

# Automated Simulation of Modelica Models with QSS Methods

## The Discontinuous Case

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# Outline

Introduction

QSS Methods

OMPD Interface

Simulation Results

Discussion

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## Goal

  
OpenMODELICA



PowerDEVS

Design and implement an interface  
between OpenModelica and  
PowerDEVS (**OMPD Interface**)

Enable the simulation of Modelica  
models with QSS methods

## Why?

Interfacing OpenModelica and PowerDEVS we take advantage of

The powerful modeling tools and market share offered by Modelica

- ▶ Users can still define their models using the Modelica language or their favorite graphical interface.
- ▶ No prior knowledge of DEVS and QSS methods is needed.

The superior performance of quantization-based techniques in some particular problem instances

- ▶ QSS methods allow for asynchronous variable updates, which potentially speeds up the computations for real-world sparse systems.
- ▶ QSS methods do not need to iterate backwards to handle discontinuities, they rather predict them, enabling real-time simulation.

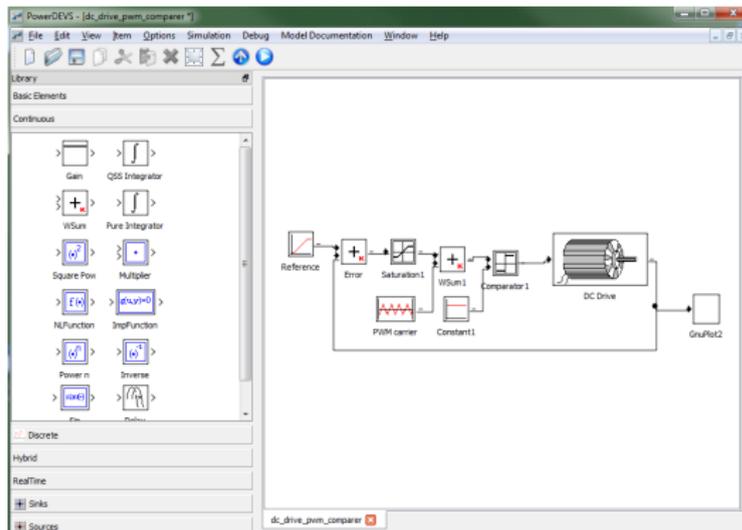


## QSS methods

Simulation of continuous systems by a digital computer requires discretization.

- ▶ Classical methods (e.g. Euler, Runge-Kutta etc.), that are implemented in Modelica environments, are based on **discretization of time**.
- ▶ On the other hand, the **Discrete Event System Specification (DEVS)** formalism, introduced by Zeigler in the 90s, enables the **discretization of states**.
- ▶ The **Quantized-State Systems (QSS)** methods, introduced by Kofman in 2001, improved the original quantized-state approach of Zeigler.
- ▶ **PowerDEVS** is the environment where QSS methods have been implemented for the simulation of systems described in DEVS.

# PowerDEVS



- ▶ Specify system structure (using DEVS formalism)
- ▶ Block implementation hidden (C++ code)
- ▶ Integrators implement the QSS methods
- ▶ Simulation using hierarchical master-slave structure and message passing

<http://sourceforge.net/projects/powerdevs/>

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## Quantized State Systems Method

### Definition

Given a system

$$\dot{x}(t) = f(x(t), t) \quad (1)$$

with  $x \in \mathbb{R}^n$ ,  $t \in \mathbb{R}$  and  $f : \mathbb{R}^{n+1} \rightarrow \mathbb{R}^n$ , the QSS approximation is given by

$$\dot{x}(t) = f(q(t), t) \quad (2)$$

where  $q(t)$  and  $x(t)$  are related componentwise by hysteretic quantization functions.

Under certain assumptions, the QSS approximation (2) is shown to be equivalent to a **legitimate** DEVS model.

## QSS Method and Perturbed Systems

Defining  $\Delta x(t) \triangleq q(t) - x(t)$ , the QSS approximation (2) can be rewritten as:

$$\dot{x}(t) = f[x(t) + \Delta x(t), t] \quad (3)$$

Notice that every component of  $\Delta x$  satisfies

$$|\Delta x_i(t)| = |q_i(t) - x_i(t)| \leq \Delta Q_i \quad (4)$$

where  $\Delta Q_i$  is the quantization width (or **quantum**) in the  $i$ -th component.

The effect of the QSS **discretization** can be studied as a problem of **bounded perturbations** over the original ODE.

At each step only one (quantized) state variable that changes more than the quantum value  $\Delta Q_i$  is updated producing a discrete event.

