenever the body element enters this region, the constraint equation of the adhesion is activated. The subtle difference is now that the added constraint equation cannot be expected to be satisfied at the time of its introduction. The constraint equation has to be enforced.

In general, one can state that sudden changes of velocity appear whenever a new constraint equation needs to be enforced at the level of velocity.¹ Sometimes this constraint equation is only valid for a certain moment (as in a collision), whereas in other examples, the constraint is more durable and changes the dynamic structure of the model.

Various methods have been developed to treat such discontinuities in a bondgraphic modeling framework. Ideal bondgraphic switches [12] have been developed for this purpose and have been successfully applied to model mechanical collisions [5, 8, 9]. Alternatively one can use modulated transformers in combination with sinks [1].

Lorenz [8] pointed out rightfully that also the transition processes between the different modes must be correctly modeled, since the enforcement of a new constraint has to be defined in a physically correct way. In the situation of complex mechanical systems, this can be a non-trivial task that requires a large system of linear equations to be solved.

Such a transition can be modeled in an implicit fashion by the propagation of Dirac pulses through the continuous differential equations. Another method is that the pulse values are calculated from the model equations of the continuous bond graph model [2]. From a physical perspective, one can derive the impulse equations by applying the corre-

¹ The enforcement of such a constraint is often the direct consequence of the introduction of a positional constraint.

sponding conservation laws [8]. Sadly, these methods are all non-graphical and are therefore only loosely connected to the original bond graph.

Contrary to these approaches, we present a method that enables the modeler to represent also the transition processes in a bondgraphic framework. Obviously, it is not possible to use standard bonds for this purpose since they are limited to continuous processes. Hence we developed a new variant of bonds that is closely related to classic bonds and that supports the discrete modeling of fast transition processes: Impulse Bond Graphs.

These graphs are based upon the observation that a sudden change of a bondgraphic variable in a storage element is accompanied by an impulse (integral amount) of its dual counterpart: In a mechanical system, a sudden change of velocity goes along with an impulse of force that assures the conservation of momentum. In an electrical capacitor, a sudden change of voltage goes along with an impulse of current that assures the conservation of charge.

Further on, a method will be presented that shows how to derive the correct impulse-bond graph out of an existing continuous bond graph model. This allows a modeler to automatically transfer the knowledge contained in the regular BG to the IBG.

2. DEFINITION OF IMPULSE BOND

Standard bond graphs are not able to model discrete events. A bond graph describes the continuous flow of power; yet, a force impulse goes along with sudden exchange of energy in a storage element. However, this transmission of energy nevertheless underlies the rules of thermodynamics and describes a physical process. To handle such energy transmissions, a new BG variant is introduced: the impulse-bond graph (IBG). An impulse-bond is represented by a two-headed harpoon, as shown in figure 1. It carries an effort and a flow variable. Yet the product of these two adjugate variables represents not power any more; it represents an amount of work.² This work is then transferred between the vertex elements at a single point of time.



Figure 1. The ordinary impulse bond and its multi-bond graph equivalent

The adjugate variables of an impulse bond are closely related to the classic effort and flow variables. The work, W, is defined to be the integral of power, P. This integral is then stepwise transformed into a product of two variables:

$$W = \lim_{a \to 0} \int_0^a P \, dt = \lim_{a \to 0} \int_0^a e \cdot f \, dt$$

where a represents the width of the impulse. The flow variable f is supposed to change discretely. The impulse of effort is then:

$$p = \int_0^a e \, \mathrm{d}t$$

It is a necessary prerequisite for any kind of impulse modeling that the shape of the integral curve *e* is completely irrelevant. Hence we can assume without further loss of generality that *e* is: e = p/a.

$$W = \lim_{a \to 0} \frac{p}{a} \int_0^a f \, dt$$

The flow is supposed to be linearly changing from a starting value f_{pre} to a final value f_{post} . This statement reduces the generality of IBGs to linear storage elements.

$$W = \lim_{a \to 0} \frac{p}{a} \int_{0}^{a} f_{pre} + \frac{t}{a} \cdot \left(f_{post} - f_{pre}\right) dt$$
$$W = \lim_{a \to 0} \frac{p}{a} \cdot a \cdot \frac{f_{pre} + f_{post}}{2}$$
$$W = p \cdot f_{m}$$

whereby f_m denotes the mean value of f: $f_m = (f_{pre}+f_{post})/2$. For example, a mechanical impulse bond carries the force impulse F (or M) as effort variable and the mean velocity v_m = $(v_{pre}+v_{post})/2$ as flow variable. The product of the redefined effort-flow pair now represents work.

