The Ideal of Circuits of a Matrix

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The circuits of an integer $d \times n$ matrix A are the integral vectors of minimal support in the kernel of A. The circuits of A are precisely the edge directions of all polyhedra $P_{\mathbf{b}} := \{\mathbf{x} \in \mathbb{R}^n \,:\, A\mathbf{x} = \mathbf{b}, \mathbf{x} \geq \mathbf{0}\}$ as $\mathbf{b}$ varies, and hence the directions used by the simplex method on these polyhedra. In this talk we examine what happens if one attempts to use the circuits of A to solve integer programs with coefficient matrix A and varying right hand side $\mathbf{b}$. We approach the problem from the algebraic point of view of comparing the toric ideal of A to the ideal generated by the circuits of A. The circuit ideal is contained in the toric ideal and its primary decomposition is a measure of how far it is from the toric ideal. We describe the distance between the two ideals in terms of certain special polytopes and their lattice points.

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