## Static Functions & Quantized Integrators

If we break (2) into the individual components we have that:

$$\begin{array}{ccc} \dot{x}_1 = f_1(x_1, \dots, x_n, t) & & \dot{q}_1 = f_1(q_1, \dots, q_n, t) \\ \vdots & \xrightarrow{QSS} & \vdots \\ \dot{x}_n = f_n(x_1, \dots, x_n, t) & & \dot{q}_n = f_n(q_1, \dots, q_n, t) \end{array} \quad (5)$$

Considering a single subcomponent we can define the "simple" DEVS models:

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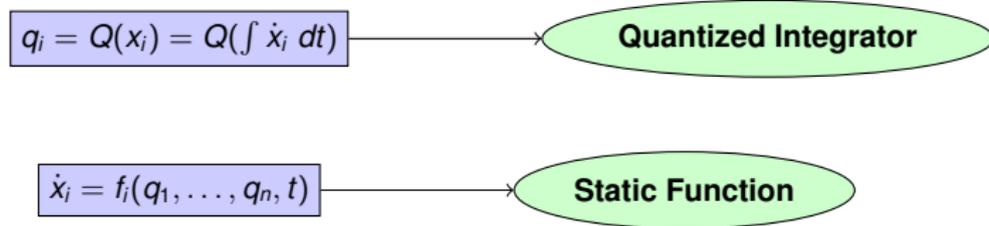


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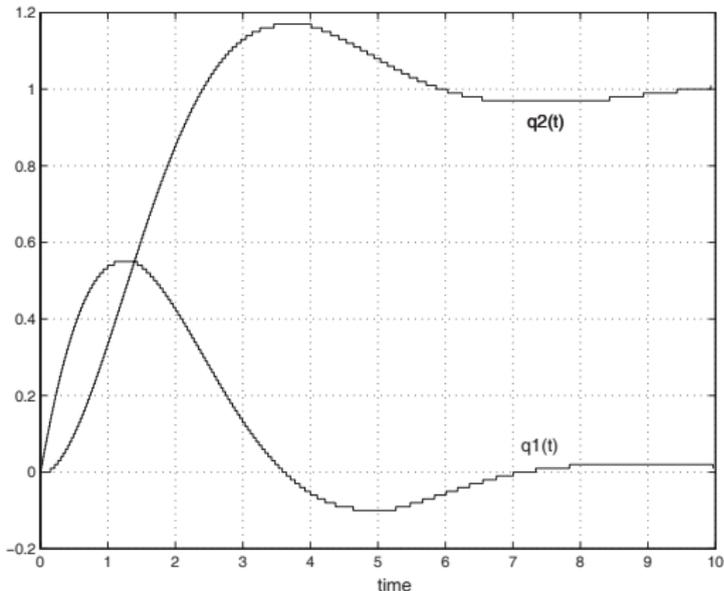
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Considering a single subcomponent we can define the "simple" DEVS models:



## QSS – Example

Solution with  $\Delta Q = 0.01$ ,  $u(t) = 1$



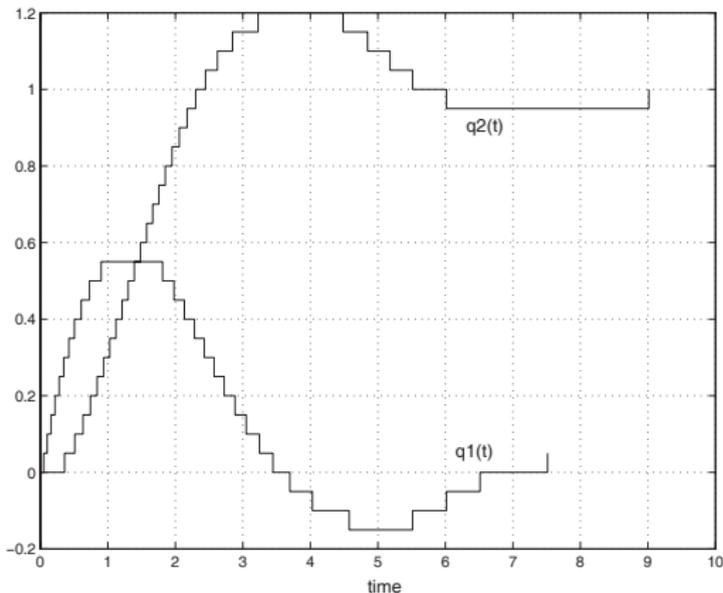
Let second order LTI system:

$$\dot{x}_1(t) = x_2(t)$$

$$\dot{x}_2(t) = -x_1(t) - x_2(t) + u(t)$$

## QSS – Example

Solution with  $\Delta Q = 0.05$ ,  $u(t) = 1$



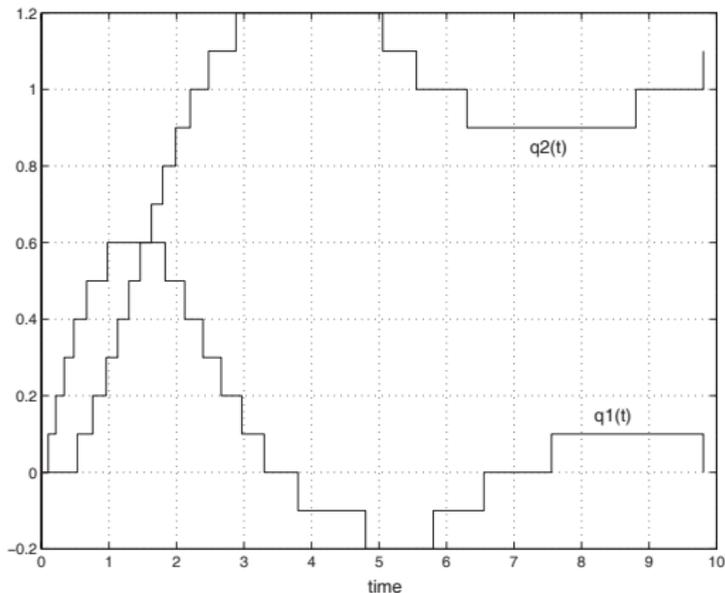
Let second order LTI system:

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## QSS – Example

Solution with  $\Delta Q = 0.1$ ,  $u(t) = 1$



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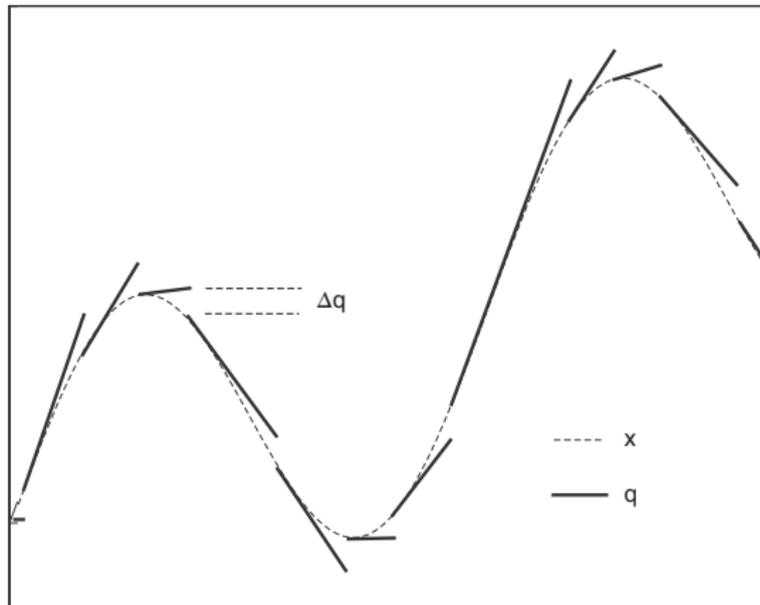
## Cost vs. Accuracy in QSS

In QSS, we know that the quantum is proportional to the global error bound.  
Thus,

- ▶ If we want to increase the global accuracy for a factor of 100, we should divide the quantum by that factor.
- ▶ Since the number of steps is inversely proportional to the quantum, that modification would increase the number of computations by a factor of 100.

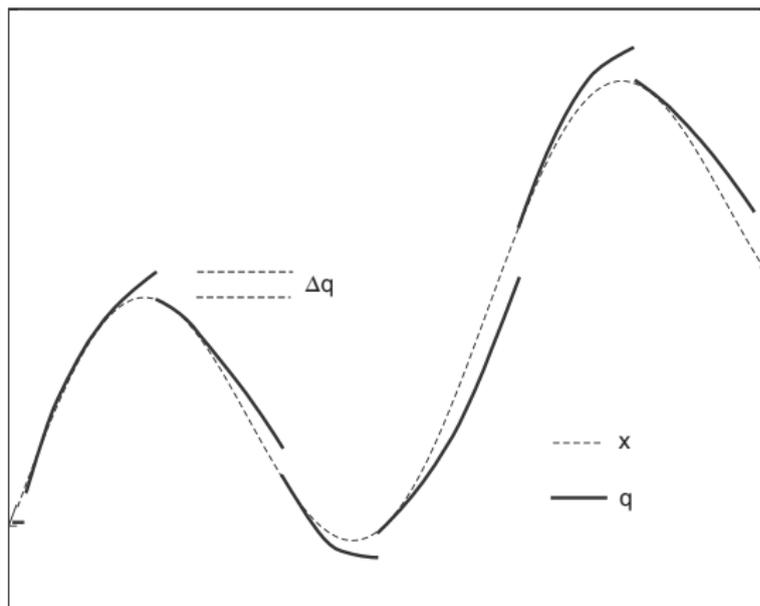
This problem is due to the fact that QSS is only **first order accurate**, i.e. it does not use information about the derivatives of  $f$ .