This type of impulse bond is more precisely called effort impulse bond. Of course, there also exists a dual variant: The flow impulse bond. The mean effort and the impulse of flow form the corresponding pair of adjugate variables. The two variants can be distinguished by noting down the effort-flow pair (cf. figure 2). However, most modeling tasks require only one kind of impulse bonds. It is therefore usually sufficient to determine the used variant at a single point in the graph.



Figure 2. Different kinds of impulse bonds

² Strictly speaking, impulse-bond graphs are pseudo-bond graphs, but they nevertheless provide a representation that observes the energy conservation laws.

3. IMPULSE-BOND GRAPHS IN ME-CHANICAL SYSTEMS

Impulse-bond graphs are a supplemental tool. They are mostly associated with their corresponding continuous models. Hence the most convenient way to create an impulsebond graph is to derive it from a regular bond graph. This derivation process mostly consists of replacing certain vertex elements in the bond graph by their impulse-bond graph equivalents. Certain vertex elements are thrown out altogether, and the resulting impulse-bond graph assumes therefore a simpler structure than the original regular bond graph.



Figure 3. Two colliding spheres, both are attached to different devices



Figure 4. Bond graph model of a planar pendulum and a spring-damper system

To study this issue in more detail, let us consider the example presented in figure 3, showing a pendulum colliding with a body that is attached to a spring-damper system. Figure 4 presents the corresponding continuous bond graph. The pendulum is modeled in the upper half of the bond graph, whereas the spring-damper system is modeled in the lower half. The positional difference is measured at the 0-junction between the two sub-bond graphs using a Dq sensor element. The collision block triggers an event whenever the positional difference crosses through zero. The resulting collision is defined to be completely elastic and is modeled by the impulse-bond graph shown in figure 5.



Figure 5: Corresponding impulse-bond graph

This impulse-bond graph was derived from the continuous model. The following replacement rules³ were applied:

- Effort sources, capacitive and resistive elements do neither cause nor transmit any effort impulse and can therefore be neglected if they are connected to a 1junction. If they are connected to a 0-junction, they have to be replaced by a source of zero effort.
- All sensor elements can be removed.
- Sources of flow determine the flow variable and consequently also the average flow variable f_m . Therefore these elements remain unchanged.

³ These rules apply for effort-impulse bond graphs. The rules for flow-impulse bond graphs are very similar. One simply substitutes in the replacement rules each bond graph element with its dual counterpart.

- Linear transformers or gyrators also project the impulse variable and the average by the same linear factor. Thus, also these elements remain unchanged.
- All junctions remain.
- All modulating signals must be replaced by a constant signal for the time of the impulse. If the value of a signal isn't constant during the impulse, the impulse-bond graph is in general not valid.
- Inductances or inductive fields are still denoted by the same symbol, but they represent now different equations. A linear inductance, *I*, is now represented by the algebraic equation:

$$p = 2 \cdot I \cdot (f_m - f_{\text{pre}})$$

The actual impulse has to be modeled by a special element that is denoted here by the symbol *ISw*. This is a mnemonic for the term "impulse switch." Like a switch, the element changes its equation at the time of an event. Unlike a normal bond graph switch, these switch elements often dissipate and sometimes generate energy. The reflection law for an elastic impact states that the mean of the velocity differences before and after the impact is zero. Hence, the *ISw* element for a fully elastic impact is defined by the following equations:

> $f_m = 0$: at the time of collision. p = 0: otherwise.

This switch does neither dissipate nor generate any energy since the product $p \cdot f_m$ is always zero. The resulting impulse-bond graph (simplified by the removal of superfluous junctions) represents a linear system of equations and contains no differential equations any longer. The linearity of the storage and transformer elements is of importance, because otherwise the product of effort and flow does not represent the correct amount of work, and the IBG might be invalid. The assumptions that were made in the definition of impulse-bondgraphic effort and flow restrict the generality of IBGs to linear elements. Fortunately, the underlying impulse-bond graphs of all mechanical models are linear. This is because the inertia is always linear (in Newtonian mechanics), and all potentially non-linear elements like dampers, springs, capacitances and other force generating elements (gravitational force) disappear. The remaining nonlinear transformers change into linear ones, because the modulation by a bondgraphic position is constant during the time of the impact. Hence, there exists a valid impulsebond graph for every mechanical system.

A second example shows the application of an impulsebond graph to model the transition between rolling with slippage and ideal rolling.



Figure 6. Bond graph of a rolling marble

The multi-bond graph of figure 6 presents a simplified model of a rolling marble that is defined to be rolling on the xy-plane. Its inertia tensor is supposed to be a simple scalar. This assumption makes it feasible to resolve not only the translational but also the rotational variables with respect to the inertial system. It also assures that no gyroscopic effect will appear. The rotational motion is transformed into the corresponding translational motion at the virtual point of contact by a linear transformer *TF*. The difference between the contact point velocity and the center's velocity is derived at the 0-junction and determines the slippage. The resistance, *R*, models the non-linear friction force by:

$$\begin{bmatrix} e_{x} \\ e_{y} \\ 0 \end{bmatrix} = \begin{bmatrix} f_{x} \\ f_{y} \\ f_{z} \end{bmatrix} \cdot \frac{\mu_{R} \cdot g \cdot m}{\mid f \mid}$$

Obviously, these equations are very stiff for small values of f_x , f_y . Hence the flow sensor, Df, triggers an event when the slippage becomes too small and reaches a predefined range of adhesion. The event causes a transition to ideal rolling. This transition is modeled by the corresponding impulse-bond graph shown in figure 7.



Figure 7. Impulse-bond graph of a rolling marble

The impulse switch, *ISw*, dissipates friction energy and models a completely non-elastic impact:

$$f_{\rm m} = f_{\rm pre}/2$$
: at the time of transition
 $p = 0$: otherwise.

It follows that the slippage f_{post} after the transition is zero, since $f_{\text{m}} = f_{\text{pre}}/2 + f_{\text{post}}/2$. After the transition, the simulation can proceed with a modified variant of the continuous model, where the resistance has been removed.

Let us conclude this chapter with a more complex example concerning the model of a piston-engine (cf. figure 8), where the powering explosions within the cylinder are modeled by force impulses. This is an interesting example, because it includes a kinematic loop. The whole system consists of four joints (three revolute joints and one prismatic joint), but exhibits a single degree of freedom only.



Figure 8. A simple piston-engine

The model is created by the usage of planar mechanical multi-bonds. The composition of these multi-bonds is presented in figure 9. Since the rotational inertia J is a constant scalar for all possible orientations, all bond graph variables can be conveniently resolved with respect to the inertial system. The positional dependencies are modeled by a-causal signals. This methodology was previously introduced and successfully implemented in Dymola [4] by means of the MultiBondLib [13, 14].



Figure 9. Composition of a planar mechanical multi-bond

Figure 13 presents the continuous model of the piston engine. To give the bond graph a consistent look, regular bonds are also represented as multi-bonds with a cardinality of one. The crankshaft is modeled on the left hand side, and the piston is modeled on the right hand side. Both elements define inertia. In between, the model of the connecting rod is placed together with its two revolute joints. The weight of the connecting rod is neglected. The sensor of the piston's position triggers an ignition event, when the piston is on its way back. The capacity models the elasticity of the fuel gas.

The corresponding impulse-bond graph of figure 14 results from applying the corresponding derivation rules to the continuous multi-bond graph. All non-linearities of the original model disappear. The *ISw* element models the introduction of the explosion energy into the mechanical system:

$$p \cdot f_{\rm m} = E_{\rm explosion}$$
: at the time of ignition.
 $p = 0$: otherwise.

Impulses on kinematic loops usually lead to large systems of linear equations, but in this specific case, the *ISw* element introduces a non-linear (quadratic) equation⁴ into the system. Hence a non-linear solver is required.



Figure 10. Angular velocity of the crankshaft - the simulation was performed in Dymola

The equations of the two bond graph models together form a hybrid system. The resulting simulation leads to the plot shown in figure 10 that depicts the angular velocity of the crankshaft as a function of time.

4. OTHER DOMAINS

As bond graphs serve as a domain-independent tool for modeling physical systems, impulse-bond graphs do as well. Yet not all possible impulse bonds are meaningful. Table 1 lists a number of meaningful variants.

⁴ The quadratic equation has two solutions that are both physically correct. A restriction on $f_{\rm m}$ by the absolute function is helpful to determine, which of these two solutions is found by the non-linear equation solver. For example: $p \cdot |f_{\rm m}| = E_{\rm explosion}$

domain	effort	flow
translational	force impulse	m. velocity
	F or M	v _m
rotational	torque impulse	m. ang. velocity
	Т	ω _m
electrical	m. voltage	charge
	u _m	Q
electrical	magnetic flux	m. current
	Φ	i _m
hydraulic/	pressure impulse	m. vol. flow
acoustic	P or Γ	$q_{\rm m}$
hydraulic/	av. pressure	volume
acoustic	$p_{\rm m}$	V
chemical	m. ch. potential	moles
	$\mu_{ m m}$	Ν
thermo-	m. temperature	entropy
dynamic	T _m	S

Table 1. Effort-flow pairs of impulse-bond graphs for various application domains

Mostly, the need for impulse modeling in other domains results from an interaction with the mechanical domain. A jack-hammer with an integrated pressure pump (cf. figure 11) is such an example. The reflection of the actual hammer leads to an impulse in the pneumatic domain. However the resulting bond graph model would be fairly poor, since the actual shockwave that is reflected to the piston of the pressure pump cannot be modeled accurately using bond graphs. This is a general problem that occurs when partial differential equations are discretized (compartmentalized) for the purpose of approximating them by a bond graph.