## Second Order QSS (QSS2 Method)



- ▶ Same definition and properties as QSS.
- ▶ **Second order** accurate method.
- ▶ The number of steps grows with the **square root** of the accuracy.
- ▶ The quantized variables have piecewise linear trajectories thus the state derivatives are also piecewise linear and the state variables piecewise parabolic.

## Third Order QSS (QSS3 Method)



- ▶ Same definition and properties as QSS.
- ▶ **Third order** accurate method.
- ▶ The number of steps grows with the **cubic root** of the accuracy.
- ▶ The method of choice for simulating real-world systems.

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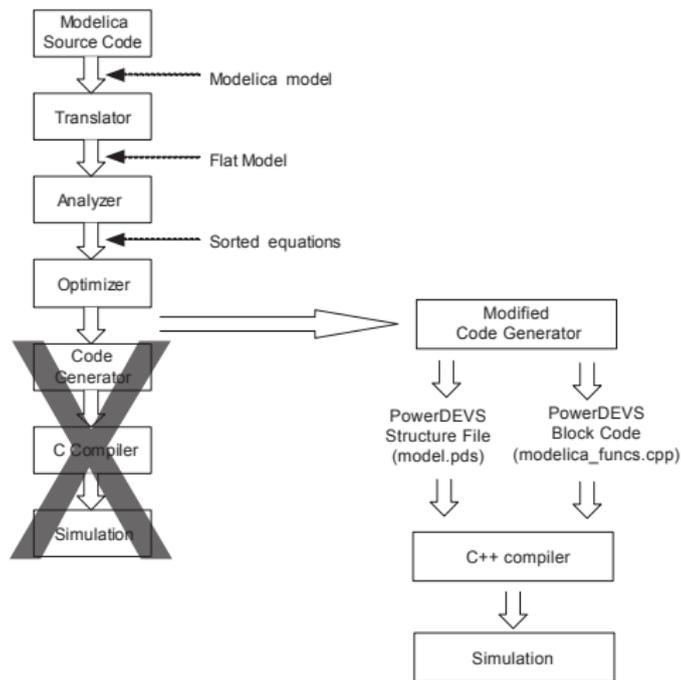
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# OpenModelica Compiler Modifications



# The Bouncing Ball Model

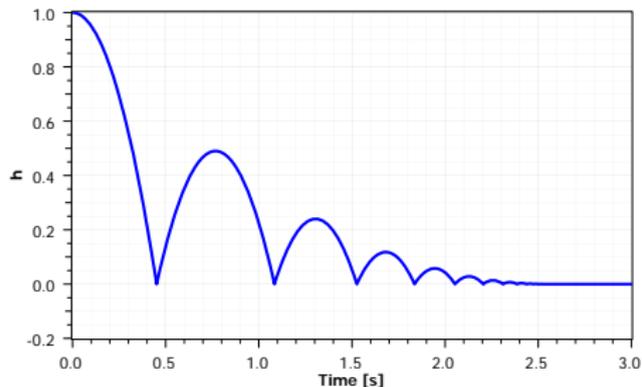
```
model BouncingBall
  parameter Real e=0.7 "coefficient of restitution";
  parameter Real g=9.81 "gravity acceleration";
  Real h(start=1) "height of ball";
  Real v "velocity of ball";
  Boolean flying(start=true) "true, if ball is flying";
  Real v_new;
  Boolean impact;
  Real dummy;
  Boolean dummy2;

equation
  der(dummy) = if (dummy>0 and h<=0) then
    dummy else h*v; // Dummy part 1
  when {sample(0,1)} // Dummy part 2
    dummy2 = false;
  end when

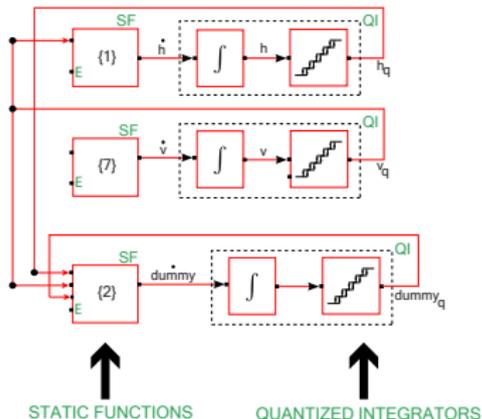
  impact = h <= 0.0;
  der(v) = if flying then -g else 0;
  der(h) = v;

  when {h <= 0.0 and v <= 0.0, impact} then
    v_new = if edge(impact) then -e*v else 0;
    flying = v_new > 0;
    reinit(v, v_new);
  end when;

end BouncingBall;
```