Figure 11. A jack-hammer with integrated pressure pump

Pure non-mechanical examples of impulses are hard to find. Such models usually describe a rapid balance or fast diffusion processes. Completely inelastic impacts are a mechanical analogon for this. The sudden charge exchange between two resistor-less connected ideal capacitors serves as an academic example and has been examined also by [1, 8].



Figure 12. Rapid exchange of charge between two capacitors

5. LIMITATIONS

Impulse-bond graphs consist of storage elements, transformers (or gyrators), and junctions. Whereas these IBGelements are always linear in mechanical systems, this is not necessarily true for other domains. Hence this section discusses the occurrence of non-linearities in these elements and how this impairs the validity of the impulse-bond graph. In practice, non-linear IBGs are rare, but it is important to be aware of the theoretical limitations.

Transformers and gyrators have to be linear. Otherwise an impulse modeling is meaningless. The original continuous transformer must transform any curve x(t) to y(t) in such a way that there is a direct transformation for $\int x(t)$ to $\int y(t)$ that holds for all possible shapes of x(t). Without further proof, this implies that the transformation must be linear at the moment of the impulse.⁵ The impulse transformation represents then consequently the same linear transformation.

Modeling impulses can be meaningful also for nonlinear storage elements. For example, the capacitance, C_I , in figure 12 might have a non-linear characteristic. The kind of

⁵ Remark: For every non-linear transformation, a pair of curves $(x_1(t), x_2(t))$ can be found that share the same integral value but are transformed into a pair $(y_1(t), y_2(t))$ that differs in its integral values.

non-linearity is indeed restricted: The non-linear differential equation for the original storage element:

$$de/dt = g(f)$$

must be integrable into the form:

$$e = h(q)$$

where $q = \int f dt$, and *h* is a non-linear function of *q* that must be fully contained in the first and third quadrant of the (e,q)plane.

Unfortunately, such non-linearities violate the assumptions made in the definition of the impulse-bondgraphic effort and flow variables.⁶ Hence the product of effort and flow does not represent the correct amount of work any longer. This reduces drastically the usability of IBGs in such cases.

Junctions define by definition only linear equations. However, their semantic implications might be impaired by non-linearities that occur from non-linear storage elements. If the product of effort and flow does not represent the correct amount of work, the behavior of a junction cannot be expected to be neutral with respect to energy.

The equations of the *ISw* elements define the discrete jump in the system. In principle, these equations can be chosen arbitrarily. A non-linear equation for the *ISw* element was presented in the example of the piston-engine discussed in this paper.

6. CONCLUSIONS

Concerning the modeling of complex continuous physical systems, bond graphs offer a suitable balance between specificity and generality [7]. The interdisciplinary concept of energy and power flows creates a semantic level that helps the modeler avoid many types of modeling errors and to find an adequate solution for his or her task. If one wishes to maintain this semantic layer for the modeling of discrete transition processes in physical systems, impulse-bond graphs provide an adequate extension. In combination with classic continuous bond graphs they form a solid foundation for the modeling of hybrid systems.

The derivation rules that describe the construction of an IBG out of a continuous model allow an inheritance of knowledge. Experienced modelers should therefore be able to quickly acquaint themselves with this new modeling paradigm.

Whereas IBGs represent a good method for sketching a transition model in a graphical way, their implementation in a graphical modeling environment faces a number of practical difficulties and hence has not been attempted. The problem is not the actual simulation itself, as there exist a number of simulation systems that are capable of performing hybrid simulations. The problem consists in the need for an advanced graphical modeling environment.

IBGs go mostly along with their continuous bondgraphic counterparts, and information is needed from the regular bond graph that concerns the state right before the discrete event. So the pair of graphs is closely interrelated, and often the simultaneous graphical displays of these two bondgraphic models obstruct each other, which makes them hard to read. A graphical modeling environment that includes multiple drawing layers would be helpful, but is currently not supported in any simulation system known to us.

Nevertheless, IBGs were successfully applied during the development of the MultiBondLib [13, 14]. The Multi-BondLib is a library for the modeling and simulation of multi-bond graphs. It includes also mechanical sub-libraries that consist of an extensive set of wrapped bondgraphic submodels. Those continuous models were extended to hybrid models by means of equations that were derived and verified by the use of IBGs.

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⁶ One might be able to modify the original definition for certain applications. This might offer an interesting approach to finding a bond graph solution to the modeling of relativistic impulses (cf. [7]).



Figure 14. Corresponding impulse-bond graph of the piston engine

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BIOGRAPHIES



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