## Add Static Blocks for State Variables

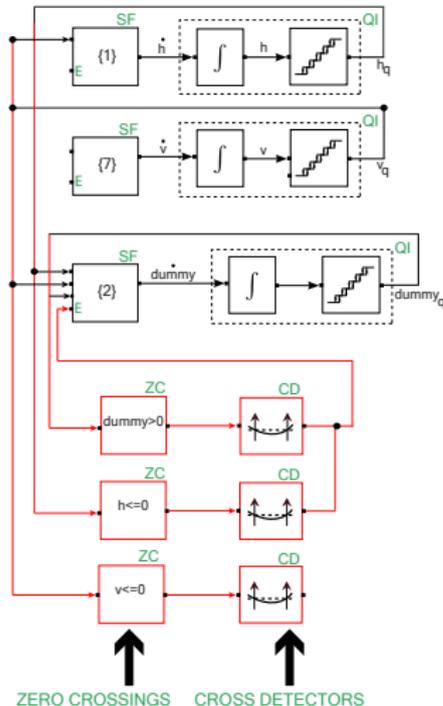


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    v_new = if edge(impact) then
        -e*v else 0; (Eq. 5)
    flying = v_new > 0; (Eq. 6)
    reinit(v, v_new);
end when;
der(v) = if flying then -g else 0; (Eq. 7)
  
```

- ▶ Extract equations (BLT blocks) needed to compute state derivative variables.
- ▶ Place the splitted equations in respective static function blocks.
- ▶ Resolve dependencies in the inputs/outputs.

## Add Zero Crossing Functions

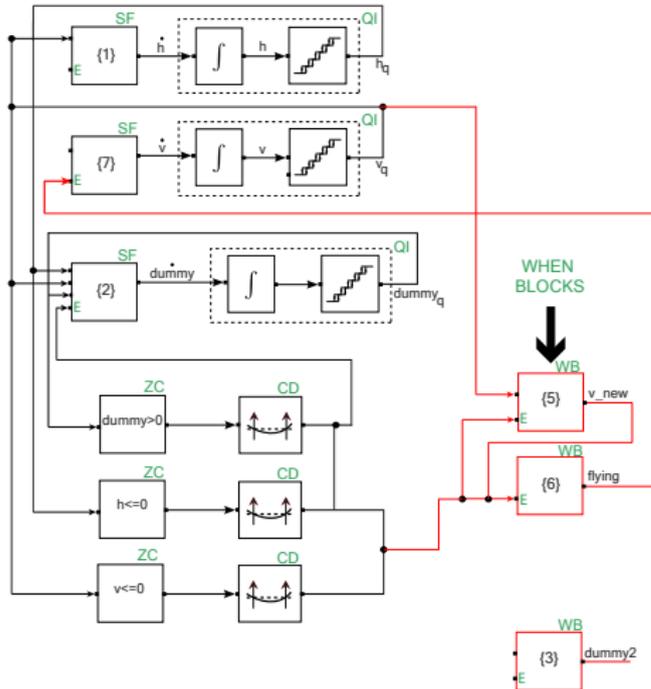


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```

- ▶ Add zero-crossing functions and the corresponding zero-cross detectors.
- ▶ Resolve dependencies in the inputs/outputs.
- ▶ The zero-cross detectors produce events at discontinuities and propagate them to the corresponding static blocks.

## Add When Blocks



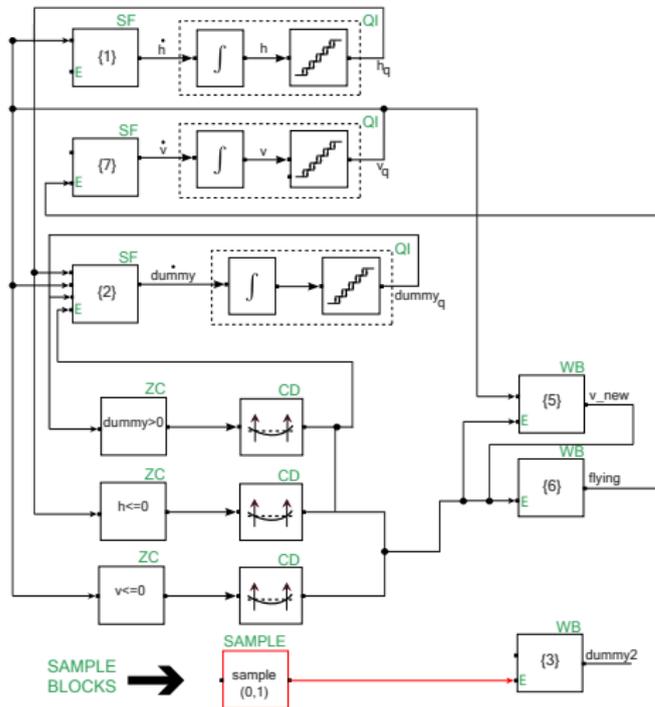
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- ▶ Add when-blocks for each generated when-clause and resolve dependencies.
- ▶ If a static function depends on a discrete variable calculated in a when-block (e.g. flying) an event is sent to the corresponding static block.
- ▶ When a cross detector fires, all the discrete variables are updated via calling the OMC function `updateDepend()`.

## Add Sample Blocks



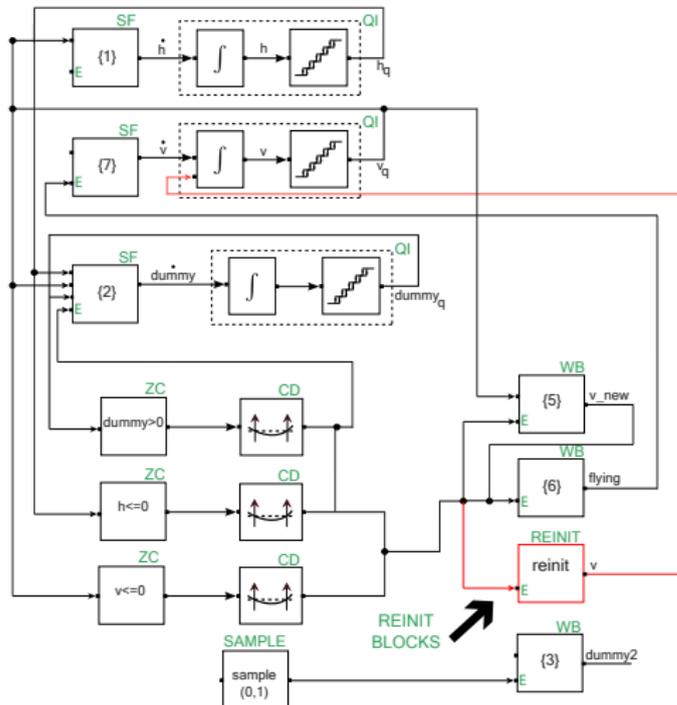
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- ▶ Add one sample block for each sample statement.
- ▶ Connect the sample blocks to the dependent when-clauses.

## Add Reinit Blocks



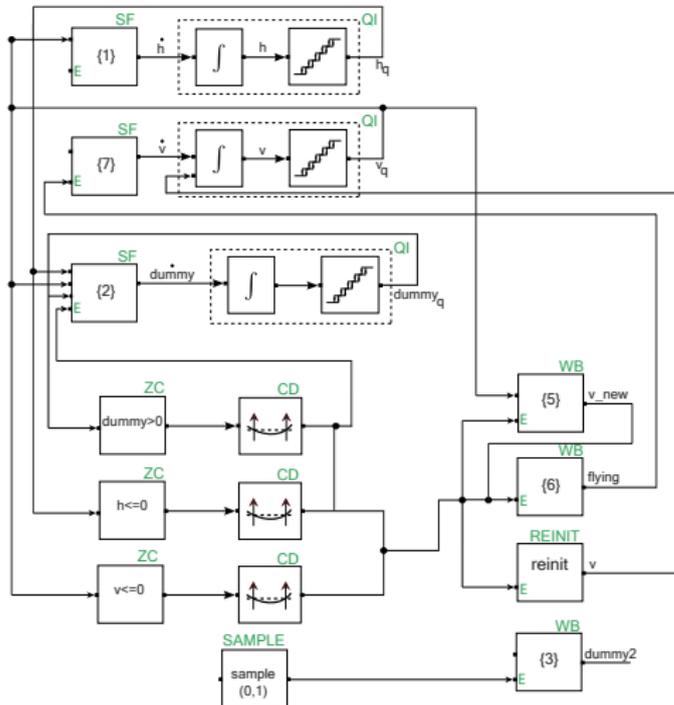
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```

- Add reinit blocks for the reinit statements and connect them to the corresponding integrators.

## Final Structure



```

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- ▶ **Radau Ila** in Dymola v7.4
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  - ▶ A single-step (Runge-Kutta) algorithm is supposed to be more efficient than a multi-step algorithm when dealing with discontinuities (due to step-size control for the latter methods).
- ▶ **Dopri45** in Dymola v7.4
  - ▶ An explicit Runge-Kutta method which could be more efficient when simulating non-stiff systems.

## Run-time Efficiency (Execution Time)

### Problem

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- ▶ But the **relative ordering of the algorithms** is expected to remain the same.

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## Reference Trajectories

- ▶ The default DASSL solver both in Dymola and OpenModelica was used with
  - ▶ a very tight tolerance of  $10^{-12}$  and
  - ▶ requesting  $10^5$  output points.
- ▶ The difference between both reference trajectories was on the order of  $10^{-6}$  therefore we report only the simulation error against the Dymola solution.

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- ▶ Then, the mean absolute error is calculated as:

$$\text{error} = \frac{1}{|\mathbf{t}^{\text{ref}}|} \sum_{i=1}^{|\mathbf{t}^{\text{ref}}|} |y_i^{\text{sim}} - y_i^{\text{ref}}| \quad (6)$$

## Half-Wave Rectifier

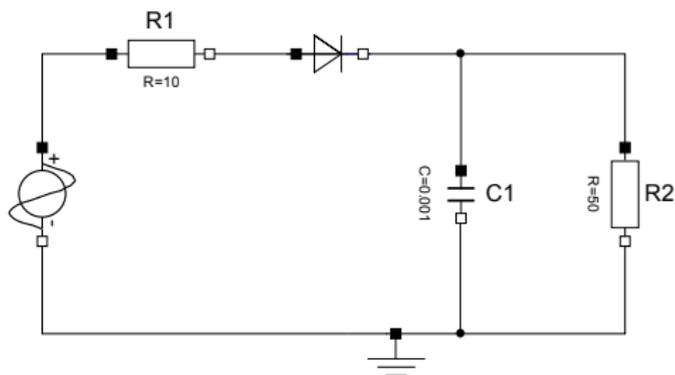
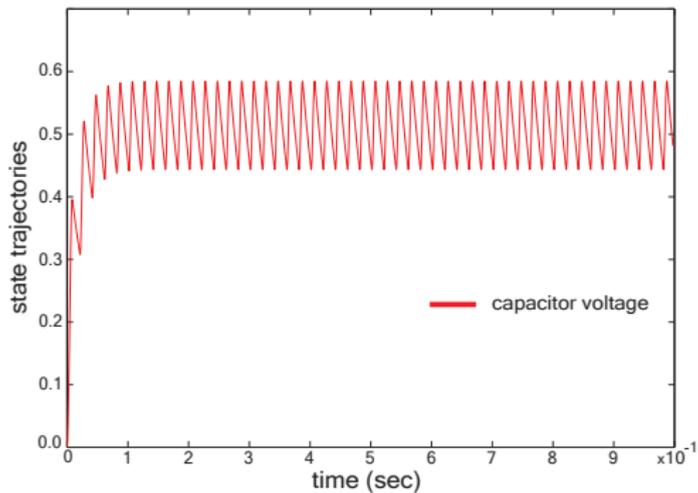


Figure: Graphical representation of the half-wave rectifier in Dymola

# Simulated trajectories for the half-wave rectifier



## Half-Wave Rectifier (Simulated for 1 sec)

			CPU time (sec)	Simulation Error
Dymola	DASSL	$10^{-3}$	0.019	1.45E-03
	DASSL	$10^{-4}$	<i>0.022</i>	<i>2.35E-04</i>
	Radau Ila	$10^{-7}$	0.031	2.20E-06
	Dopri45	$10^{-4}$	0.024	4.65E-05
PowerDEVS	QSS3	$10^{-3}$	<b>0.014</b>	<b>2.59E-04</b>
	QSS3	$10^{-4}$	0.026	2.23E-05
	QSS3	$10^{-5}$	0.041	2.30E-06
	QSS2	$10^{-2}$	0.242	3.02E-03
	QSS2	$10^{-3}$	0.891	3.04E-04
	QSS2	$10^{-4}$	3.063	3.00E-05
OpenModelica	DASSL	$10^{-3}$	0.265	3.80E-03
	DASSL	$10^{-4}$	<i>0.281</i>	<i>5.40E-04</i>

## Switching Power Converter

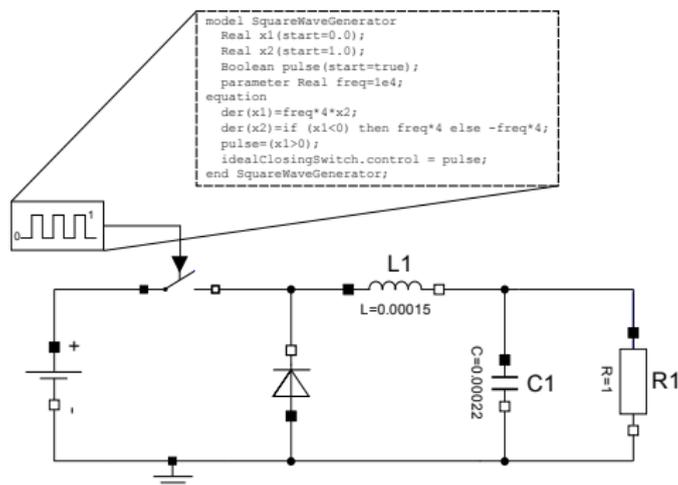
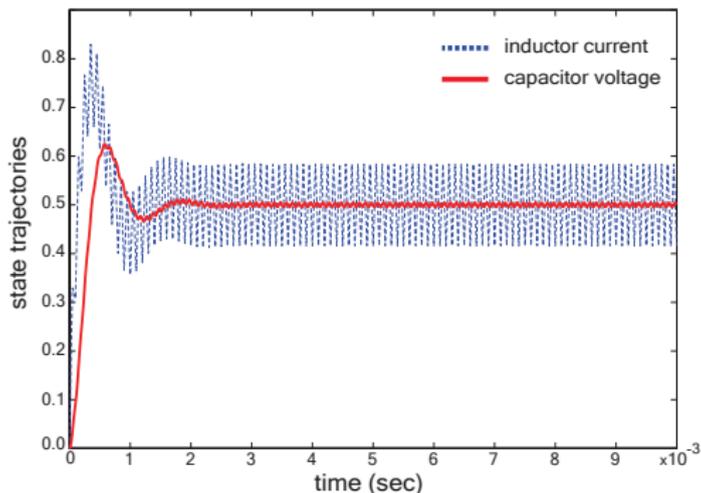


Figure: Graphical representation of the switching power converter in Dymola

# Simulated state trajectories for the switching power converter



## Switching Power Converter (Simulated for 0.01 sec)

			CPU time (sec)	Simulation Error
<b>Dymola</b>	<b>DASSL</b>	$10^{-3}$	0.051	1.82E-04
	<b>DASSL</b>	$10^{-4}$	<i>0.063</i>	<i>7.18E-05</i>
	<b>Radau Ila</b>	$10^{-3}$	0.064	1.11E-07
	<b>Radau Ila</b>	$10^{-4}$	0.062	1.11E-07
	<b>Dopri45</b>	$10^{-3}$	0.049	6.38E-06
	<b>Dopri45</b>	$10^{-4}$	0.047	9.76E-06
<b>PowerDEVS</b>	<b>QSS3</b>	$10^{-3}$	0.049	1.41E-03
	<b>QSS3</b>	$10^{-4}$	<i>0.062</i>	<i>1.68E-05</i>
	<b>QSS3</b>	$10^{-5}$	0.250	8.96E-06
<b>OpenModelica</b>	<b>DASSL</b>	$10^{-3}$	50.496	-
	<b>DASSL</b>	$10^{-4}$	<i>1.035</i>	<i>2.62E-02</i>

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## Conclusions

- ▶ An interface between OpenModelica and PowerDEVS is presented and analyzed.
- ▶ The OMPD interface successfully handles discontinuities allowing the simulation of real-world Modelica models using QSS solvers.
- ▶ Comparing QSS3 and DASSL in OpenModelica, a **20-fold decrease** in the required CPU time was achieved for the example models.
- ▶ Furthermore in our discontinuous examples, QSS3 is as efficient as DASSL in Dymola, in spite of the fact that Dymola offers a much more sophisticated model preprocessing than OMC.

## Future Work

- ▶ Provide support for stiff QSS solvers.
- ▶ Perform more extensive simulations of benchmark problems in order to test the correctness of the interface and the performance of QSS methods.
- ▶ Incorporate QSS solvers in future official OpenModelica releases.
- ▶ Investigate the parallel simulation capabilities of QSS methods.

Questions?